# EXPERIMENTAL VERIFICATION OF THE CHARGE INDEPENDENCE PRINCIPLE IN THE $d + d \rightarrow He^4 + \pi^0$ REACTION FOR 400-Mev DEUTERONS

Yu. K. AKIMOV, O. V. SAVCHENKO, and L. M. SOROKO

Joint Institute for Nuclear Research

Submitted to JETP editor April 21, 1961

J. Exptl. Theoret. Phys. (U.S.S.R.) 41, 708-724 (September, 1961)

Experiments with a 400-Mev deuteron beam are described in which the  $d + d \rightarrow He^4 + \pi^0$  reaction, forbidden by the law of conservation of total isotopic spin, is investigated. It is shown that the total cross section for the reaction is less than  $1.1 \times 10^{-32}$  cm<sup>2</sup> on the 90% confidence level. The upper limit of the cross section is compared with the cross section of the  $d + d \rightarrow He^4 + \gamma$  reaction and with the cross section of the  $p + He^3 \rightarrow He^4 + \pi^+$  reaction, which is allowed by the isotopic spin selection rules. The data obtained in the present study, together with the theoretical estimates, indicate that the  $d + d \rightarrow He^4 + \pi^0$  reaction is strongly forbidden and hence confirm the validity of the law of conservation of total isotopic spin in processes of pion production by nucleons and light nuclei.

## 1. INTRODUCTION

THE law of conservation of total isotopic spin is the most important consequence of the principle of charge independence and leads to fully determined relations between the cross sections for different processes and to the formulation of additional selection rules. In the present study, we consider only one of the possible classes of phenomena in which the principle of charge independence leads to important consequences, namely, pion production by nucleons and light nuclei. In this case it is possible to verify the charge independence principle in the following ways:

A. It is known that the total cross sections for various processes of the form

$$N + N \rightarrow N + N + \pi$$
 (1)

satisfy the relation

$$\mathfrak{s}(pp \to pn\pi^+) + \mathfrak{s}(pn \to nn\pi^+) + \mathfrak{s}(pn \to pp\pi^-)$$
  
= 2 [ \mathfrak{s}(pp \to pp\pi^0) + \mathfrak{s}(pn \to pn\pi^0) ]. (2)

This relation has been verified experimentally for incident nucleon energies from the pion production threshold to 660 Mev.<sup>[1]</sup> At 600 Mev, where the cross sections were measured with the smallest error, relation (2) is satisfied to an accuracy of 8-10%.

B. Between the differential cross sections of the two charge-conjugate reactions

$$p+p \rightarrow d+\pi^+,$$
 (3)

$$n + p \rightarrow d + \pi^0 \tag{4}$$

the following relation holds:

$$d\mathfrak{s} \left( p + p \rightarrow d + \pi^{+} \right) = 2d\mathfrak{s} \left( n + p \rightarrow d + \pi^{0} \right). \tag{5}$$

Verification of this relation at 400 Mev<sup>[2]</sup> and 590 Mev<sup>[3]</sup> showed that no significant deviation is observed. A stricter verification of relation (5) requires, along with an increase in the accuracy of the measurements, the consideration of a number of perturbation corrections resulting from the nonidentical properties of different charge states of the nucleon and pion as well as the Coulomb forces in reaction (3).

C. Verification of the relations between differential cross sections of the reactions

$$p+d \to \mathrm{H}^3 + \pi^+, \qquad \qquad | \textbf{ (6)}$$

$$p + d \rightarrow \mathrm{He}^3 + \pi^0,$$
 (7)

representing two independent channels from the same initial state. This method has certain advantages from the experimental viewpoint, since it permits the elimination of a number of systematic errors. Calculation of the Clebsch-Gordan coefficients<sup>[4]</sup> for the  $(H^3\pi^+)$  and  $(He^3\pi^0)$  states leads to the relation

$$d\mathfrak{s} (p + d \to \mathrm{H}^3 + \pi^+) = 2d\mathfrak{s} (p + d \to \mathrm{He}^3 + \pi^0).$$
 (8)

But as in the case of reactions (3) and (4), a perturbation effect arises here, too, owing to the nonidentical properties of the neighboring charge states of the H<sup>3</sup> and He<sup>3</sup> nuclei and of the  $\pi^0$  and  $\pi^+$  mesons. The size of the possible perturbations in reactions (6) and (7) were calculated for a proton energy of 600 Mev.<sup>[5]</sup> It turned out that these perturbations alter the relation between the cross sections for processes (6) and (7), so that, instead of (8), we have

$$d \mathfrak{s} (p + d \to \mathrm{H}^3 + \pi^+) = 2 (1.10 \pm 0.035) \, d \mathfrak{s} (p + d \to \mathrm{He}^3 + \pi^0). \tag{9}$$

The limited accuracy of similar calculations places a limit of sensitivity on this method of verifying relation (9). Reactions (6) and (7) have been studied at 370 Mev<sup>[6]</sup> and 450 Mev.<sup>[7]</sup> The most accurate results were obtained at 600 Mev<sup>[8]</sup> and gave

$$d\sigma (p + d \to H^3 + \pi^+) = 2 (1.065 \pm 0.03) d\sigma (p + d \to He^3 + \pi^0).$$
(10)

D. The study of the forbidden reaction

$$d + d \rightarrow \mathrm{He}^4 + \pi^0$$
, (11)

as suggested by Lapidus, [9] occupies a special place. Reaction (11) involves particles in only one charge state, and therefore the perturbation resulting from the difference between neighboring charge states is automatically absent. The study of reaction (11) also attracts interest, since it allows a strict verification of the hypothesis that there exists a  $\pi_0^0$  meson<sup>[10]</sup> with a scalar isotopic spin.

We present below new data on reaction (11), which was studied with 400-Mev deuterons. The first results on the study of this reaction, obtained by us earlier, have already been published [see<sup>[11]</sup> and also the proceedings of the conferences on highenergy physics at Kiev (1959) and Rochester (1960)]. Furthermore, we report the results of the cross section measurements for the electromagnetic process

$$d + d \rightarrow \text{He}^4 + \gamma$$
 (12)

at the same deuteron energy and for the reaction

$$p + \text{He}^3 \rightarrow \text{He}^4 + \pi^+$$
 (13)

produced by 670-Mev protons. We also give theoretical estimates of the cross section for pion production by nucleons and light nuclei, which make it possible to draw additional conclusions on the degree of forbiddenness of reaction (11).

