SYMMETRY PROPERTIES OF STRONG INTERACTIONS

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A method is proposed which makes it possible to find all symmetry properties of the strong interactions which are possible at least in principle, provided the most general form of the Lagrangian is given (in the sense of the values of the bare coupling constant and particle masses). The method is applied to the Lagrangian in the Yukawa form and to a compound model with a four-fermion interaction.

1. INTRODUCTION

In recent years many papers have been devoted to the study of the possible symmetry properties of the strong interactions. Many types of symmetry have been proposed, but all of them, with the exceptions of the conservation of the number of baryons, and of strangeness and isotopic invariance, are to some extent in contradiction with experiment. But both the question as to whether some types of symmetry can hold approximately and that of the nature of the approximation involved in such a case are still unclear. This is due to the fact that as a rule it is only by a difficult quantitative analysis that one can distinguish a violation of a symmetry property that is due to its approximate nature from the complete absence of the property. Nevertheless, the question of the symmetries of strong interactions remains as interesting as before, and the search for such properties continues.

Theoretical work devoted to the invariance properties of the strong interactions is most often done in the following way. One writes down a Lagrangian "in the most general form." Its free-particle part contains the bare masses of the particles, and the interaction contains the coupling constants. Both sets of quantities are essentially unknown parameters of the theory. The next step is to equate some of these parameters to each other, and some to zero. The new Lagrangian has additional symmetry properties as compared with the original Lagrangian.

The change from the "general" to the "special" Lagrangian is more natural in some cases than in others, and is usually due to the inventiveness of the author. In the present paper we propose a rather simple and in some sense standard method for this change, which guarantees the obtaining of all "special" Lagrangians which are possible in principle and of the corresponding symmetry properties, as soon as the Lagrangian of most general form has been written down. This method has been used earlier by the writer,[1] and has also been used recently by Behrends and Sirlin.[2]

The method is applied to the Lagrangian written in the Yukawa form. If we assume that the parities of Λ and Σ, and also of N and Ξ, are the same, and that the parities of K and Λ are different (other possibilities are discussed in Sec. 4), then in the most general case the strong-interaction Lagrangian which conserves baryon number, strangeness, and electric charge, and also satisfies the requirement of isotopic invariance, has the well known form

$$L = \{g_1(\overline{N}i\gamma_5\tau N) + g_2(\overline{\Lambda}i\gamma_5\Sigma + \overline{\Sigma}i\gamma_5\Lambda) - ig_3[\overline{\Sigma}i\gamma_5\Sigma] + g_4(\overline{\Xi}i\gamma_5\tau\Xi)\}\boldsymbol{\pi} + \{g_5(\overline{N}i\gamma_5\Lambda)K + g_6(\overline{N}i\gamma_5\tau\Sigma)K + g_7K(\overline{\Lambda}i\gamma_5 i\tau_2\Xi) + g_8K(\overline{\Sigma}i\gamma_5 i\tau_2\boldsymbol{\tau}\Xi) + \text{Herm. adj.}\}, \quad (1)$$

where N, Λ, and so on denote the operators for the corresponding particles in the Heinsenberg representation, the bold-face letters are isotopic vectors, τ_i are the isotopic Pauli matrices, $\pi^{\pm} = (\pi_1 \mp i\pi_2)/2^{1/2}$, $\Sigma^{\pm} = (\Sigma_1 \mp i\Sigma_2)/2^{1/2}$, and g_i are the real, and in general unequal, bare strong-interaction constants.

If there exists in nature a ρ meson which is pseudoscalar in ordinary space and scalar in isotopic space, then we must add to L

$$L' = \{g_9(\overline{N}i\gamma_5 N) + g_{10}(\overline{\Lambda}i\gamma_5\Lambda) + g_{11}(\overline{\Sigma}i\gamma_5\Sigma) + g_{12}(\overline{\Xi}i\gamma_5\Xi)\}\rho. \quad (2)$$

The ρ meson may decay very rapidly, and therefore be practically unobservable, but the addition of the expression (2) to the Lagrangian (1) increases the number of parameters g_j which can be varied, and consequently increases the number of classes of symmetry possible in principle.

As has already been remarked, for certain relations between the g_j, and also between the bare masses, which appear in the free-particle Lagrangian, the theory is invariant under corresponding transformations of the operators for the particles. The problem is to find all possible relations of this kind and the corresponding classes of symmetry of the theory with the Lagrangian (1)—(2).*

The method proposed for the solution of this problem is essentially very simple. As is well known, to every symmetry of the theory there must correspond a certain current j_λ, bilinear in the operators that appear in Eqs. (1) and (2), and having a vanishing divergence ($\partial j_\lambda/\partial x_\lambda = 0$). Therefore the finding of all possible symmetry classes is equivalent to the problem of finding all the currents whose divergences vanish under various assumptions about the values of the constants g_j and masses m_i. Because of the isotopic invariance of the theory these currents must be either scalars, or vectors, or spinors in the isospace. When written in the most general form, each current of this type contains a certain number of arbitrary constants. By calculating the divergence of this current by means of the equations of motion, which are direct consequences of Eqs. (1) and (2), and equating it to zero, one can get a system of homogeneous algebraic equations for these constants, and thus for the g_j and m_i. By finding all the nontrivial solutions of such a system one can solve the problem of constructing currents with vanishing divergence, and consequently also the problem of determining the possible symmetry classes of the theory with the Lagrangian (1)—(2).

Among the solutions obtained in this way will of course be the "obvious" ones, which are valid for arbitrary g_j and m_i. They correspond to conservation of baryon number, strangeness, and charge and isotopic invariance, since these properties of the theory are already contained in the expressions (1) and (2). Strictly speaking all of the "nonobvious" solutions are in contradiction with experiment, because if they hold then, as will be shown below, either certain observed processes must be forbidden, or the masses of different particles must coincide, which again disagrees with experiment. Therefore "nonobvious" symmetry properties can hold only approximately. As has already been noted, the question of the degree of validity of such an approximation is a difficult one, which must be given separate consideration in each concrete case and is not in the scope of this paper. We can only remark at once that the symmetry classes considered earlier by Gell-Mann,[3] Pais,[4] and other authors are among the "nonobvious" solutions.

As has already been noted, the problem of finding the "nonobvious" solutions has been treated in a paper by Behrends and Sirlin.[2] These authors do not introduce the ρ meson into the Lagrangian and confine themselves to solutions that hold either when all the g_i ($i = 1, 2, \ldots, 8$) are different from zero or when $g_5 = g_6 = g_7 = g_8 = 0$, $g_1, g_2, g_3, g_4 \neq 0$. For this reason they do not get the solutions given in Eqs. (19), (20), (30), (31), and (32) and in the Appendix. In the remaining cases our results agree.

