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FIELD THEORY WITH NONLOCAL INTERACTION.
I. CONSTRUCTION OF THE UNITARY S MATRIX

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It is found that the impossibility of solving the nonlocal Tomonaga-Schwinger equations and the absence of analogous difficulties in the Lagrangian approach can be explained by the fact that these two approaches correspond to two essentially different nonlocal field theories, and not to two different representations of the same theory. By means of a simple representation found in this paper for the local S matrix in terms of retarded commutators, a unitary S matrix is constructed which corresponds to a nonlocal interaction. The matrix is completely relativistically invariant, and in the limit of local theory goes over into the usual S matrix.

1. INTRODUCTION

FOR a long time there has been interest in a field theory with a nonlocal interaction containing the product of field operators taken at noncoinciding points of space-time. Besides hopes for escape from the difficulties of existing field theory, and for the possibility of treating unrenormalizable interactions, there has recently been a basis for this interest in the expectation of experimental results on tests of quantum electrodynamics at small distances. It is not excluded that these experiments may give a negative result, i.e., that deviations from the predictions of existing theory may be detected. The introduction of a nonlocal interaction is evidently the simplest possibility for describing such deviations.

We shall here be discussing formal nonlocal theory, in which one uses a "rigid" form-factor, which is a prescribed function of the differences of the coordinates. The form of the function can be fixed as an additional condition (if the experiments mentioned earlier give a negative result, some characteristics of the form-factor can be found directly from experiment). Of course such a theory is of a preliminary, and at best phenomenological, nature. In spite of the existence of an extensive literature devoted to nonlocal theory, however, there is no definite answer to the question of the possibility of constructing even such a preliminary theory.

There are a number of problems that arise in the statement of the nonlocal problem with rigid form-factor; the most important of them will now be mentioned.

First is the problem of the freedom of the theory from mathematical contradictions.[1] By this we mean the possibility of satisfying Bloch's consistency condition. In this connection it must be noted that from the very beginning there is some inconsistency in the theory in question. This is that in a formally relativistically invariant theory there are inevitable motions at small distances with speeds larger than the speed of light. In itself this is not in contradiction with experiment, but the important question is whether or not this formal inconsistency will develop into a mathematical contradiction. The next problem, in order of importance, bears on the macroscopically causal character of the theory.[2] The existence of signals faster than light unavoidably brings with it violations of causality. The question is whether one can localize this violation in a small space-time region, and thus make it in a reasonable sense unobservable. A related question is that of the possibility of constructing a dispersion apparatus in nonlocal field theory.

Next, the requirement that the S matrix be unitary is extremely important. Examples have been given in the literature of nonlocal theories[3] in which this requirement was violated.

Finally, there are arguments[4] about the possible ineffectiveness of the form-factor in higher orders of perturbation theory. It is necessary that the form-factor lead to an effective removal of the divergences.

The remaining problems—renormalization, gauge invariance in electrodynamics, and so on—are not of such central significance.

In a series of papers, of which this is the first, an attempt will be made to show that the difficulties associated with the problems just listed can quite definitely be overcome and are not so deep as is generally supposed. In the present paper we consider the problems of consistency and unitarity, which can be studied without taking a concrete form for the form-factor.

3. THE MATHEMATICAL CONSISTENCY OF THE NONLOCAL THEORY

For definiteness we shall consider the pseudoscalar theory with pseudoscalar interaction; in the local case the interaction Hamiltonian is[1)]

$$H = g\int d\,1\,(\bar{\psi}(1)\gamma_5\psi(1))\,\varphi(1) = -L. \tag{1}$$

The corresponding interaction Lagrangian differs from this expression only in sign.

