RADIATIVE CORRECTIONS IN PION DECAYS

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Radiative corrections to pion decays are calculated. The spectra of real photons emitted together with μ mesons and electrons are different in shape. The ratio of the decay probabilities, therefore, depends on the common cut-off parameter for the photon spectrum in both channels. The contribution of radiation effects to the total decay probability (decay with emission of any photon) is 3.93%. This correction is primarily determined by the ratio of the photon emission probabilities from the electron and the μ meson. Expressions for the lepton and photon spectra in π decays are given.

RADIATIVE corrections in weak interactions have been the subject of many papers.[1-9] It was shown in this work that the form of these corrections is different in different processes. In β decay, where there are two charged particles of the same helicity in the final state, one finds integrals which diverge at the upper limit owing to the interaction between these particles (together with the effect of their proper masses). In μ decay, on the other hand, in which the charged particles also have the same helicity but one is absorbed in the beginning and the other created in the end of the process, the corresponding integrals compensate each other* and the corrections can be calculated.

The divergence in the first case is apparently connected with general difficulties of the four-fermion interaction. However, in the second case we should expect that the major part of the effect is due to the emission of real photons with comparatively low energies, so that the magnitude of the correction is, for the most part, simply determined by the probability for such a decay. It is natural that the emission of quanta will have a comparatively large effect on the angular and energy distribution of the reaction products, but will affect the magnitude of the total probability only weakly. Thus, in the μ — e decay, the Michel parameter, which determines the spectrum of the electrons, is changed by 5% owing to the radiative corrections, whereas the lifetime of the μ meson changes only by 0.5%. The radiative corrections should play a similar role in the case of the two possible types of pion decay.

Recent investigations of the pion decay[10,11] have shown that the experimental value of the ratio of the electron and the muon decay probabilities of the π meson is close to the value predicted by the Feynman-Gell-Mann theory. The theoretical value is, without radiative corrections,[12]

$$R_0 = \frac{(W_{e\nu})_0}{(W_{\mu\nu})_0} = \frac{(m_\pi^2 - m_e^2)^2}{(m_\pi^2 - m_\mu^2)^2} \frac{m_e^2}{m_\mu^2} = 1.282 \cdot 10^{-4}, \qquad (1)$$

where $(W_{e\nu})_0$ and $(W_{\mu\nu})_0$ are the uncorrected probabilities for the electron and muon decay of the π meson, respectively. Radiative corrections to both decay types have been calculated by Berman[6] (also by Kinoshita[8]), who showed that the correction to the ratio (1) is surprisingly large and reaches the value of 14%.

Berman calculated, in the usual manner, the sum of the "radiative corrections" and the "probability for the emission of real soft photons." This quantity depends on the cut-off energy of the quanta or, what is the same thing, on the observed energy loss of the electrons ΔE (the result computed below corresponds to ΔE = 0.25 Mev). It is clear that the correction can be large in this case if the shape of the photon spectrum is very different in the two decays. In this case the cut-off will separate out different parts of the lepton spectrum. The calculations show that this is indeed the reason for the large size of the correction. This circumstance has also been noted by Kinoshita;[8] but he did not give all the formulas.

In the present paper we shall repeat all these calculations; our formula for the correction to the

*If the graphs are drawn such that the decay is described as the transformation of one charged particle into another (μ into e, p into e), the divergent integral occurs when the helicity of the charged particle changes.

decay probability differs somewhat from that of Kinoshita (but the numerical value of our correction is almost the same as Kinoshita's). We shall also give formulas for the photon spectra. The lepton spectrum obtained by us agrees with that calculated by Ioffe and Rudik,[14] Vaks and Ioffe,[15] and Bludman and Ruderman.[16].

The calculations are carried out in the standard manner. The emission of virtual nucleons is neglected, since the corresponding terms contain the nucleon mass in the denominator (see, for example, [17]). Choosing a coordinate system in which the π meson is initially at rest, we have for the radiative decay probability[8]

$$\frac{W_{e\nu\gamma}}{(W_{e\nu})_0} = \frac{\alpha}{\pi}\Big\{b(\mu)\Big[\ln\frac{\lambda}{m_\pi} - \ln(1-\mu^2) - \frac{1}{2}\ln\mu + \frac{3}{4}\Big] - \frac{\mu^2(10-7\mu^2)}{2(1-\mu^2)^2}\ln\mu + \frac{2(1+\mu^2)}{(1-\mu^2)}L(1-\mu^2) + \frac{(15-21\mu^2)}{8(1-\mu^2)}\Big\}, \qquad (2)$$

$$L(x) = \int_0^x \frac{\ln(1-t)}{t}\,dt = -\sum_{k=1}^{\infty}\frac{x^k}{k^2}\quad(|x|\leqslant 1),$$

$$b(\mu) = 2\Big(\frac{1+\mu^2}{1-\mu^2}\ln\mu + 1\Big). \qquad (3)$$

In these formulas λ is the infrared cut-off parameter and μ is the ratio of the lepton and pion masses.