### 2. EXPERIMENTAL ARRANGEMENT

The method of experimental verification of the charge independence principle by the measurement of the degree of forbiddenness of process (11) has the advantage that by formulating the problem in this way we do not require accurate measurements of the total or differential cross sections, as is the case for the comparison of two charge-conjugate



FIG. 1. Kinematics of the reactions  $d + d \rightarrow He^4 + \pi^0$ ,  $d + d \rightarrow He^4 + \gamma$ , and  $d + d \rightarrow He^3 + n$  for an incident deuteron energy of 404 Mev ( $\theta$  is the angle of emission of the He<sup>4</sup> and He<sup>3</sup> nuclei; P<sub>eff</sub> is the effective momentum of these particles and is equal to p/Z, where Z is the charge of the nucleus). The numbers along the curves are the angles of emission of the He<sup>4</sup> and He<sup>3</sup> nuclei in the c.m.s. of the colliding deuterons. The dotted line corresponds to the angle at which the heavy particles were recorded, i.e.,  $5.8^{\circ}$  1.s.

reactions. The experimentally determined cross section for the forbidden process or the estimate of its upper limit is a measure of the violation of the forbiddenness. This method is essentially a null method of measurement. We can draw conclusions as to the degree of forbiddenness of reaction (11) if, in addition to this reaction, we study the electromagnetic process (12) and reaction (13); the latter is allowed by the selection rules resulting from the law of conservation of total isotopic spin.

The kinematics of reactions (11), (12), and (13) for 404-Mev deuterons is shown in Fig. 1. The limiting angle of emission of the  $\alpha$  particle is 8° in reaction (11) and  $10^{\circ}$  in reaction (12) in the laboratory system (l.s.). The  $\pi^0$  energy in the center-of-mass system (c.m.s.) is 82 Mev, which allows us to assume that the  $\pi^0$  meson is emitted primarily in the s and p states. This conclusion follows from the fact that occurrence of d waves in reaction (3) becomes appreciable in the polarization effects beginning with a  $\pi^+$ -meson energy of ~ 100 Mev, <sup>[12]</sup> and the effect of the d wave on the angular distribution as a result of the  $\cos^4 \theta$ terms is not significant even for very high energies.<sup>[13]</sup> On the other hand, the emission of  $\pi^0$ mesons in reaction (11) in the s and p states is not forbidden by any additional selection rules,



FIG. 2. Experimental setup: 1 - beam deflector; 2 - deuteron or proton beam; 3 - magnetic quadrupole lenses; 4 - gaseous or liquid deuterium target or carbon target; 5 - lead shield; 6 - monitor; 7 - trajectory of secondary charged particles; 8 - bending electromagnet; 9 - focusing shims; 10 - concrete shield; 11 - telescope consisting of five scintillation counters; 12 - shielding wall; 13 - evacuated tube; 14 - scintillation counters for separating particles on basis of time of flight.

apart from the law of conservation of total isotopic spin, and in the case of the violation of the latter, the differential cross section of reaction (11) has the form  $A + B \sin^2 \theta$ . The measurements of reaction (11) were therefore carried out for an isotropic angle  $(55^{\circ} \text{ in the c.m.s. or } 5.8^{\circ})$ in the l.s.).

The basic transition in reaction (12) is the electric quadrupole transition  ${}^{1}S \rightarrow {}^{1}D, [{}^{14}]$  which is responsible for an angular distribution of the form  $\sin^2 \theta \cos^2 \theta$ .\* The c.m.s. angle of observation of the  $\alpha$  particles is 41°, i.e., quite close to the maximum of the differential cross section.

$$d + d \to \mathrm{He}^3 + n, \tag{14}$$

for a l.s. angle of emission  $5.8^{\circ}$  were used for the determination of the resolving power of the recording apparatus, for resetting the apparatus, and for control measurements. The efficiency of the apparatus was checked by comparison of reactions (6) and (7) produced by 670-Mev protons. The absolute cross sections for all the foregoing processes were determined from measurements of the deuteron yield from reaction (3), whose cross section is well known over a wide energy range. [13,15]

### 3. CONDITIONS OF THE EXPERIMENT

The experimental setup is shown in Fig. 2. The deuteron or proton beam was focused on the target by means of a two-sector magnetic quadrupole lens with an 80-mm aperture. The secondary charged particles produced in the target were separated by a brass collimator set at an angle of 5.8° with respect to the beam axis and were deflected in the magnetic field by an angle 27°, after which they passed through a steel collimator in a concrete shielding wall and were recorded by a system of scintillation counters.

At the exit of the synchrocyclotron chamber, the deuteron beam had a mean energy of 405.3  $\pm$  0.5 Mev and a dispersion of 1.7  $\pm$  0.5 Mev.<sup>[16]</sup> The deuteron energy at the target dropped to 404 Mev. Photographic paper was exposed at different points along the deuteron trajectory to aid in the adjustment of the beam and the control of its shape. The total intensity of the deuteron beam was  $\sim 0.6$ Measurements of the He<sup>3</sup> yield from the reaction  $\times 10^{11}$  sec<sup>-1</sup>. The beam was monitored by an emission chamber located behind the target. We used 220-Mev protons to calibrate the emission chamber for the deuteron beam by comparison of the currents from the emission and the ionization chambers. In this way, we were able to find the cross section of reaction (14), which was used later on for the estimate of the cross sections of processes (11) and (12).

### 4. RECORDING APPARATUS

The total cross section for dd collisions is  $\sim 10^{-24} \text{ cm}^2$  and hence for the measurement of cross sections less than  $10^{-31}$  cm<sup>2</sup> the recording apparatus should possess a degree of selection so as to separate one "favorable" event from a background of several million other particles. The charged particles from reactions (3), (6), (7), (11) - (14) were identified by the effective momentum p/Z (Z is the charge of the particle), specific ionization, and range; in later experiments, selection was also made on the basis of the time of flight.

<sup>\*</sup>The angular distribution of reaction (12) reported in the Proceedings of the Tenth Annual Conference on High Energy Physics at Rochester (p. 51) is in error.

The secondary charged particles with a given effective momentum were separated by an electromagnet with a pole diameter of 100 cm and gap of 13 cm. Iron shims of trapezoidal shape and thickness 3.2 cm were placed between the poles of the magnet. The lateral bevel of the shims provided horizontal focusing of the particles at the point of location of the apparatus.

To record charged particles of high ionization and small range in a background of other radiation of lower ionization we used the method of separating particles on the basis of their ionization in several scintillation counters.<sup>[17]</sup> This method makes it possible to decrease appreciably the contribution from particles of low ionization produced in the "tail" in the ionization spectrum and also to suppress the background caused by starformation processes in the scintillator material and by random superposition of pulses in the electronic equipment.

