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SCATTERING OF GAMMA RAYS IN LIQUID He^3

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Detailed formulas are derived for the frequency and angular distributions of the scattered radiation. Numerical estimates indicate a quite substantial magnitude for the effect, which makes possible its use in determining the velocity of "zero sound" in He^3.

SOME time ago, we investigated the problem of the scattering of light in liquid He^3.[1] One of the objects of this work was to examine the possibility of using Rayleigh scattering of light to determine the velocity of the so-called "zero," or high-frequency, sound ($\omega_s\tau \gg 1$, where τ is the time between collisions of the quasi-particles). The occurrence of this form of sound was predicted by Landau on the basis of his theory of a Fermi liquid.[2,3] Calculations[1] showed that the scattering of visible light is extremely small, and that detection of this effect would evidently lie beyond the bounds of experimental feasibility. However, since the scattering increases rapidly with frequency ($\sim \omega^4$ or ω^5), it was clear, even then, that an increase in the frequency of the electromagnetic radiation might significantly improve the conditions for observation.

Now, thanks to the discovery of the Mössbauer effect and the possibility it provides for the measurement of relatively insignificant variations in the energy of γ quanta, one may raise the question of the feasibility of determining the velocity of zero sound in He^3 using Rayleigh scattering of γ quanta. As we shall see below, this allows the effect to be enhanced by approximately five orders of magnitude, as compared with visible light.

Inasmuch as the change in the wave vector of the γ quantum equals the wave vector of the sound quantum, it is the angular range for which this latter quantity is sufficiently small which interests us:

$$q = 2\,(\omega/c)\,\sin\,(\theta/2) \ll p_0/\hbar, \tag{1}$$

where p_0 is the Fermi boundary momentum. Only in this region does the concept of sound have meaning. The quantity $p_0/\hbar$ is of the same order as the reciprocals of the interatomic distances. To fulfill condition (1), therefore, we may employ the formulas already derived,[1] except that for the dielectric constant D we must use the expression $1 - 4\pi N_e e^2/m_e\omega^2$, where N_e and m_e are the number of electrons per unit volume and their mass, respectively, while ω is the frequency of the γ quanta. The derivative of D with respect to the density of atoms, $\partial D/\partial N$, clearly equals $-2\pi e^2/m_e\omega^2$, and in consequence, the coefficient of ω^4 becomes simplified while the extinction coefficient dh ceases to depend upon the frequency of the incident radiation (or, for a given q and $\Delta\omega$, upon the change in frequency of the γ quantum).

It is especially desirable to obtain the entire dispersion curve from ordinary ($\omega_s\tau \ll 1$) to zero sound ($\omega_s\tau \gg 1$). In this case the final equation (23) of[1] is sufficient, since it was derived under the assumption that $\tau\Delta\omega \gg 1$.

In view of this fact, we have made several changes in the derivations given previously.[1] In place of the form used in that paper for the collision integral, $I(n) = -\delta n/\tau$, where δn is the change in the distribution function, we have taken it in the form

$$I\,(n) = -\frac{1}{\tau}\Big(\delta n - \int \delta n \frac{d\Omega_1}{4\pi} - 3\cos\theta \int \delta n \cos\theta_1 \frac{d\Omega_1}{4\pi}\Big). \tag{2}$$

This form for the collision integral insures that the conservation laws for momentum and particle number will be fulfilled, and makes it possible to go over to the hydrodynamic approximation (i.e., the case $\omega_s\tau \ll 1$).

We have already used this kinetic equation to investigate the dispersion of sound.[4] In addition it is assumed, as in[4], that the function $f(\chi)$ introduced in Landau's theory,[2] includes not only the zeroth, but also the first harmonic: $f = f_0 + f_1 \cos\chi$. The rest of the calculations corresponded completely to those performed in[1].

