NUCLEON CORRELATIONS AND PHOTONUCLEAR REACTIONS. II. (γ, p) *AND* (γ, n) *REACTIONS IN THE NONRESONANCE REGION* ($E_\gamma \gtrsim 30$ Mev)

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Submitted to JETP editor February 6, 1961

J. Exptl. Theoret. Phys. (U.S.S.R.) **41**, 451-455 (August, 1961)

It is shown that the inclusion of pair correlation of nucleons is absolutely necessary for explaining the cross sections for single nucleon photonuclear reactions [i.e., (γ, p) and (γ, n) reactions] in the energy region beyond the giant resonance.

1. In the present paper we shall consider the question of the effect of pair correlation of nucleons on the cross section for photoreactions with the emission of a single nucleon [i.e., (γ, p) and (γ, n) reactions] in light nuclei, under the condition that the product nucleus is left in the ground state or weakly excited states ($E^* \lesssim 10$ Mev). All the assumptions concerning the properties of correlators which were made by us previously[1] are retained here, including the assumption that the correlators are small.

The wave function of the nucleus Ψ^A can be written as*

$$\Psi^A \approx \Psi^A_{IPM} + \sum_{ij} \{Z^1_{ij}\}^{-1} \hat{q}\chi_{ij}\hat{Q}^1\Psi^A_{IPM} \qquad (\chi_{ij} \equiv \chi^1_{ij}), \qquad (1)$$

where we include only triplet correlators. A simple estimate shows that if $r_c^0 \leq r_c^1/3$, the contribution of the singlet correlators to the reaction cross section does not exceed 10% even for $E_\gamma \sim 100$ Mev.

2. The basis of the method proposed here is the expansion of the nuclear wave function, written taking account of pair correlation of nucleons, in terms of a complete set of suitably chosen functions of the independent motion. For this purpose it is first necessary to separate in Ψ^A_{IPM} the wave function of the pair of correlating nucleons. Suppose that in the LS-coupling approximation the initial nucleus has the configuration $S^{n_1}[\lambda_1]p^{n_2}[\lambda_2]$ where $n_1 + n_2 = n \equiv A$. Then

$$\Psi^A_{IPM}(L\Lambda, S\Sigma, TM_T)$$
$$= \sqrt{\frac{n_2(n_2-1)}{n(n-1)}} \sum_{\xi'\xi_0} \mathcal{R}^{pp}_{\xi'\xi_0} \{\Psi^{A-2}_{p^{n_2-2}}(\xi'), \Psi^{p^2}(\xi_0)\}_{LST} + (-)^{n_2-1}$$
$$\times \sqrt{\frac{2n_1n_2}{n(n-1)}} \sum_{\xi'\xi_0} \mathcal{R}^{sp}_{\xi'\xi_0} \{\Psi^{A-2}_{s^{n_1-1}p^{n_2-1}}(\xi'), \Psi^{sp}(\xi_0)\}_{LST} + \ldots . \qquad (2)$$

*The notation is the same as in[1].

Here $\mathcal{R}^{pp}_{\xi'\xi_0}$ and $\mathcal{R}^{sp}_{\xi'\xi_0}$ are fractional parentage coefficients of the type $\langle s^{n_1}p^{n_2-2}, p^2 | \} s^{n_1}p^{n_2}\rangle$ and $\langle s^{n_1-1}p^{n_2-1}sp | \} s^{n_1}p^{n_2}\rangle$ respectively; $\xi' \equiv L'S'T'$ are the quantum numbers of the system of $A-2$ nucleons; $\xi_0 \equiv LST$ are the quantum numbers of the pair which are selected, and $\{\ ,\ \}_{LST}$ denotes the vector coupling of the angular momenta ξ' and ξ_0 in the state LST. In formula (2) we have omitted the term corresponding to the separating out of two s-nucleons, since correlations of such pairs lead to final states of the nucleus $A-1$ with high excitation energy. (These will be states $s^{n_1-1}p^{n_2}$.) We shall not consider such excitations. Furthermore

$$\Psi(\xi_0) \equiv |n_{i0}l_{i0}, n_{j0}l_{j0}; \xi_0\rangle$$
$$= \sum C^{L_0\Lambda_0}_{l_{i0}m_{i0}l_{j0}m_{j0}} |n_{i0}l_{i0}m_{i0}\rangle |n_{j0}l_{j0}m_{j0}\rangle X_{S_0\Sigma_0}\Omega_{T_0\tau}, \qquad (3)$$

where $|n_{i0}l_{i0}m_{i0}\rangle$ are the single particle wave functions from which Ψ^A_{IPM} is constructed, and X and Ω are respectively the spin and isotopic spin functions for the pair of nucleons.