In the next section we shall find in explicit form the currents that conserve strangeness and have zero divergence. For each such current there are indicated, at least partially, the symmetry properties following from the equation $dj_\lambda/dx_\lambda = 0$ that are in contradiction with experiment (without counting the ascribing of equal masses to different particles). An analogous study of the properties of currents with parity changes is made in Sec. 3. The results obtained are discussed briefly in Sec. 4. The problem of the connection of the results of Sec. 2 and 3 with the properties of conservation of the vector currents involved in the weak interactions is not considered, since the writer has already devoted a paper to this.[1] The symmetry properties of the strong interactions in the Sakata-Okun' model[5-7] are investigated in Sec. 5.

2. CURRENTS WITH CONSERVATION OF STRANGENESS

If we begin with isotopic scalar currents, in the most general form they can be written as follows (a_i are arbitrary constants):

$$j_\lambda^S = a_1(\overline{N}\gamma_\lambda N) + a_2(\overline{\Lambda}\gamma_\lambda\Lambda) + a_3(\overline{\Sigma}\gamma_\lambda\Sigma) + a_4(\overline{\Xi}\gamma_\lambda\Xi) - a_5(\overline{K}\partial_\lambda K), \tag{3}$$

where the symbol ∂_λ has the meaning

$$(\Phi_2\partial_\lambda\Phi_1) \equiv \Phi_2\frac{\partial\Phi_1}{\partial x_\lambda} - \frac{\partial\Phi_2}{\partial x_\lambda}\Phi_1. \tag{4}$$

A direct calculation of the divergence of the current (3) with the use of Eqs. (1) and (2) leads to the expression

*It must be stated at the start that we are concerned with invariance properties of the strong interactions under continuous transformations of the type of rotations in the isospace; discrete transformations of the type of reflections are not considered.

$$\partial j_\lambda^S / \partial x_\lambda = [(\overline{\Lambda} j\gamma_5\Sigma)\pi - \pi(\overline{\Sigma} i\gamma_5\Lambda)] g_2 (a_2 - a_3)$$
$$+ [(\overline{N} i\gamma_5\Lambda) K - \overline{K}(\overline{\Lambda} i\gamma_5 N)] g_5 (a_1 - a_2 - a_5)$$
$$+ [(\overline{N} i\gamma_5\tau\Sigma) K - \overline{K}(\overline{\Sigma}\tau i\gamma_5 N)] g_6 (a_1 - a_3 - a_5)$$
$$+ [K(\overline{\Lambda} i\tau_2 i\gamma_5\Xi) + (\overline{\Xi} i\tau_2 i\gamma_5\Lambda)\overline{K}] g_7 (a_2 - a_4 - a_5)$$
$$+ [K(\overline{\Sigma} i\tau_2\tau i\gamma_5\Xi) + (\overline{\Xi}\tau i\tau_2 i\gamma_5\Sigma)\overline{K}] g_8 (a_3 - a_4 - a_5). \quad (5)$$

If we require that $\partial j_\lambda^S/\partial x_\lambda = 0$, then in Eq. (5) we must equate to zero the coefficients of all derivatives of Heisenberg operators, since these derivatives are independent. This gives the system of algebraic equations

$$g_2(a_2 - a_3) = 0, \qquad g_5(a_1 - a_2 - a_5) = 0,$$
$$g_6(a_1 - a_3 - a_5) = 0,$$
$$g_7(a_2 - a_4 - a_5) = 0, \qquad g_8(a_3 - a_4 - a_5) = 0. \quad (6)$$

The system (6) is satisfied for arbitrary g_j if

$$a_2 = a_3 = a_1 - a_5 = a_4 - a_5. \quad (7)$$

The relation (7) gives three equations for five unknowns, and consequently has two linearly independent solutions, which we can take to be

$$\begin{array}{llll} 1) & a_1 = 1, & a_5 = 0, & a_2 = a_3 = a_4 = 1, \\ 2) & a_1 = 0, & a_5 = 1; & a_2 = a_3 = a_4/2 = -1. \end{array} \quad (8)$$

Thus for arbitrary values of g_j we see that currents with vanishing divergence are

$$j_\lambda^S = (\overline{N}\gamma_\lambda N) + (\overline{\Lambda}\gamma_\lambda\Lambda) + (\overline{\Sigma}\gamma_\lambda\Sigma) + (\overline{\Xi}\gamma_\lambda\Xi),$$
$$j_\lambda^{S'} = -(\overline{\Lambda}\gamma_\lambda\Lambda) - (\overline{\Sigma}\gamma_\lambda\Sigma) - 2(\overline{\Xi}\gamma_\lambda\Xi) - (\overline{K}\partial_\lambda K), \quad (9)$$

which corresponds to the conservation of baryon number and strangeness, which was obvious from the beginning.

If we return to Eq. (6) and exclude the obvious solutions (8)—to do so it is enough to set $a_1 = a_5 = 0$—then there remains

$$g_2(a_2 - a_3) = g_5 a_2 = g_6 a_3 = g_7(a_2 - a_4) = g_8(a_3 - a_4) = 0. \quad (10)$$

Any solution of Eq. (10) is in contradiction with experiment. In fact, if in Eq. (10) $g_5 = g_6 = 0$ or $g_7 = g_8 = 0$, then either the KN or the KΞ interactions are absent. This means that either the number of particles N or the number of Ξ is conserved, and this does not agree with experiment. The other possibilities in Eq. (10) are

$$\begin{array}{lllll} 1) & a_2 = 0, & a_3 \neq 0, & g_2 = g_6 = 0, & a_4 = g_8 = 0, \\ 2) & a_2 = 0, & a_3 \neq 0, & g_2 = g_6 = 0, & g_7 = a_3 - a_4 = 0, \\ 3) & a_3 = 0, & a_2 \neq 0, & g_2 = g_5 = 0, & a_4 = g_7 = 0, \\ 4) & a_3 = 0, & a_2 \neq 0, & g_2 = g_5 = 0, & g_8 = a_2 - a_4 = 0. \end{array} \quad (11)$$

These four cases correspond to constancy of N_Σ, $N_\Sigma + N_\Xi$, N_Λ; and $N_\Lambda + N_\Xi$ (N_R is the number of particles R). This means that observed reactions of the types $K^- + p \to \Sigma^\pm + \pi^\mp$ or $K^- + p \to \Lambda + \pi^0$ would be forbidden. Consequently, strictly speaking, there can be no isoscalar currents except (9) which have vanishing divergence.