The telescope designed for the selection of particles by this method consisted of six scintillation counters. The first five counters, connected in coincidence, were spectrometric counters, and the sixth counter was connected in anticoincidence with the first five. Connected in this way, the apparatus selected only those events in which all counters, apart from the anticoincidence counter, gave simultaneous pulses whose amplitudes at the output of the amplifiers exceeded the discrimination threshold. In the selection of particles on the basis of their range, aluminum and copper filters were placed between counters 4 and 5 of the telescope. The differential range interval was fixed by a filter placed before the anticoincidence counter. In the later series of measurements, an additional selection of the particles was made on the basis of the time of flight. The pulses from the first two counters, 3.3 m apart, were applied to a coincidence circuit with a resolving time of (5-7) $\times 10^{-9}$  sec.

## 5. ADJUSTMENT OF THE APPARATUS

The basic characteristics of the recording system were determined with a beam of  $\alpha$  particles of different energy<sup>[17]</sup> and by the identification of heavy charged particles which are produced in reactions (6), (7), and (14) with comparatively large cross section ( $\geq 10^{-29}$  cm<sup>2</sup>). The overall efficiency of the apparatus was checked by comparison of the yields of H<sup>3</sup> and He<sup>3</sup> nuclei produced in reactions (6) and (7). Reaction (6) was studied earlier<sup>[18]</sup> with a 670-Mev proton beam. The He<sup>3</sup> yield from reaction (7) was measured with a deu-

terium gas target filled to a pressure of 3.5 atm. The experimental arrangement and the method of selection of the particles were the same as those described  $in^{[18]}$ .

To determine the absolute cross section of reaction (7) under the same conditions, we measured the deuteron yield from reaction (3). It was found that

$$\frac{d\sigma}{d\Omega} \left( \theta_{\text{He}^{3}} = 5.8^{\circ} \right) = (20.1 \pm 1.8) \cdot 10^{-30} \,\text{cm}^{2}/\text{sr} \tag{15}$$

while the c.m.s. differential cross section with respect to the angle of emission of the  $\pi^0$  meson turned out to be

$$\frac{ds^{\bullet}}{d\Omega^{\bullet}} \left( \theta_{\pi^{\circ}}^{*} = 12^{\circ} \right) = (4.5 \pm 0.4) \cdot 10^{-30} \text{ cm}^{2}/\text{sr}$$
(16)

The triton yield from reaction (6) was measured a second time with an additional selection made on the basis of the time of flight. The results obtained in this way were in agreement with the first series of measurements<sup>[18]</sup> within the limits of experimental error.

These measurements made it possible to find the ratio of the cross sections of reactions (6) and (7) for a  $\pi^0$  c.m.s. angle ~ 12°:

$$d\sigma (p + d \to H^3 + \pi^+) = (2.04 \pm 0.32) \cdot d\sigma (p + d \to He^3 + \pi^0).$$
(17)

The obtained ratio of cross sections was in quite good agreement with the calculated value 2.20  $\pm$  0.07,<sup>[5]</sup> within the limits of accuracy of the measurements. By comparing our results with more accurate measurements,<sup>[8]</sup> we can conclude that the recording apparatus had practically the same efficiency of registration for both singly and doubly charged particles.

## 6. THE $d + d \rightarrow He^3 + n$ REACTION

Before each series of measurements with the deuteron beam, we checked the apparatus by measuring the He<sup>3</sup> yield from reaction (14). The He<sup>3</sup> particles were identified by the momentum, ionization, range, and time of flight (in the last series of measurements). The comparatively large cross section for this reaction ( $\sim 10^{-28}$  cm<sup>2</sup>) made it possible to measure the He<sup>3</sup> yield quite reliably and comparatively quickly. The conditions of recording the He<sup>3</sup> nuclei during measurements with different targets and with different methods of selection are shown in Figs. 3, 4, and 5.

The differential cross section of reaction (14) for a l.s. angle 5.8° was determined independently in several runs by relating it to the cross section of reaction (3) when the synchrocyclotron was switched from deuteron acceleration to proton ac-



FIG. 3. Conditions of recording  $\alpha$  particles from the d+d  $\Rightarrow$  He<sup>4</sup> +  $\pi^0$  reaction and He<sup>3</sup> nuclei from the d + d  $\Rightarrow$  He<sup>3</sup> + n reaction with a liquid deuterium target. O – background readings of the recording apparatus set for  $\alpha$  particles from the d + d  $\Rightarrow$  He<sup>4</sup> +  $\pi^0$  reaction as a function of the magnet current J; the dotted curve corresponds to a possible cross section of 2  $\times 10^{-32}$  cm<sup>2</sup> for this reactions;  $\bullet$  – He<sup>3</sup> yield from the filled target; X – He<sup>3</sup> yield from the empty target. Here and in the following figures N and J are reduced to relative units (readings of the individual instruments).

celeration. The differential cross section of reaction (14) averaged over four runs without corrections for the absorption and multiple scattering in the copper decelerating filters of the telescope turned out to be

$$\eta \frac{d\tau}{d\Omega}(5.8^\circ) = (2.18 \pm 0.15) \cdot 10^{-28} \text{ cm}^2/\text{sr}$$
 (18)

The value of the correction  $\eta$  was measured by comparison of the He<sup>3</sup> from the liquid deuterium target with and without the filter. Taking into account this correction, we obtain

$$\frac{d\sigma}{d\Omega}$$
 (5.8°) = (3.3 ± 0.23) · 10<sup>-28</sup> cm<sup>2</sup>/sr, (19)

or, in the c.m.s.,

$$\frac{d\sigma^*}{d\Omega^*} (16^\circ) = (4.3 \pm 0.3) \cdot 10^{-29} \text{ cm}^2/\text{sr.} (20)$$

## 7. MEASUREMENTS WITH A LIQUID DEUTERIUM TARGET

The first data on reaction (11) were obtained with deuterium-polyethylene and carbon targets.<sup>[11]</sup> It was found that the upper limit of the cross section for reaction (11) was

$$\sigma_t (d + d \rightarrow \text{He}^4 + \pi^0) < 1.0 \cdot 10^{-31} \text{ cm}^2$$
 (21)

on the 90% confidence level. An appreciable yield of  $\alpha$  particles from collisions between deuterons and carbon nuclei did not permit a significant improvement in the accuracy of the results by a direct



FIG. 4. Conditions of recording  $\alpha$  particles from the d + d  $\rightarrow$  He<sup>4</sup> +  $\pi^{0}$  and d + d  $\rightarrow$  He<sup>4</sup> +  $\gamma$  reactions; a and b – counting responses of the recording apparatus set for  $\alpha$  particles from the d + d  $\rightarrow$  He<sup>4</sup> +  $\pi^{0}$  reaction in measurements with a heavy polyethylene target (V<sub>d</sub> – discrimination threshold, R – thickness of the decelerating filter in the telescope); c – results of the yield measurements from the d + d  $\rightarrow$  He<sup>3</sup> + n, d + d  $\rightarrow$  He<sup>4</sup> +  $\pi^{0}$ , and d + d  $\rightarrow$  He<sup>4</sup> +  $\gamma$  reactions with a deuterium gas target.

increase in the duration of the measurements. The use of a liquid deuterium target made it possible to reduce the  $\alpha$ -particle background to approximately  $\frac{1}{35}$  of the background for a heavy polyethylene target.