Nonlocal generalizations of the theory can be made in two ways—by introducing the form-factor either in the Hamiltonian

$$H = g\int d\,(1)\,(\bar{\psi}\,(1')\,\gamma_5\psi\,(1''))\,\varphi\,(1'''), \tag{2}$$

or in the interaction Lagrangian

$$\mathbf{L} = -\frac{g}{2}\int d\,(1)\,(\bar{\boldsymbol{\psi}}\,(1')\,\gamma_5\boldsymbol{\psi}\,(1'')\,\boldsymbol{\varphi}\,(1''') + \text{Herm. adj.} \tag{3}$$

Here $d(1) = d1'd1''d1'''F(1', 1'', 1''')$; F is the form-factor, which satisfies the condition

$$F\,(1', 1'', 1''') = F^*\,(1'', 1', 1'''),$$

so as to assure that the expression (2) is Hermitian, and which goes over into $\delta(1' - 1'')\,\delta(1' - 1''')$ in the limit of local theory. Here bold-face letters denote operators in the Heisenberg representation.

In the nonlocal theory based on the Hamiltonian (2) we immediately encounter a violation of the well known condition of consistency of the Tomonaga-Schwinger equation

$$[\mathscr{H}(1),\ \mathscr{H}(2)] = 0, \tag{4}$$

where the points 1 and 2 are separated by a spacelike interval. We recall that Eq. (4) is the condition for the existence of the S matrix $S(\sigma)$ as a definite functional of the spacelike surface σ. Starting from the equation

$$i\delta S\,(\sigma)/\delta\sigma\,(1) = \mathscr{H}\,(1 \mid \sigma)\,S\,(\sigma) \tag{5}$$

and equating to each other the derivatives $\delta^2 S/\delta\sigma(1)\,\delta\sigma(2)$ and $\delta^2 S/\delta\sigma(2)\,\delta\sigma(1)$, we arrive at the consistency condition in general form[5]

$$[\mathscr{H}\,(1 \mid \sigma),\ \mathscr{H}\,(2 \mid \sigma)] = i\left\{\frac{\delta\mathscr{H}\,(2 \mid \sigma)}{\delta\sigma\,(1)} - \frac{\delta\mathscr{H}\,(1 \mid \sigma)}{\delta\sigma\,(2)}\right\}. \tag{6}$$

Here and in the preceding equations $\mathcal{H}$ is the interaction Hamiltonian density. If this quantity does not depend on σ, we come back to the condition (4).[2)]

Furthermore one can also not avoid difficulties in the framework of the usual one-time formalism. Although the corresponding solution does exist, its relativistic invariance is violated in a fundamental way. The point is that the S matrix involves retarded commutators of the type

$$\theta\,(1 - 2)\,[\mathscr{H}\,(1),\ \mathscr{H}\,(2)],$$

which do not have invariant meaning because the commutator does not vanish outside the light cone (the function θ is zero when point 1 is earlier than point 2, and unity in the opposite case). Thus a nonlocal theory based on the use of the Hamiltonian (2) cannot be constructed at all.

On the other hand, a direct treatment of the Heisenberg equations that are based on the Lagrangian (3) does not encounter this sort of difficulties. Corresponding examples relating to the classical theory are well known.[3)] In quantum theory the solution by perturbation theory is always possible, and no violations of relativistic invariance occur in it.[4] In individual cases it may also be possible to find exact solutions of the field equations, which are also free from difficulties.[4)]

It is important to note that although the Heisenberg operators obtained with such a statement of the problem do not commute outside the light cone, this fact has no bearing on the difficulties under discussion, but indicates only a violation of microcausality.

The situation we have presented is often interpreted in the following rather unsatisfactory way. It is supposed that the two approaches that have been indicated correspond to the treatment of the same problem in two different representations—the interaction representation and the Heisenberg

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1)In the equations that follow space-time arguments are denoted by numbers.

2)We emphasize that Eq. (6) is a direct consequence of Eq. (5). Therefore one can construct an infinite number of nonlocal Hamiltonians which satisfy the condition (6); it suffices to take a unitary operator $S(\sigma)$ and find $\mathcal{H}$ from Eq. (5). In this process, however, the causality condition is in general sharply violated. In the general case this condition is in no way connected with the condition of consistency. Only for the simplest Hamiltonian of the type (2) are the two conditions equivalent.