If we now go on to isovector currents, in the most general case we have

$$j_\lambda^V = b_1(\overline{N}\gamma_\lambda\tau N) + b_2(\overline{\Lambda}\gamma_\lambda\Sigma + \overline{\Sigma}\gamma_\lambda\Lambda) - ib_3[\overline{\Sigma}\gamma_\lambda\cdot\Sigma]$$
$$+ b_4(\overline{\Xi}\gamma_\lambda\tau\Xi) - ib_5\left[\pi, \frac{\partial}{\partial x_\lambda}\pi\right] - b_6(\overline{K}\tau\partial_\lambda K)$$
$$- b_7(\rho\partial_\lambda\pi) + b_8(\overline{\Lambda}\gamma_\lambda\Sigma - \overline{\Sigma}\gamma_\lambda\Lambda). \quad (12)$$

As in the case of Eqs. (5) and (6), the equation $\partial j_\lambda^V/\partial x_\lambda = 0$ leads to the system of equations

$$b_7 g_1 = b_7 g_2 = b_7 g_3 = b_7 g_4 = b_7 g_9 = b_7 g_{10} = b_7 g_{11} = b_7 g_{12} = 0,$$
$$b_8 g_2 = b_8 g_5 = b_8 g_6 = b_8 g_7 = b_8 g_8 = b_8(g_{10} - g_{11})$$
$$= b_8(m_\Sigma - m_\Lambda) = 0,$$
$$\begin{array}{ll} (2b_1 + b_5) g_1 = 0, & (b_1 - b_6) g_5 - b_2 g_6 = 0, \\ (2b_4 + b_5) g_4 = 0, & (b_1 - b_6) g_6 - b_2 g_5 = 0, \\ (b_3 + b_5) g_2 + b_2 g_3 = 0, & g_6(b_1 + b_6 - b_3) = 0, \\ (b_3 + b_5) g_3 + b_2 g_2 = 0, & (b_4 - b_6) g_7 - b_2 g_8 = 0, \\ b_2(m_\Sigma - m_\Lambda) = 0, & (b_4 - b_6) g_8 - b_2 g_7 = 0, \\ b_2(g_{10} - g_{11}) = 0, & g_8(b_4 + b_6 - b_3) = 0. \end{array} \quad (13)$$

The system (13) has the obvious solution

$$b_1 = b_3/2 = b_4 = -b_5/2 = b_6,$$
$$b_2 = b_7 = b_8 = 0, \quad (14)$$

since the vanishing of the divergence of the current

$$j_\lambda^V = (\overline{N}\gamma_\lambda\tau N) - 2i[\overline{\Sigma}\gamma_\lambda\cdot\Sigma] + (\overline{\Xi}\gamma_\lambda\tau\Xi)$$
$$+ 2i[\pi\partial\pi/\partial x_\lambda] - (\overline{K}\tau\partial_\lambda K) \quad (15)$$

(instead of $2[\pi\partial\pi/\partial x_\lambda]$ we can of course write $[\pi, \partial_\lambda\pi]$) is equivalent to electric charge conservation and isotopic invariance. Components of the current (15) occur in the Lagrangian of the electromagnetic interaction and in the scheme of weak interactions proposed by Feynman and Gell-Mann.[8]

In order to eliminate from Eq. (13) the obvious solution (14), we can set $b_5 = 0$ in Eq. (13). In the solution of the system that results from this we shall not consider cases in which even any one of the equations

$$g_5 = g_6 = 0, \quad g_2 = g_5 = g_7 = 0, \quad g_2 = g_6 = g_8 = 0,$$
$$g_7 = g_8 = 0, \quad g_1 = g_2 = g_3 = g_4 = 0, \quad (16)$$

holds; these equations mean the unobserved conservation of the numbers of particles N, Λ, Σ, Ξ, or π, respectively. The cases (16) can be considered separately; the results of such a treatment are given in the Appendix (the fact that the cross section for Ξ production is small experimentally[9] might indeed mean that g_7 and g_8 are small; the vanishing of g_5, g_6, g_7, g_8 corresponds approximately to the "very strong" coupling of Gell-Mann[3]). After the possibilities (16) are ruled out the system (13) with $b_5 = 0$ has only three solutions:

$$1)\ g_2 = \varepsilon g_3,\ g_5 = \varepsilon g_6,\ g_7 = \varepsilon g_6,\ \varepsilon = \pm 1;\ g_{10} = g_{11},\quad m_\Lambda = m_\Sigma,\ b_7 = b_8 = b_1 = b_4 = 0,\quad b_3 = b_6 = -\varepsilon b_2, \tag{17}$$

$$j_\lambda^{V_1} = -\varepsilon(\bar\Lambda\gamma_\lambda\Sigma + \bar\Sigma\gamma_\lambda\Lambda) - i[\bar\Sigma\gamma_\lambda\cdot\Sigma] - (\bar K\boldsymbol{\tau}\partial_\lambda K).$$

$$2)\ g_1 = g_4 = 0,\quad g_2 = -\varepsilon g_3,\quad g_5 = \varepsilon g_6,\quad g_7 = \varepsilon g_8,\quad \varepsilon = \pm 1,\quad g_{10} = g_{11},\quad m_\Lambda = m_\Sigma;$$

$$b_7 = b_8 = b_6 = 0,\qquad b_1 = \varepsilon b_2 = b_3 = b_4, \tag{18}$$

i.e., with $b_1 = 1$

$$j_\lambda^{V_2} = (\bar N\gamma_\lambda\boldsymbol{\tau}N) + \varepsilon(\bar\Lambda\gamma_\lambda\Sigma + \bar\Sigma\gamma_\lambda\Lambda) - i[\bar\Sigma\gamma_\lambda\cdot\Sigma] + (\bar\Xi\gamma_\lambda\boldsymbol{\tau}\Xi).$$

$$3)\ g_1 = g_4 = g_6 = g_8 = 0,\ b_7 = b_8 = b_2 = b_3 = 0,\quad b_1 = b_4 = b_6, \tag{19}$$

i.e., with $b_1 = 1$

$$j_\lambda^{V_3} = (\bar N\gamma_\lambda\boldsymbol{\tau}N) + (\bar\Xi\gamma_\lambda\boldsymbol{\tau}\Xi) - (\bar K\boldsymbol{\tau}\partial_\lambda K).$$

The appearance of sign factors ϵ in Eqs. (17) and (18) and in subsequent formulas is due to the possibility of arbitrary changes of sign of the operators for the particles and of the strong-interaction constants. The question of the physical meaning of the relative signs of these constants has been discussed by Kobzarev and Okun'.[10]