As a result, the extinction coefficient turns out to be

$$dh = \left(\frac{e^2}{m_e c^2}\right)^2 \frac{p_0 m^*}{\pi^2 \hbar^3} \hbar\Delta\omega n(\Delta\omega) \frac{(1+F_1/3)^2}{\tau (qv)^2} \xi^{-2} \frac{(\xi^* w^* - \xi w)/(\xi - \xi^*) - |w|^2 (3|\xi|^2+1)}{\left|\left(1+\frac{1}{\xi\sigma}\right)\left(1+\frac{F_1}{3}\right) - w\left[\left(F_0 - \frac{1}{\xi\sigma}\right)\left(1+\frac{F_1}{3}\right) + \xi^2\left(F_1 - \frac{3}{\xi\sigma}\right)\left(1+\frac{1}{\xi\sigma}\right)\right]\right|^2}$$

$$\times \frac{1}{2}(1+\cos^2\theta)\frac{d\Omega}{4\pi} d\Delta\omega;$$

$$n(\Delta\omega) = \left[\exp\frac{\hbar\Delta\omega}{T} - 1\right]^{-1}, \quad \xi = \frac{i\Delta\omega\tau - 1}{iqv\tau}, \quad \sigma = iqv\tau,$$

$$v = p_0/m^*, \quad w = \frac{\xi}{2}\ln\frac{\xi+1}{\xi-1} - 1, \quad F_0 = f_0 p_0 m^*/\pi^2\hbar^3; \qquad (3)$$

m^* is the effective mass of an excitation in He^3,

$$m^* = m_{He^3}(1 + F_1/3).$$

In the limit $qv\tau \ll 1$, $\Delta\omega\tau \ll 1$, we obtain from Eq. (3)

$$dh = \frac{4}{45}\left(\frac{e^2}{m_e c^2}\right)^2 \frac{p_0 m^*}{\pi^2\hbar^3}\hbar\Delta\omega n(\Delta\omega)$$

$$\times \frac{(1+F_1/3)^2 \tau (qv)^4}{[\Delta\omega^2 - (u_1 q)^2]^2 + [{}^4/_{15}(1+F_1/3)\Delta\omega\tau q^2 v^2]^2}$$

$$\times \frac{1}{2}(1+\cos^2\theta)\frac{d\Omega}{4\pi} d\Delta\omega;$$

$$u_1 = v\,[{}^1/_3(1+F_0)(1+F_1/3)]^{1/2}, \qquad (4)$$

u_1 is the velocity of sound in He^3 (see[2]). It can readily be seen from Eq. (4) that the frequency distribution corresponds to two narrow Rayleigh satellites whose frequencies are displaced relative to that of the fundamental line by $\Delta\omega = \pm u_1 q$. The absence of a central line corresponding to the entropy fluctuations is due to our having neglected effects of order $1 - c_p/c_V$ in choosing a collision integral of the form (2). Letting $\tau \to 0$, we obtain from (4)

$$an = \frac{1}{8}\left(\frac{e^2}{m_e c^2}\right)^2 \frac{p_0 m^*}{\pi^2\hbar^3}\hbar\Delta\omega n(\Delta\omega)$$

$$\times [\delta(\Delta\omega - u_1 q) + \delta(\Delta\omega + u_1 q)]$$

$$\times (1+F_0)^{-1}\frac{1}{2}(1+\cos^2\theta)\, d\Omega d\Delta\omega. \qquad (5)$$

In the opposite limit, $qv\tau \gg 1$, we obtain two narrow satellites displaced from the fundamental line by $\Delta\omega = \pm u_2 q$, where u_2 is the zero sound velocity, and a central plateau for $|\Delta\omega| < qv$. For the satellites we have

$$dh_1 = \left(\frac{e^2}{m_e c^2}\right)^2 \frac{p_0 m^*}{\pi^2\hbar^3}\hbar\Delta\omega n(\Delta\omega)\frac{w^2(s)s^2}{\tau}\left(1+\frac{F_1}{3}\right)^2$$

$$\times \frac{A(s)}{B^2(s)}\left\{(|\Delta\omega| - u_2 q)^2\right.$$

$$\left. + \frac{1}{\tau^2}\left[\frac{A(s)}{B(s)} s^2\left(1+\frac{F_1}{3}\right)\right]^2\right\}^{-1}\frac{1}{2}(1+\cos^2\theta)\frac{d\Omega}{4\pi}d\Delta\omega, \qquad (6)$$

where

$$A(s) = \frac{1}{s^2-1} - \left(\frac{w(s)+1}{s}\right)^2 - 3w^2(s),$$

$$B(s) = \left(\frac{1}{s^2-1} - w(s)\right)\left(1+\frac{F_1}{3}\right) - 2w^2(s)s^2 F_1,$$

$$s = \frac{u_2}{v}, \quad w(s) = \frac{s}{2}\ln\frac{s+1}{s-1} - 1,$$

while the quantity s satisfies the equation

$$w(s) = [F_0 + F_1 s^2/(1+F_1/3)]^{-1}. \qquad (7)$$

In the limit as $\tau \to \infty$ we obtain from (6)