The wave function (3) of the pair is expressed in the coordinates $\mathbf{r}_i$, $\mathbf{r}_j$. We go over to the coordinates $\mathbf{r} = \mathbf{r}_i - \mathbf{r}_j$, $\mathbf{R} = (\mathbf{r}_i + \mathbf{r}_j)/2$. For oscillator functions this transformation is known as the Talmi transformation; the properties of the coefficients of this transformation have been treated in papers by Moshinsky[2] and Arima and Terasawa.[3] We find

$$|n_{i0}l_{i0}, n_{j0}l_{j0}; \xi_0\rangle$$
$$= \sum_{\tilde{n}\tilde{l}\tilde{N}\tilde{L}} \langle n_{i0}l_{i0}, n_{j0}l_{j0}; \xi_0 | \tilde{n}\tilde{l}, \tilde{N}\tilde{L}; \xi_0\rangle |\tilde{n}\tilde{l}, \tilde{N}\tilde{L}; \xi_0\rangle;$$
$$|\tilde{n}\tilde{l}, \tilde{N}\tilde{L}; \xi_0\rangle = \sum C^{L_0\Lambda_0}_{\tilde{l}\tilde{m}\tilde{L}\tilde{\Lambda}} |\tilde{n}\tilde{l}\tilde{m}\rangle |\tilde{N}\tilde{L}\tilde{\Lambda}\rangle X_{S_0\Sigma_0}\Omega_{T_0\tau}. \qquad (4)$$

Here $\tilde{n}\tilde{l}\tilde{m}$ and $\tilde{N}\tilde{L}\tilde{\Lambda}$ are the quantum numbers of the relative motion and the motion of the center of gravity of the pair respectively.

We now apply the operator $\hat{q}\chi_{ij}\hat{Q}^1$ to the function (4) and expand the result in the complete set of functions $|n_i l_i, n_j l_j; \xi_0\rangle$.* We then obtain

$$\hat{q}\chi_{ij}\hat{Q}^1 |n_{i0}l_{i0}, n_{j0}l_{j0}; \xi_0\rangle = \sum_{n=1}^{\infty}\sum_{n_i l_i n_j l_j}^{q} c_n \langle n_{i0}l_{i0}, n_{j0}l_{j0}; \xi_0 | 00, \tilde{N}_0\tilde{L}_0; \xi_0\rangle \times \langle n0, \tilde{N}_0\tilde{L}_0; \xi_0 | n_i l_i, n_j l_j; \xi_0\rangle | n_i l_j, n_j l_j; \xi_0\rangle \delta_{\xi_0\xi_0'}, \quad (5)$$

where ξ_0' corresponds to $S_0 = 1$, $T_0 = 0$ and the coefficients c_n are given by the expansion

$$\chi_{ij}|000; r\rangle = \sum_{n=0}^{\infty} c_n |n00; r\rangle.$$

Substituting (2) — (5) in formula (1), we get

$$\Psi^A(L\Lambda, S\Sigma, TM_T) = \Psi^A_{IPM}(L\Lambda, S\Sigma, TM_T) + \{Z^1\}^{-1}\sum_{ij}\Big[\sqrt{\frac{n_2(n_2-1)}{n(n-1)}}\sum_{\xi'\xi_0'}\mathcal{R}^{pp}_{\xi'\xi_0'} \times \sum C^{L\Lambda}_{L'\Lambda' L_0\Lambda_0} C^{S\Sigma}_{S'\Sigma' 1\Sigma_0}\Psi^{A-2}_{p^{-2}}(\xi') \times \sum_{n=1}^{\infty}\sum_{n_i l_i n_j l_j}^{q} c_n d^n_{n_i l_i n_j l_j} |n_i l_i, n_j l_j; \xi_0'\rangle + (-)^{n_2-1}\sqrt{\frac{2n_1n_2}{n(n-1)}}\sum_{\xi'\xi_0'}\mathcal{R}^{sp}_{\xi'\xi_0'}\sum C^{L\Lambda}_{L'\Lambda' L_0\Lambda_0} C^{S\Sigma}_{S'\Sigma' 1\Sigma_0} \times \Psi^{A-2}_{s^{-1}p^{-1}}(\xi')\sum_{n=1}^{\infty}\sum_{n_i l_i n_j l_j}^{q} c_n f^n_{n_i l_i n_j l_j} |n_i l_i, n_j l_j; \xi_0'\rangle\Big], \quad (6)$$

where the superscript q on the summation sign indicates that it is necessary to exclude states which are occupied in Ψ^A_{IPM}, and we have introduced the notation

$$d^n_{n_i l_i, n_j l_j} \equiv \langle 01, 01; \xi_0' | 00, \tilde{N}_0\tilde{L}_0; \xi_0'\rangle \langle n0, \tilde{N}_0\tilde{L}_0; \xi_0'' | n_i l_i^3, n_j l_j; \xi_0'\rangle,$$

$$f^n_{n_i l_i, n_j l_j} \equiv \langle 00, 01; \xi_0' | 00, 0\tilde{L}_0; \xi_0'\rangle \langle n0, 0\tilde{L}_0; \xi_0' | n_i l_i, n_j l_j; \xi_0'\rangle.$$