The current $j_\lambda^{V_1}$ and the conditions on g_j in Eq. (17) are equivalent to the symmetry of Pais,[4] and the author has discussed them earlier[1] in more detail [in[1] the signs of all the baryon currents must be changed, and also the sign of ϵ' in Eq. (17)]. Once again, strictly speaking, the conditions for the constants g_j in Eqs. (18) and (19) are in contradiction with experiment. In fact, the vanishing of the divergences of the currents $j_\lambda^{V_2}$ and $j_\lambda^{V_3}$ means the conservation of the respective operators

$$\hat N_p - \hat N_n - \hat N_{\Sigma^+} + \varepsilon(\hat N_{Y^0} + \hat N_{Z^0}) - \hat N_{\Sigma^-} + \hat N_{\Xi^0} - \hat N_{\Xi^-},\qquad \hat N_p - \hat N_n + \hat N_{\Xi^0} - \hat N_{\Xi^-} + \hat N_{K^+} - \hat N_{K^0}, \tag{20}$$

where the particle-number operators $\hat N_\psi$ and $\hat N_\Phi$ for the spinor ψ and the boson Φ are defined by

$$\hat N_\psi = \int \psi^+(x)\psi(x)\,d^3x,\qquad \hat N_\Phi = -\int \Phi^+(x)\partial_4\Phi(x)\,d^3x, \tag{21}$$

and $Y^0 = (\Lambda^0 - \Sigma^0)/2^{1/2}$, $Z^0 = (\Lambda^0 + \Sigma^0)/2^{1/2}$. The conservation of either of the operators (20) forbids, for example, π-meson charge transfer ($\pi^- + p \to \pi^0 + n$) and a number of other observed reactions.

3. CURRENTS WITH CHANGE OF STRANGENESS

An isoscalar current with change of strangeness by two units can consist of only two terms:

$$j_\lambda^B = c_1(\bar N\gamma_\lambda\Xi) + \tfrac{1}{2}c_2(\bar K\,i\tau_2\partial_\lambda\bar K) = c_1(\bar N\gamma_\lambda\Xi) + c_2(\bar K i\tau_2\,\partial\bar K/\partial x_\lambda). \tag{22}$$

The divergence condition $\partial j_\lambda^B/\partial x_\lambda = 0$ holds for

$$c_1(m_N - m_\Xi) = c_2 m_K^2 = 0,\quad c_1(g_1 - g_4) = c_1(g_9 - g_{12}) = 0,\quad c_1g_5 - c_2g_7 = 0,\quad c_1g_6 - c_2g_8 = 0,\quad c_1g_7 - c_2g_5 = 0,\quad c_1g_8 - c_2g_6 = 0. \tag{23}$$

The system (23) has two solutions:

$$\text{a)}\ g_5 = \varepsilon g_7,\quad g_6 = \varepsilon g_8,\quad \varepsilon = \pm 1;\quad g_1 = g_4,\quad g_9 = g_{12},\quad m_N = m_\Xi,\quad m_K = 0;\quad c_1 = \varepsilon c_2,$$

$$\text{b)}\ g_5 = g_6 = g_7 = g_8 = 0,\quad g_1 = g_4,\quad g_9 = g_{12},\quad m_N = m_\Xi;\quad c_1 \neq 0, \tag{24}$$

the first of which has been considered earlier by Feinberg and Behrends.[11] The second solution requires the absence of K-meson interactions.

The only isovector current with change of strangeness by 2, $(\bar N\gamma_\lambda\boldsymbol{\tau}\Xi)[(\bar K i\tau_2\boldsymbol{\tau}\partial_\lambda\bar K) \equiv 0]$, has zero divergence only under the senseless condition that N and Ξ have no strong interactions.

Currents with strangeness change by unity can have only half-integral isotopic spins ($1/2$ or $3/2$), since only expressions of the form $(\bar N\gamma_\lambda\Lambda)$ or $(\bar N\gamma_\lambda\Sigma)$ can be constructed from the quantities of the theory.

Let us first consider the current with isospin $1/2$. In the most general case it is of the form

$$j_\lambda^\psi = d_1(\bar N\gamma_\lambda\Lambda) + d_2(\bar N\gamma_\lambda\boldsymbol{\tau}\Sigma) + d_3(\bar\Lambda\gamma_\lambda\,i\tau_2\,\Xi) + d_4(\bar\Sigma\gamma_\lambda\,i\tau_2\,\boldsymbol{\tau}\Xi) + d_5(\bar K\boldsymbol{\tau}\partial_\lambda\boldsymbol{\pi}) + d_6(\bar K\partial_\lambda\rho), \tag{25}$$

or, in terms of components

$$(j_\lambda^\psi)_{1/2} = d_1(\bar p\gamma_\lambda\Lambda) + d_2[(\bar p\gamma_\lambda\Sigma^0) + \sqrt2(\bar n\gamma_\lambda\Sigma^-)] + d_3(\bar\Lambda\gamma_\lambda\Xi^-) + d_4[-(\bar\Sigma^0\gamma_\lambda\Xi^-) + \sqrt2(\bar\Sigma^+\gamma_\lambda\Xi^0)] + d_5[(\bar K^+\partial_\lambda\pi^0) + \sqrt2(\bar K^0\partial_\lambda\pi^-)] + d_6(\bar K^+\partial_\lambda\rho), \tag{26}$$

$$(j_\lambda^\psi)_{-1/2} = d_1(\bar n\gamma_\lambda\Lambda) + d_2[-(\bar n\gamma_\lambda\Sigma^0) + \sqrt2(\bar p\gamma_\lambda\Sigma^+)] - d_3(\bar\Lambda\gamma_\lambda\Xi^0) + d_4[-(\bar\Sigma^0\gamma_\lambda\Xi^0) - \sqrt2(\bar\Sigma^-\gamma_\lambda\Xi^-)] + d_5[-(\bar K^0\partial_\lambda\pi^0) + \sqrt2(\bar K^+\partial_\lambda\pi^+)] + d_6(\bar K^0\partial_\lambda\rho). \tag{27}$$

Just as in the preceding cases, the requirement $\partial j_\lambda^\psi/\partial x_\lambda = 0$ leads to a system of equations

$$\begin{array}{ll}
-d_1g_1 + d_2g_2 + d_5g_5 = 0, & -d_3g_4 + d_4g_2 + d_5g_7 = 0,\\
d_1g_2 - d_2g_1 + d_5g_6 = 0, & d_3g_2 - d_4g_4 + d_5g_8 = 0,\\
d_1g_5 - d_2g_6 - 2d_5g_1 = 0, & d_3g_7 - d_4g_8 - 2d_5g_4 = 0,\\
d_2(g_1 - g_3) + d_5g_6 = 0, & d_4(g_4 - g_3) + d_5g_8 = 0,\\
-d_2g_6 - d_4g_8 + d_5g_3 = 0, & d_1(g_{10} - g_9) + d_6g_5 = 0,\\
d_2g_5 + d_3g_8 + d_5g_2 = 0, & d_2(g_{11} - g_9) + d_6g_6 = 0,\\
d_1g_6 + d_4g_7 + d_5g_2 = 0, & d_3(g_{12} - g_{10}) + d_6g_7 = 0,\\
d_1g_7 + d_2g_8 + 2d_4g_6 = 0, & d_4(g_{12} - g_{11}) + d_6g_8 = 0,\\
2d_2g_8 + d_3g_5 + d_4g_6 = 0, & 2d_2g_6 + d_5g_1 - d_6g_9 = 0,\\
-d_1g_5 + d_3g_7 - d_6g_{10} = 0, & -d_2g_6 + d_4g_8 - d_6g_{11} = 0,
\end{array}$$

$$d_5(m_\pi - m_K) = d_6(m_\rho - m_K) = 0,\ 2d_4g_8 + d_5g_4 - d_6g_{12} = 0,\quad d_1(m_N - m_\Lambda) = d_2(m_N - m_\Sigma) = d_3(m_\Lambda - m_\Xi) = d_4(m_\Sigma - m_\Xi) = 0. \tag{28}$$