$$dh_1 = \frac{1}{4}\left(\frac{e^2}{m_e c^2}\right)^2 \frac{p_0 m^*}{\pi^2\hbar^3}\hbar\Delta\omega n(\Delta\omega)\left(1+\frac{F_1}{3}\right)w^2(s)$$

$$\times\left[\frac{1}{s^2-1} - w(s)\left(1+\frac{F_1}{3}\right) - 2w^2(s)s^2F_1\right]^{-1}$$

$$\times [\delta(\Delta\omega - u_2 q)$$

$$+ \delta(\Delta\omega + u_2 q)]\frac{1}{2}(1+\cos^2\theta)\, d\Omega d\Delta\omega. \qquad (8)$$

The central plateau depends little upon τ for $qv\tau \gg 1$. We shall give, therefore, only the expression for the limit $\tau \to \infty$:

$$dh_2 = \frac{1}{8}\left(\frac{e^2}{m_e c^2}\right)^2 \frac{p_0 m^*}{\pi^2\hbar^3}\hbar\Delta\omega n(\Delta\omega)\left(1+\frac{F_1}{3}\right)\frac{\Theta(qv - |\Delta\omega|)}{qv}$$

$$\times\left\{\left[1+\frac{F_1}{3} - \tilde{w}(\xi)\left(F_0\left(1+\frac{F_1}{3}\right)+\xi^2F_1\right)\right]^2\right.$$

$$\left. + \frac{\xi^2\pi^2}{4}\left[F_0\left(1+\frac{F_1}{3}\right)+\xi^2F_1\right]^2\right\}^{-1}\frac{1}{2}(1+\cos^2\theta)\, d\Omega d\Delta\omega, \qquad (9)$$

where

$$\xi = \frac{\Delta\omega}{qv}, \quad \tilde{w} = \frac{\xi}{2}\ln\frac{1+\xi}{1-\xi} - 1, \quad \Theta(x) = \begin{cases}1 & x>0\\ 0 & x<0\end{cases}.$$

It should be pointed out that whether one or the other of the limiting cases occurs depends solely upon the relation between qv and $1/\tau$; i.e., upon the scattering angle of the light. This is clear in the case $qv\tau \ll 1$, since $u_1 \sim v$ and $\Delta\omega = qu_1 \sim qv \ll 1/\tau$. For $qv\tau \gg 1$, however, both $\tau\Delta\omega \gg 1$ (for the satellites, since $u_2 \sim v$, and for the edge of the Doppler plateau) and $\tau\Delta\omega \ll 1$ are possible. It can be shown with the aid of Eq. (3) that the case $qv\tau \gg 1$, $\tau\Delta\omega \ll 1$ is in actuality not a special one, but is described by Eq. (9), obtained under the assumption that $\Delta\omega \sim qv \gg 1/\tau$.

Let us now make some numerical estimates. According to the most recent specific heat measurements on He^3 at low temperatures [5] the ratio of the effective mass to the mass of the He^3 atom $m^*/m = 2$. The remaining parameters (see [6]) are: $p_0/\hbar = 0.76 \times 10^8$ cm^{-1}, and $u_1 = 183$ m/sec, whence $F_0 = 6.95$, $F_1 = 3$, $v = 79.5$ m/sec, $s = u_2/v = 2.45$, $u_2 = 195$ m/sec, and $w(s) = 0.0625$.

For observation of the satellites it is essential that condition (1) be fulfilled. Inasmuch as we are dealing with small angles, this condition may be written in the form

$$\theta \ll \theta_1 = cp_0/\hbar\omega. \tag{10}$$

For $\hbar\omega \sim 10$ kev this yields $\theta_1 \sim 0.15 \approx 9°$.

Further, the degree of "quantization" of the process is of interest. This may be established by comparing $\hbar\Delta\omega$ with T. For the acoustical satellites quantum effects begin to play a part for angles exceeding

$$\theta_2 \sim cT/\hbar\omega u = 10^{-2}T \text{ (T in degrees)}. \tag{11}$$

It may therefore be presumed that a purely quantum situation prevails for the acoustical satellites at temperatures below 0.1° K and γ-quantum energies greater than 10 kev. It follows from this that only the Stokes satellite with $\hbar\Delta\omega \gg T$ will be observed.