3. We are interested in reactions where the product nucleus is produced in a state with configuration $s^{n_1}[\lambda_1]p^{n_2-1}[\lambda]$. In the approximation of weak correlators which we are using, it is not necessary to take account of the correlation in the final state wave function; it can therefore be written in the form

$$\Psi^{A-1}_{IPM}(L_1\Lambda_1, S_1\Sigma_1, T_1M_{T_1}) = \sum_{\xi_1'} Q^{s_1 p}_{\xi_1'}\sum C^{L_1'\Lambda_1'}_{L_1\Lambda_1\bar{l}\bar{m}} C^{S_1'\Sigma_1'}_{S_1\Sigma_1 1/2\bar{\sigma}} C^{T_1M_{T_1}}_{T_1'M_{T_1'}1/2\bar{\tau}} \times \Psi^{A-2}(\xi_1')|\bar{n}\bar{l}\bar{m}\rangle \chi_{1/2\bar{\sigma}}\omega_{1/2\bar{\tau}}. \quad (7)$$

Here $Q^p_{\xi_1'}$ and $(-1)^{n_2-1}Q^s_{\xi_1'}$ are fractional parentage coefficients of type $\langle s^{n_1}p^{n_2-2}, p|\}\, s^{n_1}p^{n_2-1}\rangle$ and $\langle s^{n_1-1}p^{n_2-1}, s|\}\, s^{n_1}p^{n_2-1}\rangle$; $\bar{n}\bar{l}\bar{m}\bar{\sigma}\bar{\tau}$ are the quantum numbers of the final state of that nucleon of the correlating pair which remains in the nucleus; $\xi_1' \equiv L_1'S_1'T_1'$ are the quantum numbers of the system of A − 2 nucleons. In formula (7) one should keep the index p or s depending on whether one is calculating the matrix element with the second or third term of the wave function (6). In the first case $\bar{n} = 0$, $\bar{l} = 1$, and in the second $\bar{n} = \bar{l} = 0$.

The second nucleon of the correlating pair makes a transition to the continuous spectrum. Its wave function is

$$\Psi^{(-)}_{\mathbf{k}} = (4\pi k)^{-1} \times \sum i^l\sqrt{4\pi(2l+1)}\, e^{-i\delta_l}|El\rangle Y_{l0}(\cos\theta)\chi_{1/2\sigma}\omega_{1/2\tau}, \quad (8)$$

where $\hbar\mathbf{k}$ is the relative momentum in the final state, $|El\rangle$ are the radial wave functions normalized to $\delta(k-k')$. The total wave function of the system in the final state can be taken as a product $\Psi^{A-1}_{IPM}\Psi^{(-)}_{\mathbf{k}}$.

In calculating the total reaction cross section, we can restrict ourselves to the E1 term in the interaction operator H_ζ, i.e., we can set

$$H_\zeta \sim e_1\sum h_i O_+ + e_2\sum h_i O_-,$$

$$h_i \sim P\sum_\mu D^1_{\mu P}(\alpha\beta) Y_{1\mu}(\theta, \varphi)\varkappa r_i, \quad P = \pm 1, \quad (9)$$

where e_1 and e_2 are the effective charges, and $O_\pm$ are the projection operators for the proton and neutron respectively. Standard computations give the matrix elements as linear combinations of the quantities

$$M^+_{El0, n_i l_i m_i}\langle\bar{n}\bar{l}\bar{m}|n_j l_j m_j\rangle, \qquad M^-_{\bar{n}\bar{l}\bar{m}, n_j l_j m_j}\langle El0|n_i l_i m_i\rangle,$$

where we use the notation $M^+ = \langle|e_1 h_i O_+|\rangle$ and $M^- = \langle|e_2 h_i O_-|\rangle$. Omitting the complicated formulas, we remark that the various terms in the wave function (6) give different types of transitions, which are shown schematically in Fig. 1, and different sets of final states of the product nucleus. Thus, for example, if the state of the initial nucleus is described by the Young pattern [44], the transitions of type α and β give only states [43], while the transitions γ_1 also give states [421] and [331].

*We should point out the following: we are assuming that the wave functions corresponding to bound states of particles in Ψ^A_{IPM} are well approximated by oscillator functions with some value of the parameter $\hbar\omega$; then it is convenient to choose the complete set of functions $|n_i l_i, n_j l_j; \xi_0\rangle$ with this same value of the parameter $\hbar\omega$. Under this condition the further computations are significantly simplified.