3)We note that difficulties also occur in the classical theory based on a Hamiltonian of the type (2) (cf. reference 6).

4)This situation occurs in the nonlocal analogue of the relativistic model which has been considered by Smolyanskii and the writer.[7]

representation. The presence of the difficulties in one representation and their absence in the other is explained either by a complete lack of equivalence of the representations, or by the concealed nature of difficulties in the second approach, based on the Lagrangian (3).

We must, however, pay attention to one essentially trivial fact, in the light of which the situation that has been described becomes completely clear. The point is that the nonlocal Hamiltonian (2) and Lagrangian (3) by no means correspond to the same problem. In fact, the Lagrangian (3) actually depends on higher derivatives of the fields, as one readily sees on expanding the integrand in Eq. (3) in series around one of the argument values. Along with this it is well known that even in a theory with the Lagrangian containing just the first derivative of the field φ the corresponding Hamiltonian necessarily includes a contact four-fermion term. In the present case there are derivatives of all orders, and therefore the Hamiltonian corresponding to Eq. (3) must have an extraordinarily complex structure with respect to both the field operators and the coupling constants (cf. papers by Katayama[8] and Hayashi[3]). In any case this Hamiltonian has nothing in common with Eq. (2).[5] The essential point is that the Hamiltonian obtained from Eq. (3) automatically satisfies the consistency condition (6) (it actually depends on the surface σ; for details see below).

Thus there are two essentially different nonlocal theories. The first, based on the use of the Hamiltonian (2), suffers from serious difficulties and is condemned to failure from the beginning.[6] The second, based on the Lagrangian (3), is free from such difficulties and calls for further study in the light of the other problems listed in the Introduction.

3. THE HEISENBERG FIELD EQUATIONS

When we consider the second type of theory the question of the unitarity of the S matrix becomes most acute.[3] We shall give some details of the corresponding calculations, in order on one hand to give an illustration of the assertions that have been made, and on the other, to introduce a number of concepts needed for what follows.

Considering for simplicity only the equation for the field

$$(\Box-\mu^2)\boldsymbol{\varphi}(1)=\mathrm{j}(1),$$

let us use the Yang-Feldman method of quantization. We have

$$\boldsymbol{\varphi}(1)=\varphi(1)-\int d2\theta(1-2)D(1-2)\,\mathrm{j}(2), \tag{7}$$

where $\varphi(1)\equiv\varphi_{\text{in}}(1)$ is a solution of the free equation. We require that this operator shall also satisfy the free commutation rules $[\varphi(1),\varphi(2)]=iD(1-2)$; by this we uniquely determine also the commutation rules of the operator $\boldsymbol{\varphi}$.

Let us then introduce the extremely convenient quantity[7]

$$\varphi(1,\sigma)=\varphi(1)-\int d2\theta(\sigma,\ 2)\ D(1-2)\,\mathrm{j}(2), \tag{8}$$

which coincides with φ_{in} for $\sigma\to-\infty$, with φ_{out} for $\sigma\to+\infty$, and with the Heisenberg operator $\boldsymbol{\varphi}(1)$ when the point 1 lies on σ (the symbol for this is $1|\sigma$). The operator $\varphi(1,\sigma)$ satisfies the free equation, but its commutation rules are by no means the free commutation rules [the actual commutation rules are automatically determined from Eq. (8)]. Therefore we separate out from $\varphi(1,\sigma)$ the part $\varphi_0(1,\sigma)$ that satisfies the free commutation rules:

$$\varphi(1,\sigma)=\varphi_0(1,\sigma)+\chi(1,\sigma),$$

where by definition $\chi(1,-\infty)=0$. Then we can write

$$\varphi_0(1,\sigma)=U^{-1}(\sigma)\,\varphi(1)\,U(\sigma),$$

where U is a unitary operator.