The particles were identified by their effective momentum, ionization, and range as in the case of the first series of measurements.<sup>[11]</sup> Reaction (14)



FIG. 5. Yield from the  $d + d \rightarrow He^3 + n$ ,  $d + d \rightarrow He^4 + \pi^0$ , and  $d + d \rightarrow He^4 + \gamma$  reactions as a function of the magnet current. O - measurements with a deuterium gas target, • - measurements with a hydrogen target.

Reaction	$d + d \rightarrow He^3 + n$	$d + d \rightarrow He^4 + \gamma$	$d + d \rightarrow He^4 + \pi^0$
Effective momentum $p/Z$ at the center of the deflecting magnet, Mev/c	743	689	656
Spread of the effective momentum due to the target thickness, Mev/c	0.6	5	6
Bending-magnet current, arbitrary units	60	55.6	52.5
Thickness of filter in the telescope	10,7 g/cm <sup>2</sup> Cu	1.0 cm A1	0.75 cm A1
Thickness of filter in front of the anticoincidence counter	1.7 g/cm <sup>2</sup> Cu	0.3 cm A1	0.3 cm A1
Mean ionization loss of particles dE/dx in carbon, Mev/g-cm <sup>2</sup>	24	44	49
Time of flight of particles over 3.35-m base, 10 <sup>-9</sup> sec	24	33	35
Time of flight of protons of the same effective momentum, 10 <sup>-9</sup> sec	17	18	19
Initial energy of heavy particles for 1.s. angle of 5.8°, Mev	374	254	232
Final energy of heavy particles after passing through five scin- tillation counters of the telescope, Mev	343	207	180

was used to adjust the apparatus. The considerable reduction of the background made it possible to obtain a better separation of the monoenergetic group of He<sup>3</sup> nuclei with the aid of all the counting responses (Fig. 3). The conditions of recording the  $\alpha$  particles were similar to the conditions in the first series of measurements.<sup>[11]</sup> The constancy of the difference in readings of the apparatus with two different filters indicated that, with a discrimination threshold above 18 v, practically only single  $\alpha$  particles were recorded. These  $\alpha$  particles were emitted under the action of deuterons from the brass foil of the deuterium target container. The  $He^3$  yield from reaction (14) is shown in Fig. 3 in the scale 1:240 for a l.s. angle 5.8°. Also shown there are the background readings for bending-magnet currents corresponding to  $\alpha$  particles from reaction (11). The straight line determined by the method of least squares for the experimental points located to the right and left of the calculated point for the  $\alpha$ -particle peak from reaction (11) corresponds to the mean background level.

Comparison of the results of the measurements with this line makes it possible to estimate the upper limit of the cross section for the d + d $\rightarrow$  He<sup>4</sup> +  $\pi^0$  reaction. If we exclude the change in the momentum interval in the transition from one bending-magnet current to another (k<sub>1</sub> = 1.15) and introduce a correction for the absorption of  $\alpha$  particles in the decelerating filter (k<sub>2</sub> = 1.10) and for the scattering in air (k<sub>3</sub> = 1.16), then

$$\frac{ds}{d\Omega}(5.8^{\circ}) < 1.35 \cdot 10^{-31} \text{ cm}^2/\text{sr}$$
 (22)

on the 90% confidence level, or, in the c.m.s.,

$$\frac{ds^{\bullet}}{d\Omega^{\bullet}}(55^{\circ}) < 1.5 \cdot 10^{-33} \text{ cm}^2/\text{sr}$$
 (23)

If the  $\pi^0$  angular distribution has the form  $A + B \sin^2 \theta$ , then

$$\sigma_t (d + d \rightarrow He^4 + \pi^0) < 1.9 \cdot 10^{-32} \text{ cm}^2$$
 (24)

on the 90% confidence level. By way of illustration, the dotted line in Fig. 3 corresponds to a possible cross section of  $2 \times 10^{-32}$  cm<sup>2</sup> for this reaction. The estimate (24) also applies to the upper limit of the cross section for the production of the  $\pi_0^0$  meson with a scalar isotopic spin and a rest mass of 100 - 150 Mev.

We give below some kinematical characteristics of  $\alpha$  particles for different possible masses of the  $\pi_0^0$  meson:

Possible values of the  $\pi_0^0$ -

meson mass, Mev	190	176	134.8	98	0
Corresponding effective					
momentum of the $\alpha$ par-					
ticles, Mev/c	575	596	635	652	669
Bending-magnet current,					
arbitr. units	46.5	48.3	51.5	52.6	54.5
Thickness of the decelerat-					
ing filter in the telescope,					
cm Al	0.265	0.384	0.62	0.71	0.87

## 8. MEASUREMENTS WITH A DEUTERIUM GAS TARGET

A further decrease in the background of  $\alpha$  particles knocked out of the brass foil of the target container and also of fragments produced in the decelerating filters and scintillation counters was obtained with a deuterium gas target and the introduction of the additional selection of particles on the basis of the time of flight. Such a target consisted of a copper tube of 12.5-cm diameter and 150 cm long. The target was set in the beam in such a way as to prevent particles produced in the front and rear windows of the target from entering the collimator located in front of the bending magnet. The front window of 9 cm diameter was covered by a brass foil  $200 \mu$  thick. The exit flange of the target had two openings covered by a terylene film  $200 \,\mu$  thick. The deuterium gas was

supplied to the target from an arrangement containing activated carbon which was cooled to the temperature of liquid nitrogen. This procedure prevented, to a considerable degree, heavier gases from reaching the target. With a collimator entrance diameter of 40 mm and various positions of the recording apparatus, such a gas target is equivalent to a liquid deuterium target 2 to 3 mm thick.

The air was evacuated over almost the entire path of the particles from the target to the counters.

The table gives the most important kinematical characteristics of the recorded heavy particles at a 1.s. angle  $5.8^{\circ}$  for 404-Mev deuterons.

The counting response of the apparatus set for  $\alpha$  particles from reaction (11) was determined from measurements of  $\alpha$  particles from the heavy polyethylene target (Figs. 4a, b). The stability of the apparatus during the measurements of  $\alpha$  particles from reactions (11) and (12) was checked periodically by putting the heavy polyethylene target in place of the deuterium gas target. The quantity checked was the  $\alpha$ -particle yield from this target. During the measurements with the deuterium gas target, no pulses were recorded from  $\alpha$ particles which could have been produced in reactions (11) and (12). No change in the efficiency of the apparatus was observed. The results are shown in Fig. 4c in which the He<sup>3</sup> yield from reaction (14) is shown in a scale 1:600;  $\alpha$  particles from reactions (11) and (12) are also shown. These data, together with the corrections mentioned above, permit us to determine the upper limit of the cross section for reaction (11):

$$\sigma_t (d + d \rightarrow He^4 + \pi^0) < 1.9 \cdot 10^{-32} \text{ cm}^2$$
 (25)

on the 90% confidence level.