The solution of the system (28) is rather cumbersome, although elementary. Therefore we give here only the final result. If we confine ourselves to the condition that Eqs. (16) do not hold (otherwise there are too many solutions), the divergence of the current j_λ^ψ is zero in only four cases:

1) $g_1 = -\varepsilon g_2 = -g_3 = g_4 = \varepsilon'' g_5 = \varepsilon\varepsilon'' g_6 = \varepsilon'\varepsilon'' g_7 = \varepsilon\varepsilon'\varepsilon'' g_8, \quad \varepsilon, \varepsilon', \varepsilon'' = \pm 1,$

$g_9 = g_{10} = g_{11} = g_{12}, \quad m_N = m_\Lambda = m_\Sigma = m_\Xi, \quad m_\pi = m_K,$

$$d_6 = 0, \quad d_1 = -3\varepsilon d_2 = \varepsilon' d_3 = -3\varepsilon\varepsilon' d_4 = \tfrac{3}{2}\varepsilon'' d_5, \qquad (29)$$

i.e., with $d_1 = 3$

$$j_\lambda^{\psi_1} = 3(\overline{N}\gamma_\lambda\Lambda) - \varepsilon(\overline{N}\boldsymbol{\tau}\gamma_\lambda\boldsymbol{\Sigma}) + 3\varepsilon'(\overline{\Lambda}\gamma_\lambda i\tau_2\Xi) - \varepsilon\varepsilon'(\overline{\boldsymbol{\Sigma}}\gamma_\lambda i\tau_2\boldsymbol{\tau}\Xi) + 2\varepsilon''(\overline{K}\boldsymbol{\tau}\partial_\lambda\boldsymbol{\pi}).$$

2) $g_1 = \varepsilon g_2 = g_3 = g_4, \quad g_5 = \varepsilon g_6 = -\varepsilon' g_7 = -\varepsilon\varepsilon' g_8 = \varepsilon'' g_9 = -\varepsilon'' g_{10} = -\varepsilon'' g_{11} = \varepsilon'' g_{12},$

$\varepsilon, \varepsilon', \varepsilon'' = \pm 1, \quad m_N = m_\Lambda = m_\Sigma = m_\Xi, \quad m_\rho = m_K,$

$$d_5 = 0, \quad d_1 = \varepsilon d_2 = \varepsilon' d_3 = \varepsilon\varepsilon' d_4 = \varepsilon'' d_6/2, \qquad (30)$$

i.e., with $d_1 = 1$

$$j_\lambda^{\psi_2} = (\overline{N}\gamma_\lambda\Lambda) + \varepsilon(\overline{N}\boldsymbol{\tau}\gamma_\lambda\boldsymbol{\Sigma}) + \varepsilon'(\overline{\Lambda}\gamma_\lambda i\tau_2\Xi) + \varepsilon\varepsilon'(\overline{\boldsymbol{\Sigma}}\gamma_\lambda i\tau_2\boldsymbol{\tau}\Xi) + 2\varepsilon''(\overline{K}\partial_\lambda\rho).$$

3) $g_2 = \varepsilon(g_1 - g_4)/\sqrt{3}, \quad g_3 = g_1 + g_4,$

$g_5 = \varepsilon''(2g_1 + g_4)/\sqrt{3}, \quad g_6 = \varepsilon\varepsilon'' g_4,$

$g_7 = \varepsilon'\varepsilon''(g_1 + 2g_4)/\sqrt{3}, \quad g_8 = -\varepsilon\varepsilon'\varepsilon'' g_1,$

$g_9 = \varepsilon'''(g_1 + 2g_4)/\sqrt{3}, \quad g_{10} = -\varepsilon'''(g_1 - g_4)/\sqrt{3},$

$g_{11} = \varepsilon'''(g_1 - g_4)/\sqrt{3},$

$g_{12} = -\varepsilon'''(2g_1 + g_4)/\sqrt{3}, \quad \varepsilon, \varepsilon', \varepsilon'', \varepsilon''' = \pm 1;$

$m_N = m_\Lambda = m_\Sigma = m_\Xi, \quad m_\rho = m_\pi = m_K,$

$$d_1 = \sqrt{3}\varepsilon d_2 = \varepsilon' d_3 = -\sqrt{3}\varepsilon\varepsilon' d_4 = \sqrt{3}\varepsilon'' d_5 = \varepsilon''\varepsilon''' d_6, \qquad (31)$$

i.e., with $d_1 = 3^{1/2}$

$$j_\lambda^{\psi_3} = \sqrt{3}(\overline{N}\gamma_\lambda\Lambda) + \varepsilon(\overline{N}\gamma_\lambda\boldsymbol{\tau\Sigma}) + \sqrt{3}\varepsilon'(\overline{\Lambda}\gamma_\lambda i\tau_2\Xi) - \varepsilon\varepsilon'(\overline{\boldsymbol{\Sigma}}\gamma_\lambda i\tau_2\boldsymbol{\tau}\Xi) + \varepsilon''(\overline{K}\boldsymbol{\tau}\partial_\lambda\boldsymbol{\pi}) + \sqrt{3}\varepsilon\varepsilon'''(\overline{K}\partial_\lambda\rho).$$

4) $g_1 = g_3 = g_4 = g_6 = g_8 = 0,$

$g_2 = -\varepsilon g_5 = -\varepsilon' g_7, \quad \varepsilon, \varepsilon' = \pm 1,$

$g_9 = g_{11} = g_{12}, \quad m_N = m_\Sigma = m_\Xi, \quad m_\pi = m_K,$

$$d_1 = d_3 = d_6 = 0, \quad d_5 = \varepsilon d_2 = \varepsilon' d_4, \qquad (32)$$

i.e., with $d_5 = 1$

$$j_\lambda^{\psi_4} = \varepsilon(\overline{N}\boldsymbol{\tau}\gamma_\lambda\boldsymbol{\Sigma}) + \varepsilon'(\overline{\boldsymbol{\Sigma}}\gamma_\lambda i\tau_2\boldsymbol{\tau}\Xi) + (\overline{K}\boldsymbol{\tau}\partial_\lambda\boldsymbol{\pi}).$$

The last condition on the g_i is a special case of Eq. (19), and consequently, strictly speaking, it is inadmissible, because the vanishing of the divergence of the current $j_\lambda^{V_3}$ is in contradiction with experiment.