The spectra of the photons and leptons are given by the formulas

$$dW_{e\nu\gamma}(\varepsilon_\gamma) = (W_{e\nu})_0\frac{\alpha}{\pi}\Big\{-\frac{2}{\varepsilon_\gamma} + \frac{(4-5\mu^2-2\varepsilon_\gamma)}{(1-\mu^2)^2} + \frac{\mu^2}{(1-2\varepsilon_\gamma)(1-\mu^2)^2} + \Big[\frac{(1+\mu^2)}{(1-\mu^2)}\frac{1}{\varepsilon_\gamma} - \frac{2(1-\mu^2-2\varepsilon_\gamma)}{(1-\mu^2)^2}\Big]\ln\frac{1-2\varepsilon_\gamma}{\mu^2}\Big\}d\varepsilon_\gamma, \qquad (4)$$

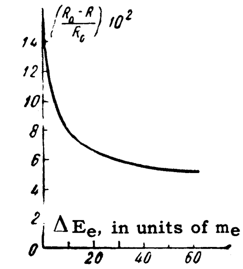
$$dW_{e\nu\gamma}(\varepsilon_e) = (W_{e\nu})_0\frac{\alpha}{\pi}\Big\{\frac{4}{(1-\mu^2)(1+\mu^2-2\varepsilon_e)}\Big[\varepsilon_e\ln\frac{\varepsilon_e+\sqrt{\varepsilon_e^2-\mu^2}}{\varepsilon_e-\sqrt{\varepsilon_e^2-\mu^2}} - 2\sqrt{\varepsilon_e^2-\mu^2}\Big] + \frac{(1+\mu^2-2\varepsilon_e)}{(1-\mu^2)^2} \times \ln\frac{\varepsilon_e+\sqrt{\varepsilon_e^2-\mu^2}-\mu^2}{\varepsilon_e-\sqrt{\varepsilon_e^2-\mu^2}-\mu^2}\Big\}d\varepsilon_e, \qquad (5)$$

where

$$\varepsilon_\gamma = E_\gamma/m_\pi,\quad \varepsilon_e = E_e/m_\pi.$$

Formula (5) agrees with the formula of Bludman and Ruderman.[16]

$\Delta E_e/m_e$		0.5	10	20	30	Total energy region
$\left(\frac{R_0-R}{R_0}\right)\cdot 10^2$	Berman	13.9	7.6	6.1	5.3	3.9[8]
	This work	14.0	7.8	6.5	5.8	3.93



Dependence of the ratio $(R_0-R)/R_0$ on the cut-off parameter.

If we take account of all Feynman graphs with emission of a virtual photon up to order e^2, we obtain for the probability of non-radiative electron (or muon) decay of the π meson

$$W_{e\nu} = (W_{e\nu})_0\Big\{1 + \frac{\alpha}{\pi}\Big[-\frac{3}{2}\ln\frac{L}{m_\pi} - b(\mu)\Big(\ln\frac{\lambda}{m_\pi} - \frac{1}{2}\ln\mu + \frac{3}{4}\Big) + \frac{(3-2\mu^2)}{(1-\mu^2)}\ln\mu - \frac{3}{8}\Big]\Big\}, \qquad (6)$$

where L and λ are the upper and lower limits of the energy of the virtual photons, respectively. The difference between Kinoshita's formula[8] and ours consists in the fact that in the former the term $-3/8$ is replaced by $-1/4$ (after the mass correction).

Summing (2) and (6), we find for the total (radiative + non-radiative) decay probability for the pion

$$\frac{W_{e\nu}+W_{e\nu\gamma}}{(W_{e\nu})_0} = 1 + \frac{\alpha}{\pi}\Big\{-\frac{3}{2}\ln\frac{L}{m_\pi} - b(\mu)\ln(1-\mu^2) + \frac{(6-20\mu^2+11\mu^4)}{2(1-\mu^2)^2}\ln\mu + \frac{2(1+\mu^2)}{(1-\mu^2)}L(1-\mu^2) + \frac{(6-9\mu^2)}{4(1-\mu^2)}\Big\}. \qquad (7)$$

In Kinoshita's work[8] the last term (after the mass correction) is replaced by $(13-19\mu^2)/8(1-\mu^2)$. With the help of formula (7) we can compute the radiative correction to (1).

For the probability of emission of a lepton with energy less than $E_{max} - \Delta E$ we find

$$\frac{W_{e\nu\gamma}(E<E_{max}-\Delta E)}{(W_{e\nu})_0} = \frac{\alpha}{\pi}\Big\{-b(\mu)\Big(\ln\frac{m_\pi}{2\Delta E} + 2\ln(1-\mu^2) - \frac{3}{4}\Big) + \frac{2(1+\mu^2)}{(1-\mu^2)}\Big[L(1-\mu^2) - L\Big(\frac{2}{1-\mu^2}\frac{\Delta E}{m_\pi}\Big)\Big] - \Big[\frac{\mu^2(10-7\mu^2)}{2(1-\mu^2)^2} + \frac{4(1-3\mu^2)}{(1-\mu^2)^3}\frac{\Delta E}{m_\pi}\Big]\ln\mu + \Big[\frac{(15-21\mu^2)}{8(1-\mu^2)} - \frac{4(1+2\mu^2)}{(1-\mu^2)^2}\frac{\Delta E}{m_\pi}\Big]\Big\}. \qquad (8)$$

Formula (8) goes over into Kinoshita's formula[8] if we neglect the terms linear in ΔE.

We can now write down the formula for the correction to the ratio of the probabilities of the electron and muon decays in which the leptons have an energy which differs from the maximal value by no more than the amount ΔE. Substituting the numerical values of the constants, we obtain

$$R(\Delta E) = R_0\left\{1 - (4.647 \cdot 10^{-3})\left[30.12 - 4.611\left(\ln\frac{2\Delta E_e}{m_e} - \frac{2\Delta E_e}{m_\pi}\right) - L\left(\frac{2\Delta E_e}{m_\pi}\right)\right]\right\}. \quad (9)$$

A comparison between our numerical results and those of Berman[6] and Kinoshita[8] is given in the table.

For an illustration, we show the dependence of the ratio on the cut-off parameter ΔE_e in the figure. We note that for $\Delta E_e = 10$ Mev, which is the value assumed in the experiments of reference 11, we have $R = 1.198 \times 10^{-4}$ [the experimental value is $R = (1.18 \pm 0.08) \times 10^{-4}$].

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