Under these conditions, a further increase in the accuracy of the measurements can be attained by increasing the statistics of the measurements. For this reason we increased the dimensions of the scintillators, as a result of which the angle subtended by the recording apparatus rose approximately sevenfold without an appreciable worsening of the resolving power for the effective momentum. The remaining conditions of the experiment were the same as in the previous series of measurements.

Figure 5 shows the results of measurements under these changed conditions. On the right is the calibration peak for the He<sup>3</sup> yield from reaction (14). The occurrence of the background is due to the worsening of the transmission and to an increase in the efficiency for the recording of fragments from stars. The background was determined with a hydrogen gas target filled to the same pressure as in the case of the deuterium target. It took two hours to change the target. During this time we measured  $\alpha$  particles from carbon in order to check the stability of the apparatus. Over a period of time totaling about 20 hours and for three values of the bending-magnet current, 14 counts from the deuterium target and 13 counts from the hydrogen target were recorded for the same number of monitor counts. The mean number of counts in the recording of  $\alpha$  particles from carbon was  $17 \pm 2$  after the measurements with the deuterium target and  $14 \pm 1.2$  after the measurements with the hydrogen target. For the evaluation of the data, we analyzed the time distribution of the counts. The observed distribution was in good agreement with a Poisson distribution and this was sufficient grounds to consider only the statistical errors in the evaluation of the data. After the introduction of the corrections, the results of this series of measurements gave

$$\sigma_t (d + d \rightarrow \text{He}^4 + \pi^0) < 1.8 \cdot 10^{-32} \text{ cm}^2$$
 (26)

on the 90% confidence level.

In the measurement of  $\alpha$  particles from reaction (12) we recorded 37 counts during a period of about 20 hours. The background under these conditions was determined in two ways: first, by measurements with a deuterium target for small and large bending magnet currents and second, with a hydrogen target. During about 20 hours of measurements, we recorded a total of 17 background counts with the same number of monitor counts. The results of the measurements of the efficiency of the apparatus obtained by putting a carbon target in place of the gas target indicated that the mean number of



FIG. 6. Conditions of recording  $\alpha$  particles from the  $p + He^3 \rightarrow He^4 + \pi^+$  reaction. a, b - counting response of the recording apparatus set for  $\alpha$  particles from the  $p + He^3 \rightarrow He^4 + \pi^+$  reaction obtained with a carbon target; c - results of the measurement of the yield for the  $p + He^3 \rightarrow He^4 + \pi^+$  reaction as a function of the magnet current: O - measurements with an He<sup>3</sup> gas target;  $\bullet$  - measurements with a deuterium gas target.

 $\alpha$ -particle counts was  $18 \pm 1.3$  after the measurements with a deuterium target and  $20 \pm 1.4$  after measurements with a hydrogen target. As before, we analyzed the time distribution of the counts. No deviation from a Poisson distribution was observed. The difference between the counts of the apparatus at the position of the calculated peak for  $\alpha$  particles from reaction (12) and the background level was  $20 \pm 7.4$ . The probability that this difference actually equals zero is less than 1%. This is sufficient cause to consider the observed positive effect from reaction (12) to be quite real. If we take into account small corrections, then the differential cross section for the  $d + d \rightarrow He^4 + \gamma$  reaction at 5.8° l.s. is

$$\frac{d\sigma}{d\Omega} (5.8^{\circ}) = (0.84 \pm 0.31) \cdot 10^{-31} \text{ cm}^2/\text{sr.} (27)$$

The c.m.s. differential cross section is

$$\frac{d5^{\circ}}{d\Omega^{\bullet}}$$
 (41.5°) = (1.6 ± 0.6) · 10<sup>-33</sup> cm<sup>2</sup>/sr. (28)

If we take into account the angular distribution of reaction (12),  $d\sigma/d\Omega \sim \sin^2 \theta \cos^2 \theta$ , then the total cross section is

$$\sigma_t (d + d \rightarrow \text{He}^4 + \gamma) = (1.1 \pm 0.4) \cdot 10^{-32} \text{ cm}^2.$$
 (29)

## 9. THE $p + He^3 \rightarrow He^4 + \pi^+$ REACTION

We measured the cross section for this reaction, which is allowed by the isotropic spin selection rules, with a 670-Mev proton beam at 5.8° l.s. The experimental setup, position of the apparatus, and the selection methods employed were the same as for the measurements described in Sec. 8. Some of the kinematical characteristics of the high-energy branch of  $\alpha$  particles produced in reaction (13) were as follows:

Angle of recording $\alpha$ particles in the l.s.	5.75°
Angle of emission of the pion in the c.m.s.	159°
Effective momentum $p/Z$	802 Mev/c
α-particle energy	329 Mev
dÊ/dx	$30 \text{ Mev/g-cm}^{-2}$
Time of flight over 3.35-m base	29.5 × 10 <sup>-</sup> ° sec
Thickness of decelerating filter	1.76 cm A1
Thickness of filter in front of anticoinci-	
dence counter	0.8 cm A1

The apparatus was set for  $\alpha$  particles from a carbon target of the same energy as the  $\alpha$  particles from reaction (13) (Fig. 6). The basic measurements were made with a gas target filled with He<sup>3</sup> to a pressure of 0.9 atm. The background was measured with a deuterium gas target filled to a pressure of 3 atm. The stability of the apparatus was checked every two hours by measurements of the  $\alpha$ -particle yield from the carbon target. During about seven hours of measurement 10 counts were recorded. When the bending-magnet current

was increased or decreased, no counts of the apparatus were recorded. Also, no counts were recorded from the deuterium target for a magnet current corresponding to  $\alpha$  particles from reaction (13). The absolute cross section of reaction (13) was calibrated by means of the triton yield from reaction (6) found under the same conditions as with the deuterium gas target.