In the most general case the current j_λ^χ with the isospin $^3/_2$ is of the form

$$(j_\lambda^\chi)_{3/2} = -h_1(\overline{p}\gamma_\lambda\Sigma^-) - h_2(\overline{\Sigma}^+\gamma_\lambda\Xi^-) - h_3(\overline{K}^+\partial_\lambda\pi^-),$$

$$(j_\lambda^\chi)_{1/2} = \{h_1[\sqrt{2}(\overline{p}\gamma_\lambda\Sigma^0) - (\overline{n}\gamma_\lambda\Sigma^-)] + h_2[\sqrt{2}(\overline{\Sigma}^0\gamma_\lambda\Xi^-) + (\overline{\Sigma}^+\gamma_\lambda\Xi^0)] + h_3[\sqrt{2}(\overline{K}^+\partial_\lambda\pi^0) + (\overline{K}^0\partial_\lambda\pi^-)]\}/\sqrt{3},$$

$$(j_\lambda^\chi)_{-1/2} = \{h_1[\sqrt{2}(\overline{n}\gamma_\lambda\Sigma^0) + (\overline{p}\gamma_\lambda\Sigma^+)] + h_2[-\sqrt{2}(\overline{\Sigma}^0\gamma_\lambda\Xi^0) + (\overline{\Sigma}^-\gamma_\lambda\Xi^-)] + h_3[\sqrt{2}(\overline{K}^0\partial_\lambda\pi^0) + (\overline{K}^+\partial_\lambda\pi^+)]\}/\sqrt{3},$$

$$(j_\lambda^\chi)_{-3/2} = h_1(\overline{n}\gamma_\lambda\Sigma^+) - h_2(\overline{\Sigma}^-\gamma_\lambda\Xi^0) + h_3(\overline{K}^0\partial_\lambda\pi^+). \qquad (33)$$

Instead of j_λ^χ it is convenient to consider the current j_λ^φ that transforms in the isotopic space according to the reducible representation $1 \times {}^1/_2$ and has the property that if we break it up according to the irreducible representations the component with $T = {}^3/_2$ is equal to j_λ^χ and that with $T = {}^1/_2$ is zero:

$$j_\lambda^\varphi = \frac{1}{3}\{h_1[2(\overline{N}\gamma_\lambda\boldsymbol{\Sigma}) - i[\overline{N}\boldsymbol{\tau}\gamma_\lambda\cdot\boldsymbol{\Sigma}]] + h_2[2(\overline{\boldsymbol{\Sigma}}\gamma_\lambda i\tau_2\Xi) - i[\overline{\boldsymbol{\Sigma}}\gamma_\lambda\cdot i\tau_2\boldsymbol{\tau}\Xi]] + h_3[\sqrt{2}(\overline{K}\partial_\lambda\boldsymbol{\pi}) - i[\overline{K}\boldsymbol{\tau}\cdot\partial_\lambda\boldsymbol{\pi}]]\}. \qquad (34)$$

If $\partial j_\lambda^\chi/\partial x_\lambda = 0$, then we also have $\partial j_\lambda^\varphi/\partial x_\lambda = 0$, and conversely. The latter equation leads to the system of equations

$$h_1 g_6 = -h_2 g_8 = -h_3 g_3 = h_3 g_4, \quad h_1 g_8 = -h_2 g_6,$$
$$h_2 g_3 = -h_2 g_4 = h_3 g_8, \quad h_1 g_1 = h_3 g_6 = -h_1 g_3, \quad h_1 g_6 = h_3 g_1,$$
$$h_2 g_2 = h_3 g_7, \quad h_2 g_7 = h_3 g_2, \quad h_1 g_2 = -h_3 g_5,$$
$$h_1(g_{11} - g_9) = h_2(g_{12} - h_{11}) = 0, \quad h_1(m_N - m_\Sigma) = h_2(m_\Sigma - m_\Xi) = 0, \quad h_3(m_\pi - m_K) = 0. \qquad (35)$$

It is easy to see that there is only one solution of the system (35) that is not forbidden by the conditions (16):

$$g_1 = -g_3 = g_4 = \tilde{\varepsilon} g_6 = \tilde{\varepsilon}\varepsilon' g_8, \quad g_2 = -\tilde{\varepsilon} g_5 = -\varepsilon'\tilde{\varepsilon} g_7,$$

$$\tilde{\varepsilon}_1\varepsilon' = \pm 1, \quad g_9 = g_{11} = g_{12}, \quad m_N = m_\Sigma = m_\Xi, \quad m_\pi = m_K,$$

$$h_1 = -\varepsilon' h_2 = \tilde{\varepsilon} h_3. \qquad (36)$$

The vanishing of the divergence of the current of Eq. (36) is compatible only with conservation of the $j_\lambda^{\psi_1}$ of Eq. (29), because only in this case (and with $\tilde{\epsilon} = \epsilon\epsilon'$, $g_2 = \epsilon g_1$, $g_{10} = g_9$, $m_\Lambda = m_N$) do the conditions for the coupling constants and masses agree in the two cases. Precisely this case was pointed out in an earlier paper,[1] where the currents with isospins $^1/_2$ and $^3/_2$ were considered together.

4. DISCUSSION OF RESULTS

In all of the arguments given above it has been assumed that the parities of Λ and Σ, and also those of N and Ξ, are the same ($P_{\Lambda\Sigma} = P_{N\Xi} = 1$), and that K and Λ have opposite parities ($P_{K\Lambda} = -1$). It is not hard to see that violation of these assumptions does not lead to the appearance of any new conserved currents and that the "obvious" currents j_λ^S, $j_\lambda^{S_1}$, j_λ^V are conserved for any relative parities of the particles. As for the current $j_\lambda^{V_1}$ of Eq. (17), for $P_{\Lambda\Sigma} = -1$ it contains $(\bar{\Lambda}\gamma_\lambda\gamma_5\Sigma) + (\bar{\Sigma}\gamma_\lambda\gamma_5\Lambda)$ instead of $(\bar{\Lambda}\gamma_\lambda\Sigma) + (\bar{\Sigma}\gamma_\lambda\Lambda)$, and owing to this the condition $m_\Sigma - m_\Lambda = 0$ is replaced by $m_\Sigma + m_\Lambda = 0$, which is not satisfied in any approximation. Consequently, for the current $j_\lambda^{V_1}$ to be conserved we must have $P_{\Lambda\Sigma} = 1$; $P_{K\Lambda}$ and $P_{N\Xi}$ can be either $+1$ or -1.