It is clear from this that the connection between the operators in the Heisenberg and interaction representations is not unitary:

$$\boldsymbol{\varphi}(1)=\varphi(1|\sigma)=U^{-1}(\sigma)\,\varphi(1)\,U(\sigma)+\chi(1|\sigma). \tag{9}$$

It is precisely for this reason that the Heisenberg operators do not commute outside the light cone.

By means of $U(\sigma)$ we can construct the interaction Hamiltonian[8]

$$\mathcal{H}(1|\sigma)=i\frac{\delta U}{\delta\sigma(1)}U^{+},$$

[5]The interaction Hamiltonian and Lagrangian differ in sign only in a local theory without higher derivatives. Already in local renormalized theory the Hamiltonian has a complex structure, owing to the presence of higher derivatives associated with the counter-terms.

[6]The relativistic Lee model relates to a theory of just this type (see reference 9).

[7]The quantity $\theta(\sigma,1)$ is equal to unity if the point 1 is earlier than σ, and to zero in the opposite case. These cases differ in the sign of the normal dropped from 1 onto σ. Because of the timelike nature of this normal the definition has an invariant meaning.

[8]In a paper by Katayama[8] it is shown that the canonical formalism also leads to the same expression for the Hamiltonian, if this formalism is modified so as to apply to theories with higher derivatives.

which automatically satisfies the consistency condition (6) (cf. footnote 2)). We can find the explicit forms of $\mathcal{H}$ and χ in the form of perturbation-theory series by expanding Eqs. (7) and (8) in power series in g. In particular, in the lowest order of perturbation theory we have

$$\mathcal{H}(1 \mid \sigma) = -\mathcal{L}(1),$$
$$\chi(1, \sigma) = \int d2d3\,(\theta(\sigma, 2) - \theta(\sigma, 3))\,D(1-3)\,\frac{\delta\mathcal{L}(2)}{\delta\varphi(3)}$$

Here and hereafter $\mathcal{L}(1)$ is the interaction Lagrangian density expressed in terms of the free operators

$$\mathcal{L}(1) = -g\int d[1]\,(\bar{\psi}(1')\,\gamma_5\psi(1''))\,\varphi(1'''),$$
$$d[1] \equiv d(1)\,\tfrac{1}{3}\,(\delta(1-1') + \delta(1-1'') + \delta(1-1''')). \tag{10}$$

The second-order term in the expansion of $\mathcal{H}$, which is given at the end of the paper [Eq. (19)], depends explicitly on the surface σ; when it is substituted in the right member of Eq. (6) it exactly compensates the nonvanishing left member of that condition, and so on. As for the operator χ, it can be seen that it vanishes both in the limit of local theory and also in the limit $\sigma \to +\infty$.

This last fact is of importance in the solution of the problem of the unitarity of the S matrix. If the condition

$$\chi(1, +\infty) = 0, \tag{11}$$

is satisfied we can write

$$\varphi_{out}(1) = S^{-1}\varphi(1)\,S,$$

where $S = U(\infty)$ plays the role of the unitary S matrix of the theory. It is important to emphasize that the field equations in themselves by no means guarantee that the condition (11) is satisfied. In some types of field theory this condition is clearly violated in the fourth order of perturbation theory. Although, as Medvedev and Hayashi[3] have shown, this defect can be eliminated by introducing a charge-symmetrical Lagrangian, the situation in the higher orders of perturbation theory remains unclear.

Let us now return to the original equation (7) and formulate the principle for constructing its solution.[4] In local theory the solution can be represented in the form (cf. e.g., [10])

$$\varphi(1) = S^{+}T(\varphi(1)\,S) = \sum_{n=0}^{\infty} R_n(\varphi(1)), \tag{12}$$

where

$$R_n(\varphi(1)) = i^n\int d2\ldots d(n+1)\,\theta(1-2)\,\theta(2-3)\ldots\theta$$
$$\times(n-(n+1))\,[\ldots[[\varphi(1)\,\mathcal{L}(2)]\,\mathcal{L}(3)]\ldots\mathcal{L}(n+1)] \tag{13}$$

is the retarded commutator integrated over the coordinates (cf. [11]).