The differential cross section of reaction (13) for a 1.s. angle 5.8° was

$$\frac{d_{5}}{d\Omega}(5.8^{\circ}) = \left(3.26 + \frac{1.35}{-1.27}\right) \cdot 10^{-30} \text{ cm}^{2}/\text{sr}$$
(30)

and in the c.m.s.

$$\frac{d\mathfrak{z}^*}{d\Omega^*} \ (\theta^*_{\pi} = 159^\circ) = \left(0.24 \frac{+0.10}{-0.09}\right) \cdot 10^{-30} \ \mathbf{cm^2/sr.}$$
(31)

## **10. DISCUSSION OF RESULTS**

A. Comparison of the  $d + d \rightarrow He^4 + \pi^0$  and the  $\frac{d + d \rightarrow He^4 + \gamma}{d + d \rightarrow He^4 + \gamma}$  reactions. If we average all the estimates of the upper limit of the cross section for reaction (11) found in the three series of measurements (24), (25), and (26), then in the l.s. we have

$$\frac{ds}{d\Omega}(5.8^{\circ}) < 8 \cdot 10^{-32} \text{ cm}^2/\text{sr},$$
 (32)

in the c.m.s.

$$\frac{ds^*}{d\Omega^*}(55^\circ) < 9 \cdot 10^{-34} \text{ cm}^2/\text{sr}$$
 (33)

and the total cross section is

$$\sigma_t (d + d \rightarrow \text{He}^4 + \pi^0) < 1.1 \cdot 10^{-32} \text{ cm}^2$$
 (34)

on the 90% confidence level. This value of the upper limit of the cross section of reaction (11) should be compared with the cross section of the electromagnetic process (12) which, on the basis of the data obtained in the present experiment, is  $(1.1 \pm 0.4) \times 10^{-32}$  cm<sup>2</sup>.

An independent estimate of this cross section can be obtained from the data on the photodisintegration of helium nuclei.<sup>[19]</sup> According to the results of [19], the integral cross section of the electromagnetic process

$$\gamma + \mathrm{He}^4 \to d + d, \qquad (35)$$

defined as

$$\int_{E_{\gamma}=23\,\mathrm{Mev}}^{190} \sigma_t(E_{\gamma}) dE_{\gamma},$$

is ~2% of the integral cross section of the reaction

$$f + \mathrm{He}^4 \to \mathrm{H}^3 + \rho.$$
 (36)

The low yield for reaction (35) is explained by the



FIG. 7. Energy dependence of the reactions  $\gamma + \text{He}^4 \rightarrow \text{H}^3 + p$  (curve 1) and  $\gamma + \text{He}^4 \rightarrow d + d$  (curve 2);  $\varepsilon_0$  – reaction threshold; O – experimental points of [19];  $\bullet$  – data of the present experiment.

fact that the electric dipole transition in this reaction is forbidden and that the electric quadrupole transition  ${}^{1}S \rightarrow {}^{1}D$  is more probable. If it is assumed that the energy dependence of the total cross section of reaction (35) is of the same form as in the case of reaction (36), then the cross section of the inverse process (12) calculated with the aid of the principle of detailed balance is ~ 0.8  $\times 10^{-32}$  cm<sup>2</sup> for incident 400-Mev deuterons.

Gorbunov and Spiridonov<sup>[19]</sup> analyzed the energy dependence of the electric quadrupole transition in reaction (36). This analysis indicated that the relative contribution from the quadrupole transition should increase with the energy of the  $\gamma$  quanta. This conclusion qualitatively agrees with the theoretical estimates<sup>[14]</sup> made for processes (35) and (36). If, finally, we take into account the increase in the cross section of photoprocesses as a result of resonances in the meson cloud, which is most clearly observed in the photodisintegration of deuterons in the 200 - 300 Mev energy region,<sup>[20]</sup> then it becomes obvious that the estimate of  $0.8 \times 10^{-32}$  $cm^2$  for the cross section of (12) is only a lower limit, and the total cross section can actually exceed this value.

The energy dependence of the cross section of reactions (35) and (36) can be considered theoretically, similarly to the treatment of Flowers and Mandl.<sup>[14]</sup> We carried out calculations with wave functions of the  $H^3$  and  $He^4$  nuclei taken in the form <sup>[21]</sup>

$$\Psi \sim e^{-\alpha R}, \qquad R = \left(\sum_{i < j} r_{ij}^2\right)^{1/2}, \qquad (37)$$

where  $r_{ij}$  is the distance between nucleons in the nucleus. For the H<sup>3</sup> nucleus, the choice of the parameter  $\alpha$  was based on the value of the energy of the Coulomb repulsion of the He<sup>3</sup> nucleus.<sup>[22]</sup> For the He<sup>4</sup> nucleus, the parameter  $\alpha$  was found from the value of the rms radius of the  $\alpha$  particle.<sup>[23]</sup> For the deuteron, we took the Hulthen function. The results of the calculation are shown in Fig. 7. Also shown in the figure are the experimental data for reaction (36) obtained in experiments on the photodisintegration of helium in the  $\gamma$ -quantum energy interval from the threshold to 190 Mev<sup>[19]</sup> and data for reaction (12) obtained in the present experiment and recalculated for the inverse reaction (35).

The calculated dependence of the cross section for reaction (36) is in qualitative agreement, within the limits of a factor of 2.5, with the experimental data in the entire energy interval. The very steep drop in the cross section in the high energy region is due to the fact that in the calculations only the electric dipole transition was taken into account, while from the data on the angular distribution of reaction (36)<sup>[19]</sup> it follows that the electric quadrupole transition begins to play an appreciable role starting with an energy of 30 Mev, and its contribution increases with the  $\gamma$ -quantum energy. The calculated integral cross section of reaction (35) is

$$\int_{E_{\gamma}=23\,\text{Mev}}^{220} \sigma_t(E_{\gamma}) dE_{\gamma} = 0,72 \cdot 10^{-27} \,\text{cm}^2 \quad \text{Mev}$$
(38)

which is in good agreement with the value 0.75  $\times 10^{-27}$  cm<sup>2</sup> Mev obtained from experiments on the photodisintegration of helium.<sup>[19]</sup> The calculated cross section of reaction (35) for 220-Mev  $\gamma$  quanta is  $1.25 \times 10^{-31}$  cm<sup>2</sup>, which, when recalculated for reaction (12), gives

$$\sigma_t (d + d \rightarrow \text{He}^4 + \gamma) = 0.7 \cdot 10^{-32} \text{ cm}^2$$
 (39)

The cross section of reaction (12) can thus be estimated in three ways: 1) experimentally (1.1  $\times 10^{-32}$  cm<sup>2</sup>); 2) by extrapolation of the data on the cross section for the photodisintegration of helium (0.8  $\times 10^{-32}$  cm<sup>2</sup>); 3) by direct calculation (0.7  $\times 10^{-32}$  cm<sup>2</sup>). Taken together, the results give

$$\sigma_t (d + d \rightarrow \text{He}^4 + \gamma) \approx 1 \cdot 10^{-32} \text{ cm}^2$$
 (40)

with an uncertainty of  $\sim 50\%$ .