By similar arguments, for the conservation of $j_\lambda^{\psi_1}$, j_λ^{φ}, $j_\lambda^{\psi_3}$ we must have $P_{\Lambda\Sigma} = P_{N\Xi} = -P_{K\Lambda} = 1$ (in the last case, we must also have $P_{N\rho} = -1$). Conservation of $j_\lambda^{\psi_2}$ requires $P_{\Lambda\Sigma} = P_{N\Xi} = 1$; $P_{\Lambda K} = P_{N\rho} = \pm 1$. The solution (24) for the current j_λ^B holds for $P_{N\Xi} = 1$.

It can be seen from the formulas of Sec. 3 that the conditions for conservation of currents with change of strangeness always include equality of masses of different particles, in contradiction with experiment (it is not hard to see that owing to the symmetry of the strong interactions in such a case not only the bare masses, but also the observed masses must be equal). There are also contradictions with experiment in certain selection rules, which are consequences of symmetry properties. In particular, this is so for all cases in which strangeness is conserved, for example, the case of Eq. (16) and, as has been noted above, of the currents (18) and (19). As Pais has shown,[4] the symmetry condition (17), which is included, in particular, in Eqs. (29) and (30), also leads to making observed reactions forbidden.

It is interesting to note that there is no such forbiddenness in the cases of Eqs. (31) and (36). Instead, one can derive a number of relations between the amplitudes for various processes. In the case of Eq. (36), for example, which allows the absence of the ρ meson, Behrends and Sirlin[2] have established a large number of relations between the amplitudes for reactions of the type meson + nucleon → meson + baryon. Namely, they have found the following equations:

$$\begin{aligned}
\Omega_1 &= \langle p\pi^+ | p\pi^+\rangle = \langle n\pi^- | n\pi^-\rangle = \langle pK^0 | pK^0\rangle = \langle nK^+ | nK^+\rangle,\\
\Omega_2 &= \langle \Sigma^+ K^+ | p\pi^+\rangle = \langle \Sigma^- K^0 | n\pi^-\rangle = -\langle nK^+ | pK^0\rangle = -\langle pK^0 | nK^+\rangle,\\
\Omega_3 &= \langle p\pi^- | p\pi^-\rangle = \langle n\pi^+ | n\pi^+\rangle = \langle p\bar{K}^0 | p\bar{K}^0\rangle = \langle nK^- | nK^-\rangle,\\
\Omega_4 &= \langle n\pi^0 | p\pi^-\rangle = -\langle p\pi^0 | n\pi^+\rangle = \langle \Sigma^+\pi^0 | p\bar{K}^0\rangle = -\langle \Sigma^-\pi^0 | nK^-\rangle,\\
\Omega_5 &= \langle \Sigma^0 K^0 | p\pi^-\rangle = -\langle \Sigma^0 K^+ | n\pi^+\rangle = -\langle \Sigma^0\pi^+ | p\bar{K}^0\rangle = \langle \Sigma^0\pi^- | nK^-\rangle,\\
\Omega_6 &= \langle \Sigma^- K^+ | p\pi^-\rangle = \langle \Sigma^+ K^0 | n\pi^+\rangle = \langle \Xi^0 K^+ | p\bar{K}^0\rangle = -\langle \Xi^- K^0 | nK^-\rangle,\\
\Omega_7 &= \langle pK^+ | pK^+\rangle = \langle nK^0 | nK^0\rangle, \quad \Omega_8 = \langle pK^- | pK^-\rangle = \langle n\bar{K}^0 | n\bar{K}^0\rangle,\\
\Omega_9 &= \langle \Sigma^+\pi^- | pK^-\rangle = \langle \Sigma^-\pi^+ | n\bar{K}^0\rangle = -\langle n\bar{K} | pK^-\rangle = -\langle pK^- | n\bar{K}^0\rangle,\\
\Omega_{10} &= \langle \Sigma^-\pi^+ | pK^-\rangle = \langle \Sigma^+\pi^- | n\bar{K}^0\rangle = \langle \Xi^0 K^0 | pK^-\rangle = -\langle \Xi^- K^+ | n\bar{K}^0\rangle,\\
\Omega_{11} &= \langle \Sigma^0\pi^0 | pK^-\rangle = \langle \Sigma^0\pi^0 | n\bar{K}^0\rangle,\\
\Omega_{12} &= \langle \Lambda K^0 | p\pi^-\rangle = \langle \Lambda K^+ | n\pi^+\rangle = -\langle \Lambda\pi^+ | p\bar{K}^0\rangle = -\langle \Lambda\pi^- | nK^-\rangle,\\
\Omega_{13} &= \langle \Lambda\pi^0 | pK^-\rangle = -\langle \Lambda\pi^0 | n\bar{K}^0\rangle,\\
\Omega_{14} &= \langle \Xi^- K^+ | pK^-\rangle = -\langle \Xi^0 K^0 | n\bar{K}^0\rangle
\end{aligned} \tag{37}$$

(Behrends and Sirlin did not have Ω_{14}, and they had different signs for the third and fourth amplitudes in Ω_2 and the second amplitude in Ω_{13}), and established five additional relations between the Ω_i. It is, however, not hard to find four more relations of this kind, so that only five of the amplitudes in Eq. (37) are independent. If we regard Ω_1, Ω_3, Ω_7, Ω_8, and Ω_{12} as independent, then

$$\begin{gathered}
\Omega_2 = \Omega_1 - \Omega_7, \quad \Omega_4 = \Omega_5 = (\Omega_1 - \Omega_3)/\sqrt{2}, \quad \Omega_6 = \Omega_3 - \Omega_7,\\
\Omega_9 = \Omega_3 - \Omega_8, \quad \Omega_{10} = \Omega_1 - \Omega_8, \quad \Omega_{11} = (\Omega_1 + \Omega_3)/2 - \Omega_8,\\
\Omega_{14} = \Omega_7 + \Omega_8 - \Omega_1 - \Omega_3, \quad \Omega_{13} = -\Omega_{12}/\sqrt{2}.
\end{gathered} \tag{38}$$

All of these relations are true if in Eq. (36) $\epsilon' = \tilde{\epsilon} = 1$ (or if in the formula IV of Behrends and Sirlin's paper[2] $\eta_1 = \eta_2 = 1$). But as before they are correct if in the definitions of the Ω_i we replace Σ by $\tilde{\epsilon}\Sigma$ and Ξ by $\epsilon'\Xi$.

Analogous equations can also be obtained in the case of Eq. (31). It may be remarked that in a recent preprint by Gell-Mann[12] the symmetry of this same case (31) is analyzed with the additional condition $g_1 = g_4$.

In the low-energy region it is hard to give relations of the type of Eqs. (37) and (38) any physical meaning, if only for the reason that because of the difference of the masses it is not clear even at what energies one is to compare the amplitudes of the various processes in these equations. In the high-energy case, on the other hand, when the mass difference is not so important, it would be interesting to get comparisons with experiment for relations of this type, and of course also for the selection rules of Pais.