An analysis of Eq. (7) shows that in nonlocal theory the retarded commutator R_n must be replaced by a quantity $\tilde{R}_n$ according to the following rules established by Bloch[4]:9)

a) One expands the complex commutator that appears in Eq. (13) according to the rules for commutators of products; one puts it in the form of a sum of terms, each of which is a product of "elementary" commutators (or anticommutators, if appropriate) of operators φ and ψ and of these operators themselves.

As an example we write out two characteristic terms which appear in R_3:

$$g^3\int d[2]\,d[3]\,d[4]\,D(1-2''')\,\{D(3'''-4''')\,(4', 4'')$$
$$\times(2', S(2''-3'), 3'') + \varphi(3''')\,\varphi(4''')$$
$$\times(3', S(3''-2'), S(2''-4'), 4'') + \ldots\}. \tag{14}$$

Here and in what follows

$$\{\psi(1)\,\bar{\psi}(2)\} = iS(1-2),\quad (n, m) \equiv (\bar{\psi}(n)\,\gamma_5\psi(m)),$$
$$(n', S(n''-m)\ldots S(k-l'), l'') \equiv (\bar{\psi}(n')\,\gamma_5 S$$
$$\times(n''-m)\,\gamma_5\ldots\gamma_5 S(k-l')\,\gamma_5\psi(l'')).$$

b) One multiplies each elementary commutator by the function θ of the difference of the corresponding arguments, the argument subtracted being that with the larger index.

The respective factors for the terms written out in Eq. (14) are:

$$\theta(1-2''')\,\theta(2''-3')\,\theta(3'''-4'''),$$
$$\theta(1-2''')\,\theta(2'-3'')\,\theta(2''-4').$$

This rule assures the relativistic invariance of the result in the sense indicated above (see Sec. 2).

c) One divides each term that occurs in $\tilde{R}_n$ by a certain integer k which is defined below.

This rule is necessary to get the right local limit for $\tilde{R}_n$ when we let $\int d[l] \to 1$, $l', l'', l''' \to l$. In the local expression (13) all of the points 1, 2, ... n + 1 are ordered in time. But the local limit of $\tilde{R}_n$ also contains unordered points, for example, 3 and 4 in the second of the terms in Eq. (14). If we multiply this term by the sum $\theta(3-4) + \theta(4-3)$, which is equal to unity, we get two ordered terms, which, as is easily seen, make equal contributions to $\tilde{R}_n$. To get the right limit it is necessary to set k = 2. In the first of the terms in Eq. (14) there are no unordered terms and k = 1.

In the general case one must find all pairs of

9)The rule c) formulated here differs from the corresponding rule of Bloch in simplicity and convenience.

unordered points, insert for each such pair a sum of θ functions, and determine the number k as the number of terms so obtained which are different from zero. For example, in a term containing $\theta(1-2)\,\theta(2-3)\,\theta(2-4)\,\theta(3-5)$ the unordered pairs are (3, 4) and (4, 5). Multiplying this term by $[\theta(3-4)+\theta(4-3)][\theta(4-5)+(5-4)]$, we get k = 3, since on being multiplied by $\theta(3-5)$ the quantity $\theta(5-4)\,\theta(4-3)$ vanishes identically.

The main properties of a nonlocal retarded commutator constructed in this way—relativistic invariance and the correct local limit—are also preserved for commutators of more complex types, for example, $\tilde{R}_n(\mathscr{L}(1))$. In the next section we shall use the rules that have been formulated to construct the unitary S matrix.