At the same time, the pion production process (11) has a cross section less than  $1.1 \times 10^{-32}$  cm<sup>2</sup> on the 90% confidence level, i.e., it does not ex-



FIG. 8. Results of the calculations of the differential cross sections of the  $p + d \rightarrow H^3 + \pi^+$  reaction for 340-Mev protons with allowance for the production of all positive pions in pp and pn collisions for a repulsive-core radius  $r_0 = 0.6 \ h/m_\pi c$ :  $1 - for a H^3$  wave function of the form (41.1),  $2 - for a H^3$  wave function of the form (41.2),  $3 - for a H^3$  wave function of the form (41.3);  $0 - data of \begin{bmatrix} 26 \\ 26 \end{bmatrix}$ .

ceed the cross section for the electromagnetic process (12). These facts are evidence that the law of conservation of total isotopic spin is valid for processes of pion production, since if reaction (11) were not forbidden the cross section would be about 100 times the cross section of process (12).

B. Comparison with the  $p + He^3 \rightarrow He^4 + \pi^+$  reaction. The upper limit found for the cross section of reaction (11) can, moreover, be compared with the cross section of reaction (13), which is allowed by the isotopic spin selection rules. In the present experiment, the cross section of reaction (13) was measured for a single 1.s. angle with a proton en-



FIG. 9. Same as Fig. 8, but for 450-Mev protons; O = data of [7].



FIG. 10. Results of calculations of the differential cross section of the  $p + He^3 \rightarrow He^4 + \pi^+$  reaction for 326-Mev(c.m.s.)  $\pi^+$  mesons: 1 – for wave functions of He<sup>3</sup> and He<sup>4</sup> of the form (37), 2 – for wave functions of He<sup>3</sup> and He<sup>4</sup> of the form (41.3); 0 – data of the present experiment.

ergy of 670 Mev, which corresponds to a  $\pi^+$  c.m.s. energy of 326 Mev. In order to estimate the cross section of this reaction for a pion energy of 80 Mev, we calculated the cross section of reaction (13) in the impulse approximation. It was assumed here that the pion is produced in an elementary nucleonnucleon collision and all the nucleons then undergo fusion into He<sup>4</sup>.

As a qualitative check of this method of calculation, we computed the cross section of reaction (6), which has been well studied experimentally for various proton energies.<sup>[6-8]</sup> Such calculations were first made for 340-Mev protons.<sup>[4]</sup> Later on,<sup>[24]</sup> the hard-core model was used in these cal-



FIG. 11. Same as Fig. 10, but for  $\pi^+$  mesons of energy 80 Mev in the c.m.s.

culations. In the present experiment, we calculated the cross section of reaction (6) for 340-Mev protons for the following four types of nuclear wave functions:

H<sup>3</sup> 
$$(R = \left(\sum_{i < j} r_{ij}^2\right)^{1/2})$$
: 1)  $\Psi \sim \exp\left(-\frac{1}{2} \alpha \sum_{i < j} |r_{ij}|\right)$ ,  
2)  $\Psi \sim e^{-\alpha R}$ , 3)  $\Psi \sim e^{-\alpha R/R}$ , 4)  $\Psi \sim e^{-\alpha^2 R^2}$  (41)

with different radii of the hard core.

The increase in the radius of the repulsive core decreases the total cross section of the reaction and increases the forward-back asymmetry in the angular distribution. For a core radius greater than  $0.4 \ h/m_{\pi}c$ , the wave functions of the first three shapes give approximately the same angular dependence, in qualitative agreement with the experimental data. The Gaussian function (4) does not give agreement with the experimental data. For this reason we did not use a wave function of this shape in the further calculations.

If similar calculations are carried out for 450-Mev protons, then it appears that the angular distribution of reaction (6) is described correctly, but the total cross section differs strongly from the observed one, and this difference increases with energy. The reason for this difference is that the calculations took into account the production of  $\pi^+$  mesons only in reaction (3) and did not at all take into account the production of  $\pi^+$  mesons in the reactions

$$p + p \to n + p + \pi^+, \tag{42}$$

$$p+n \rightarrow n+n+\pi^+,$$
 (43)

whose relative contribution <sup>[25,1]</sup> increases strongly with the proton energy. The contribution of reactions (42) and (43) can be taken into account qualitatively if we add to the cross section of reaction (3) the amount by which the sum of the total cross sections for the production of  $\pi^+$  mesons in reactions (42), (43), and (3) exceeds the total cross section of reaction (3). The differential cross sections of reaction (6) calculated in this way for 340and 450-Mev protons with a repulsive-core radius of 0.6  $\hbar/m_{\pi}c$  are in satisfactory agreement with the experimental data (Figs. 8 and 9). For 340-Mev protons the wave function of the form (37) gives the best description of the experimental data.

We calculated by the same method the differential cross section of reaction (13). The calculations were made for  $\pi^+$  mesons at 80 and 326 Mev in the c.m.s., which correspond to bombarding protons of 280 and 670 Mev, respectively. The results of the calculations are shown in Fig. 10 and 11. For 670-Mev protons quite good agreement was observed between the calculated and experimental data, within the limits of a factor of 2, if a wave function of the form (37) is used. The fact that there was agreement for quite large  $\pi^+$  energies gives reason to believe that the results of the calculations for 80-Mev  $\pi^+$  mesons are sufficiently reliable. For this energy and for a wave function of the form (37) we obtain

$$\sigma_t (p + \text{He}^3 \rightarrow \text{He}^4 + \pi^+) = 79 \cdot 10^{-30} \text{ cm}^2.$$
 (44)

This result must still be compared directly with the upper limit of the cross section of reaction (11), since these two reactions differ from one another in the structure of the colliding nuclei. It is natural, however, to assume that the difference between reactions (11) and (13) is the same as that between the reactions  $p + H^3 \rightarrow He^4 + \gamma$  and  $d + d \rightarrow He^4 + \gamma$ , i.e., the cross sections differ by a factor of  $10^2$ . In this case the reference point for the "allowed reaction (11) is the cross section  $0.01\sigma_t(p + He^3 \rightarrow He^4 + \pi^+) = 80 \times 10^{-32} \text{ cm}^2$ . This value is approximately 70 times the upper limit of the total cross section of reaction (11), which is an additional indication that reaction (11) is strongly forbidden.

C. Estimate of the cross section of the "allowed'' reaction  $d + d \rightarrow He^4 + \pi^0$ . As was indicated in the introduction, reaction (11) is forbidden only by the law of conservation of total isotopic spin, and the possible transitions in this reaction are the same as for both the forbidden process  $(T = 0) \rightarrow (T = 1)$ , and for the "allowed" process  $(T = 0) \rightarrow (T = 0)$ . By the "allowed" reaction (11) we understand a reaction in which we ascribe to the  $\pi^0$  meson an isotopic spin equal to zero, while its remaining properties are unchanged. We can then carry out the calculation in the impulse approximation using a procedure similar to that of the preceding section. It was assumed in the calculations that the  $\pi^0$  meson is produced in an elementary nucleon-nucleon collision and then all the nucleons undergo fusion into the He<sup>4</sup> nucleus. We took into account the production of  $\pi^0$ mesons in both pn and pp collisions. The total cross section of the "allowed" reaction (11) calculated for a hard-core radius 0.6 and  $\hbar/m_{\pi}c$ leading to the best agreement between calculation and experiment in reaction (6) proved to be 30  $\times 10^{-32}$  cm<sup>2</sup> for an  $\alpha$ -particle wave function of the form (37) and  $80 \times 10^{-32}$  cm<sup>2</sup> for an  $\alpha$ -particle wave function of the form (41.3). The first of these estimates is the more reliable one, since it corresponds to a wave function of exponential form (37) which gives the best agreement between

calculation and experiment for reactions (6) and (13).