5. THE MODEL OF SAKATA AND OKUN'

If we assume that all baryons, π mesons, and K mesons consist of p, n, Λ, $\bar{p}$, $\bar{n}$, and $\bar{\Lambda}$,[5,6] then in this model the most general four-fermion (strong) interaction between the basic particles is of the form

$$L = \frac{1}{2}\sum_j f_j^{(1)}(\overline{N}O_jN_j)(\overline{N}O_jN_j) + \frac{1}{2}\sum_j f_j^{(2)}(\overline{\Lambda}O_j\Lambda)(\overline{\Lambda}O_j\Lambda) + \sum_j f_j^{(3)}(\overline{N}O_jNO_j)(\overline{\Lambda}\Lambda), \quad (39)$$

where only the antisymmetric interaction types contribute to the first two sums. Arguments like those that have been given show that for the Lagrangian (39), with arbitrary values of the $f_j^{(i)}$, currents with zero divergence are

$$j_\lambda^S = (\overline{N}\gamma_\lambda N) + (\overline{\Lambda}\gamma_\lambda\Lambda), \quad j_\lambda^{S_1} = -(\overline{\Lambda}\gamma_\lambda\Lambda), \quad (40)$$

$$\mathbf{j}_\lambda^V = (\overline{N}\gamma_\lambda\boldsymbol{\tau}N).$$

Moreover, as has already been pointed out by Okun',[7] in this model it is not possible to construct either a nonconserved isovector current or a current with vanishing divergence that does not have an electromagnetic analog. In the language of Sec. 2, the Sakata model forbids isovector currents which are not the same as the $\mathbf{j}_\lambda^V$ of Eq. (15).

As for the only isospinor current,

$$j_\lambda^\psi = (\overline{N}\gamma_\lambda\Lambda), \quad (41)$$

we have

$$\frac{\partial j_\lambda^\psi}{\partial x_\lambda} = (m_N - m_\Lambda)(\overline{N}\Lambda) - \sum_j f_j^{(1)}(\overline{N}O_j\Lambda)(\overline{N}O_jN) + \sum_j f_j^{(2)}(\overline{N}O_j\Lambda)(\overline{\Lambda}O_j\Lambda) + \sum_j f_j^{(3)}[(\overline{N}O_j\Lambda)(\overline{N}O_jN) - (\overline{N}O_j\Lambda)(\overline{\Lambda}O_j\Lambda)], \quad (42)$$

and the condition $\partial j_\lambda^\psi/\partial x_\lambda = 0$ is satisfied only in the case in which $f_j^{(3)} = 0$ for the symmetric types, and for the antisymmetric

$$f_j^{(1)} = f_j^{(2)} = f_j^{(3)}, \qquad m_N = m_\Lambda. \quad (43)$$

The condition (43) means absolute identity of N and Λ, and strictly speaking is incorrect.[13] Analogs of Eq. (43) are found in Eqs. (29) and (30), where there is complete degeneracy of the baryons.

APPENDIX

THE ISOVECTOR CURRENTS (THE CASE OF EQ. (16))

The solution of (13) with $b_5 = 0$ has meaning only for $b_7 = 0$. Otherwise $g_9 = g_{10} = g_{11} = g_{12}$, i.e., the ρ mesons do not interact with baryons at all, and this makes it altogether senseless to introduce the term with $b_7 \neq 0$. For the same reason we must suppose that $b_1 = 0$ for $g_1 = g_5 = g_6 = 0$, $b_3 = 0$ for $g_2 = g_3 = g_6 = g_8 = 0$, $b_4 = 0$ for $g_4 = g_7 = g_8 = 0$, $b_5 = 0$ for $g_1 = g_2 = g_3 = g_4 = 0$, and $b_6 = 0$ for $g_5 = g_6 = g_7 = g_8 = 0$. This also applies to the same extent to the currents (14) and (17)—(19), where all such b_i must be stricken out, providing only (in the last three cases) that the vanishing of the corresponding constants g_j is not in contradiction with the conditions on the coupling constants contained in (17)—(19).

If now we write out again the currents obtained from Eqs. (14) and (17)—(19) by all possible "strikings out," then according to Eq. (13) with $b_5 = 0$ the divergence of the vector current vanishes only when one or another of the following systems of equations is satisfied:

1) $g_1 = g_6 = g_7 = g_8 = 0, \quad q_4, g_5 \neq 0,$
$b_8 = b_2 = b_3 = b_4 = 0, \quad b_1 = b_6;$

2) $g_1 = g_2 = g_3 = g_7 = g_8 = 0, \quad g_4, g_6 \neq 0,$
$b_8 = b_2 = b_4 = 0, \quad b_1 = b_6 = b_3/2;$

3) $g_7 = g_8 = 0, \quad g_5 = \varepsilon g_6 \neq 0, \quad \varepsilon = \pm 1, \quad g_{10} = g_{11},$
$m_\Sigma = m_\Lambda, \quad b_8 = b_4 = 0;$

a) $g_2 = \varepsilon g_3, \quad b_1 = 0, \quad b_6 = b_3 = -\varepsilon b_2;$

b) $g_2 = -\varepsilon g_3, \quad g_1 = 0, \quad b_6 = 0, \quad b_1 = b_3 = \varepsilon b_2,$

and also in the cases differing from those enumerated by the replacement

$$g_1 \leftrightarrow g_4, \quad g_5 \leftrightarrow g_7, \quad g_6 \leftrightarrow g_8, \quad b_1 \leftrightarrow b_4.$$

[1] V. M. Shekhter, JETP **36**, 581 (1958), Soviet Phys. JETP **9**, 403 (1959).

[2] R. E. Behrends and A. Sirlin, Phys. Rev. **121**, 324 (1961).

[3] M. Gell-Mann, Phys. Rev. **106**, 1296 (1957).

[4] A. Pais, Phys. Rev. **110**, 574 (1958).

[5] S. Sakata, Prog. Theoret. Phys. **16**, 686 (1956).

[6] L. B. Okun', JETP **34**, 469 (1958), Soviet Phys. JETP **7**, 322 (1958).

[7] L. B. Okun', Proc. 1958 Ann. Int. Conf. on High Energy Physics at CERN, Geneva, 1958, page 223.

[8] R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).

[9] L. Alvarez, Proc. Int. Conf. on High Energy Particle Physics, Kiev, 1959, AN SSSR, 1960.

[10] I. Yu. Kobzarev and L. B. Okun', JETP **39**, 210 (1960), Soviet Phys. JETP **12**, 150 (1961).

[11] G. Feinberg and R. E. Behrends, Phys. Rev. **115**, 745 (1959).

[12] M. Gell-Mann, Preprint.

[13] W. Thirring, Nuclear Phys. **10**, 97 (1959).

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