4. CONSTRUCTION OF THE UNITARY S MATRIX

It has already been stated that although the solution of the nonlocal problem based on the Lagrangian (3) is always possible in principle, the question of the unitarity of the S matrix obtained in this way remains essentially open. The difficulties of investigating this incline one to renounce the dynamical principle altogether and go over to a direct construction of the S matrix in a form such that there is no doubt of its unitarity. We shall proceed along the line of a direct generalization of the S matrix of local theory, with the requirement that the equation to be found be relativistically invariant and provide the possibility of an inverse transition to the limit of the original local expression.

It has been shown above how one must carry out the corresponding generalization of the retarded commutator. Therefore the present problem can be regarded as solved if one can succeed in expressing the local S matrix in terms of retarded commutators. Such a representation of the S matrix is always possible. Nishijima[11] has found recurrence relations connecting T products with retarded commutators. There also exists a direct representation of the S matrix in terms of retarded commutators. To derive it we shall start from the usual expression for the local S matrix

$$S = T\exp\left\{i\int d1\,\mathscr{L}(1)\right\}$$

and regard $\mathscr{L}$ as proportional to the coupling constant g.[10]

[10] In the general case one can multiply $\mathfrak{L}$ by a parameter which plays the role of g in the subsequent calculations, and which must be set equal to unity after the calculations are completed.

Differentiating this relation with respect to g, we find

$$g\frac{dS}{dg} = i\int d1T\,(\mathscr{L}(1)\,S) = iS\int d1\mathscr{L}_{\mathrm{H}}(1),$$

where, in complete analogy with Eq. (13), the Heisenberg-representation Lagrangian $\mathscr{L}_H$ can be expressed in the form

$$\int d1\mathscr{L}_{\mathrm{H}}(1) = \int d1\sum_{n=0}^{\infty} R_n(\mathscr{L}(1)) \equiv \sum_{n=0}^{\infty}\mathfrak{R}_n.$$

Integrating the resulting equation, we find finally

$$S = \tilde{T}_g\exp\left\{i\int_0^g\frac{dg}{g}\sum_{n=0}^{\infty}\mathfrak{R}_n(g)\right\}, \tag{15}$$

where $\tilde{T}_g$ is the antichronological ordering with respect to charge (which must increase from left to right), and $\mathfrak{R}_n$ is regarded as a function of the charge.

This expression has two important properties: it contains only retarded commutators, and instead of time ordering there is an ordering with respect to charge. This enables us to make a direct extension of Eq. (15) to the nonlocal theory,

$$S = \tilde{T}_g\exp\left\{i\int_0^g\frac{dg}{g}\sum_{n=0}^{\infty}\tilde{\mathfrak{R}}_n(g)\right\}, \tag{16}$$

where $\tilde{\mathfrak{R}}_n$ is the retarded commutator of the nonlocal Lagrangians of Eq. (10), calculated according to the Bloch rules.

We have still to convince ourselves that the expression we have obtained is unitary. It is known that the condition for unitarity of any T exponential is that the exponent be antihermitian. In local theory the operator $\mathfrak{R}_n$ is Hermitian by construction. It is easy to see that $\tilde{\mathfrak{R}}_n$ also has this property. In fact, a complicated commutator occurring in $\mathfrak{R}_n$ can always be broken up into pairs of terms whose components are each other's Hermitian adjoints. These components include elementary commutators with identical arguments, and therefore have a common set of θ functions and identical values of k. Therefore the transition from $\mathfrak{R}_n$ to $\tilde{\mathfrak{R}}_n$ does not involve a loss of the Hermitian property.[11]

We give the expansion of Eq. (16) to third order in g:

$$S = 1 + i\tilde{\mathfrak{R}}_0 - {}^1/_2\,((\tilde{\mathfrak{R}}_0)^2 - i\tilde{\mathfrak{R}}_1) - {}^1/_6\,i\,((\tilde{\mathfrak{R}}_0)^3 - 2i\tilde{\mathfrak{R}}_0\tilde{\mathfrak{R}}_1 - i\tilde{\mathfrak{R}}_1\tilde{\mathfrak{R}}_0 - 2\tilde{\mathfrak{R}}_2) + \dots \tag{17}$$