Hence the upper limit of the total cross section for reaction (11) is smaller by a factor of several tens than the cross section of the "allowed" reaction. The foregoing estimate for the "allowed" reaction is nothing else but an estimate of the cross section of the  $d + d \rightarrow He^4 + \pi_0^0$  reaction<sup>[10]</sup> with the emission of a  $\pi_0^0$  meson with scalar isotopic spin and mass close to the mass of an ordinary  $\pi^0$  meson with a vector isotopic spin. If it is assumed here that the hypothetical  $\pi_0^0$  meson has the same strong-interaction constant as an ordinary pion, then the fact that reaction (11) is strongly forbidden also means that the isotopically scalar  $\pi_0^0$ meson with a rest mass in the interval 100–150 Mev does not exist.

## CONCLUSIONS

1) The total cross section of the  $d + d \rightarrow He^4 + \pi^0$  reaction for  $\pi^0$  mesons of c.m.s. energy ~ 80 Mev does not exceed the cross section for the electromagnetic process  $d + d \rightarrow He^4 + \gamma$ , while the expected ratio of the cross sections of these processes should be approximately  $10^2$  if it were not forbidden.

2) The total cross section of the  $p + He^3 \rightarrow He^4 + \pi^+$  reaction for the same pion c.m.s. energy is approximately  $7 \times 10^3$  times the upper limit of the cross section for the  $d + d \rightarrow He^4 + \pi^0$  reaction, and this difference cannot be explained only in terms of the structure of the colliding nuclei.

3) The upper limit of the total cross section for the d + d  $\rightarrow$  He<sup>4</sup> +  $\pi^0$  reaction is ~ 3% of the calculated value of the cross section for the "allowed" process.

4) These facts taken together indicate that the  $d + d \rightarrow He^4 + \pi^0$  reaction is strongly forbidden and therefore the law of conservation of total isotopic spin holds for processes of pion production by nucleons and light nuclei.

5) The  $\pi_0^0$  meson with a scalar isotopic spin and rest mass of 100 - 150 Mev does not exist.

In conclusion, the authors express their gratitude to L. I. Lapidus for discussion of the proposed plan of the experiments, to V. P. Dzhelepov for his interest in the work and for assistance in carrying out the experiments, to R. M. Sulyaev and B. S. Neganov for aid in the work with the  $He^3$  gas, and to Kim Ze P'he and I. V. Puzynin of the Joint Institute of Nuclear Research computer group for performing the calculations on the electronic computer. <sup>1</sup>A. F. Dunaĭtsev and Yu. D. Prokoshkin, JETP **38**, 747 (1960), Soviet Phys. JETP **11**, 540 (1960).

<sup>2</sup> R. H. Hildebrand, Phys. Rev. 89, 1090 (1953).
 <sup>3</sup> Flyagin, Dzhelepov, Kiselev, and Oganesyan, JETP 35, 854 (1958), Soviet Phys. JETP 8, 592

(1959).
 <sup>4</sup> M. Ruderman, Phys. Rev. 87, 383 (1952).

<sup>5</sup>H. S. Köhler, Phys. Rev. **118**, 1345 (1960).

<sup>6</sup>Bandtel, Frank, and Moyer, Phys. Rev. **106**, 802 (1957).

<sup>7</sup>Creve, Ledley, Lillethan, Marcowitz, and Rey, Phys. Rev. **118**, 1091 (1960).

<sup>8</sup>Harting, Kluyver, Kusumegi, Rigopoulos, Sacks, Tibell, Vanderhaeghe, and Weber, Phys. Rev. **119**, 1716 (1960).

<sup>9</sup>L. I. Lapidus, JETP **31**, 865, (1956), Soviet Phys. JETP **4**, 740 (1957).

<sup>10</sup> A. Baldin and P. Kabir, DAN SSSR **122**, 361 (1958), Soviet Phys.-Doklady **3**, 956 (1958).

<sup>11</sup> Akimov, Savchenko, and Soroko, JETP **38**, 304 (1960), Soviet Phys. JETP **11**, 221 (1960).

<sup>12</sup> Akimov, Savchenko, and Soroko, JETP **35**, 89 (1958), Soviet Phys. JETP **8**, 64 (1959).

<sup>13</sup> B. S. Neganov and L. B. Parfenov, JETP 34,

767 (1958), Soviet Phys. JETP 7, 528 (1958).

<sup>14</sup>B. H. Flowers and F. Mandl, Proc. Roy. Soc. (London) **206**, 131 (1951).

 $^{15}$  M. G. Meshcheryakov and B. S. Neganov, DAN SSSR 100, 677 (1955).

<sup>16</sup> I. M. Vasilevskii and Yu. D. Prokoshkin,

Atomnaya Énergiya (Atomic Energy) 7, 225 (1959). <sup>17</sup> Akimov, Savchenko, and Soroko, Pribory i

Tekhnika Éksperimenta (Instrum. and Exptl. Techniques) No. 4, 71 (1960).

<sup>18</sup> Akimov, Savchenko, and Soroko, JETP **38**, 643 (1960), Soviet Phys. JETP **11**, 462 (1960).

<sup>19</sup> A. N. Gorbunov and V. M. Spiridonov, JETP

33, 21 (1957), Soviet Phys. JETP 6, 16 (1958).

 $^{20}$  Keck, Tollestrup, and Smythe, Phys. Rev. 96, 850 (1954).

<sup>21</sup> J. Irving, Phil. Mag. 42, 338 (1951).

<sup>22</sup> A. V. Tollestrup, Phys. Rev. **75**, 1947 (1949).

<sup>23</sup> R. W. McAllister and R. Hofstadter, Phys. Rev. 102, 851 (1956).

<sup>24</sup>S. Bludman, Phys. Rev. 94, 1722 (1954).

<sup>25</sup> B. S. Neganov and O. V. Savchenko, JETP 32, 1265 (1957), Soviet Phys. JETP 5, 1033 (1957).

<sup>26</sup> Frank, Bandtel, Medey, and Moyer, Phys. Rev. 94, 1716 (1954).

Translated by E. Marquit 128