To resolve the question of which form of sound will be observed it is necessary to compare qv with $1/\tau$. In accordance with [6] and Landau's paper [3] the quantity τ, which for $\hbar\Delta\omega \ll T$ has the form $\tau = 2.3 \times 10^{-12}T^{-2}$, (T in degrees), must in the case $\hbar\Delta\omega \gg T$ be replaced by

$$\tau = 1.6 \cdot 10^{12} (\Delta\omega)^{-2}, \text{ sec } (\Delta\omega \text{ in sec}^{-1}). \tag{12}$$

The limiting angles up to which the zero-sound satellite is observable are given by the relation $\Delta\omega = u_2 q \sim 1/\tau$, or

$$\theta_3 \sim 1.5 \cdot 10^{12} \; c/\omega u. \tag{13}$$

Angles smaller than θ_3 correspond to zero sound. Substitution of numerical data shows that θ_3 closely approximates θ_1, as determined by Eq. (10). (The order-of-magnitude agreement is quite obvious, but in this case there is also numerical agreement). As a consequence, only the zero-sound satellite will be observed over the whole angular range for which the study of acoustical satellites is feasible, for the energies and temperatures under consideration.

The amplitude of the scattering can be estimated with the aid of Eq. (18). As a result, one obtains

$$dI_1 = \int \frac{dh_1}{d\Delta\omega} d\Delta\omega \approx 5 \cdot 10^5 \hbar\omega\theta d\Omega, \text{cm}^{-1}. \tag{14}$$

For an energy $\hbar\omega = 10$ kev and for angles $\sim 1°$, this yields $dI_1 \approx 10^{-4} d\Omega$ cm^{-1}. This is 10^5 times as great as the amplitude found for the optical region. This should clearly be regarded as a limiting figure. An increase in the γ-quantum energy does not lead to enhancement of the effect, since, in accordance with the condition (10), it is necessary in this case to take proportionately smaller scattering angles.

One of the characteristics of scattering in the zero-sound region is the central plateau. This corresponds to $\hbar\Delta\omega < vq$; i.e., its edge is approximately half-way between the undisplaced line and the satellite. The intensity of the plateau is given by Eq. (9), and is nearly independent of $\Delta\omega$. For $\Delta\omega/vq = 1/2$, substitution of numerical values yields

$$dh_2 = 2 \cdot 10^9 d\,(\hbar\Delta\omega)\; d\Omega, \text{cm}^{-1}. \tag{15}$$

Over the whole plateau one finds, for $\hbar\omega = 10$ kev and $\theta = 1°$

$$dI_2 = \int \frac{dh_2}{d\Delta\omega} d\Delta\omega \approx 10^{-7} \; d\Omega, \text{cm}^{-1}.$$

Finally, let us consider the problem of γ-quantum scattering in liquid He^4. The appropriate formulas have been derived by Ginzburg. [7] Setting $\partial D/\partial N = -2\pi e^2/m_e\omega^2$ and taking the quantum factor into account, we obtain for the normal doublet

$$dh_1 = \frac{1}{8} (e^2/m_e c^2)^2 (\rho/m^2 u_1^2)\, \hbar\Delta\omega n\,(\Delta\omega)\, [\delta\,(\Delta\omega - u_1 q) + \delta\,(\Delta\omega + u_1 q)]\; \frac{1}{2}\,(1 + \cos^2\theta)\; d\Omega d\Delta\omega, \tag{16}$$

where ρ is the density of the He^4, m is the atomic mass, and u_1 is the first-sound velocity. The intensity of the anomalous doublet associated with second sound is smaller by the factor

$$(c_p/c_v - 1)/(1 - u_2^2/u_1^2)^2$$

The restriction (10) on the angles and the estimate (11) of the quantum limit remain approximately correct for He^4 as well. At temperatures below 1° K, therefore, one can study a purely quantum situation. In this case we find, from Eq. (16), the intensity for the Stokes satellite

$$dI_1 = \int \frac{dh_1}{d\Delta\omega} d\Delta\omega = 4 \cdot 10^4 \hbar\omega\theta d\Omega, \text{cm}^{-1}.$$

The intensity of the anomalous doublet below 1°K is practically zero.

In conclusion, we express our thanks to Academician L. D. Landau for his consideration of this work, and to V. P. Peshkov, at whose initiative this calculation was undertaken.

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