The retarded commutators themselves have the forms (the notations are the same as in Eq. (14))

[11] We note that, independently of this fact, we could have taken the Hermitian part of $\tilde{\mathfrak{R}}_n$ in the exponent of Eq. (16).

$$\widetilde{\mathfrak{R}}_0 = -g\int d(1)(1', 1'')\varphi(1'''),$$

$$\widetilde{\mathfrak{R}}_1 = -\frac{g^2}{2}\int d(1)\,d(2)\,\theta(1'''-2''')D(1'''-2''')(1', 1'')(2', 2'') + \theta(1''-2')(1', S(1''-2'), 2'')\{\varphi(1'''), \varphi(2''')\} + \text{Herm. adj.}$$

$$\widetilde{\mathfrak{R}}_2 = -\frac{g^3}{2}\int d(1)\,d(2)\,d(3)\,D(1'''-2''')\varphi(3''') \times (3\theta(1'''-2''')\theta(2''-3') - \theta(1'''-2''') - 2\theta(2''-3') + 1)\{(1', 1''), (2', S(2''-3')3'')\} + {}^1/_4(3\theta(1''-2') - 1)\theta \times (2''-3')\{\{\varphi(1'''), \varphi(2''')\}\varphi(3''')\} \times (1', S(1''-2'), S(2''-3')3'') + \text{Herm. adj.} \quad (18)$$

As can be seen particularly clearly from the expression for $\widetilde{\mathfrak{R}}_2$, the main difference between $\mathfrak{R}$ and $\widetilde{\mathfrak{R}}$ is that the order of the processes of smearing out and of ordering is changed (the functions θ go under the signs of integration over the internal coordinates). This fact is in qualitative agreement with the well known theorem of Takahashi and Umezawa (cf. e.g., [5]) on the construction of the S matrix in a theory with higher derivatives. A detailed analysis of the expression we have obtained for the S matrix and, in particular, a formulation of the corresponding Feynman rules will be given later in this series.

We note that we would have arrived at this same expression for the S matrix, but by a more complicated way, if we had used the approach of Stueckelberg and Bogolyubov,[2,12] and determined the Hermitian part of the S matrix from the condition of unitarity and the antihermitian part from considerations like those used in the derivation of Eq. (16). The axiomatic approach with the use of the macrocausality condition to determine the antihermitian part of the S matrix has also been used earlier,[13] but the arguments in question were not fully developed. A paper by Stueckelberg and Wanders[2] relating to this same group of questions will be discussed in the next paper of this series, which is devoted to the problem of causality.

In conclusion we note that formally we can set up an effective Hamiltonian to correspond to the S matrix (16). For this purpose one must construct an S matrix with a variable upper limit, introducing into the integrand $\widetilde{\mathfrak{R}}_n$ the factor $\theta(\sigma, 1)\,\theta(\sigma, 2)\ldots\theta(\sigma, n+1)$, and use the relation (5). As the result we get to the second order in g

$$\mathcal{H}(1\,|\,\sigma) = g\int d[1](1', 1'')\varphi(1''') - \frac{g^2}{2}\int d[1]\,d(2)\,\theta(\sigma, 2)\,\theta \times (2'''-1''')D(1'''-2''')(1', 1'')(2', 2'') + \theta(2'-1'') \times (1', S(1''-2'), 2'')\{\varphi(1'''), \varphi(2''')\} + \text{Herm. adj.} \quad (19)$$

The method described in Sec. 3 leads to this same expression. We may suppose that this coincidence occurs in all orders of perturbation theory, and also the S matrix (16) itself must be equal to the operator $U(\infty)$. We emphasize, however, that the identification of the S matrix of the theory based on the nonlocal Lagrangian with this operator is possible only if the condition (11) is satisfied. Therefore the approach based not on the dynamical principle but on the direct introduction of the S matrix has definite advantages, primarily from the point of view of the condition for unitarity. It is this approach that is used in our further work.

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