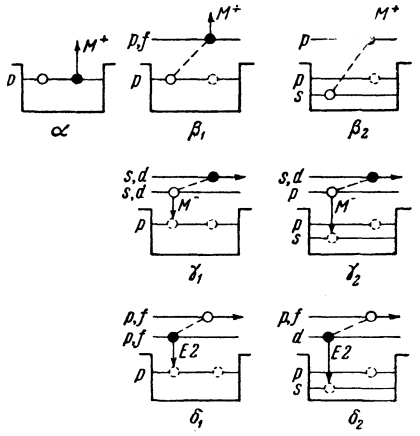

FIG. 1. Classification of terms of the wave function (6) and electromagnetic transitions: o – neutrons, ● – protons; the dashed line joins the correlating nucleons. The vertical arrows correspond to the operators M^+ or M^-, and the horizontal lines to the matrix element $< E\,l|n_j l_j >$.

4. Calculations have been made for the $C^{12}(\gamma, p)B^{11}$ reaction in the interval $30 \lesssim E_\gamma \lesssim 70$ Mev. We took for the classification for the levels of B^{11} that given by Inglis[4] for the limiting case of LS coupling. The interaction in the final state was taken into account by using the complex potential:

$$V = V_0 + iW_0 \quad \text{for} \quad R \leqslant R_0,$$
$$V = 0 \quad \text{for} \quad R > R_0;$$

here $R_0 = 1.25\,A^{1/3} \times 10^{-13}$ cm, V_0 and W_0 are chosen to depend on the proton energy in accordance with Fig. 1 of Glassgold's summary.[5] The phase shifts were computed using the approximate formulas of Pargamanik and Ul'yanov.[6] The Coulomb interaction was neglected.

The parameter $\hbar\omega$ of the single-particle oscillator functions was set equal to 16 Mev. The correlator χ_{ij} was taken in the form $\exp\{-\beta r_{ij}^2\}$. We used terms in the expansion up to n = 4 inclusive.

The results of the computation of the total cross section for the $C^{12}(\gamma, p)$ reaction for final states ^2P [43] are given in Fig. 2. Here we also give the experimental data of Penner and Leiss.[7] As we see, for $\beta \approx 0.55 \times 10^{26}$ cm^{-2}, there is satisfactory agreement of computation with experiment. Thus the radius of correlation of triplet pairs of nucleons in nuclei like He^4 and C^{12} is apparently approximately the same. In Fig. 2 we also give the result of computation omitting the correlation. It is easy to see that correlation of the nucleons has an extremely important effect on the reaction cross section, especially at high energies.

With this same value of $\beta = 0.55 \times 10^{26}$ cm^{-2} we calculated the cross section for transitions with excitation of the B^{11} nucleus to the states ^2D [43], ^2F [43], and ^4P [421]. The overall total

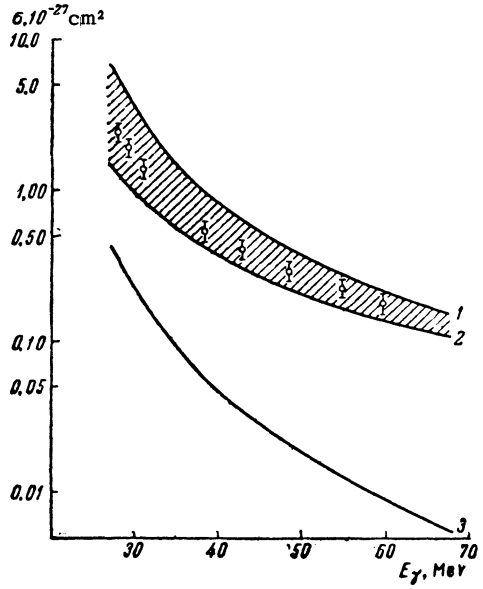

FIG. 2. Total cross section for the $C^{12}(\gamma, p)B^{11}$ reaction with transition to the state ^2P [43]. Curves 1 and 2 correspond to $\beta = 0.5 \times 10^{26}$ and 0.6×10^{26}; curve 3 is calculated omitting correlation. The experimental points are taken from the work of Penner and Leiss[7].

cross section and the experimental data of Penner and Leiss[7] are given in Fig. 3.

Calculations of the angular distributions of the protons were not made because of the complexity of the formulas. However, there is no doubt that if one includes E2 and higher multipoles, one can obtain the characteristic forward angular distribution of (γ, p) reactions. More interesting is the fact that in a model with correlations it is possible to explain the shift toward the forward direction in the angular distribution of neutrons from

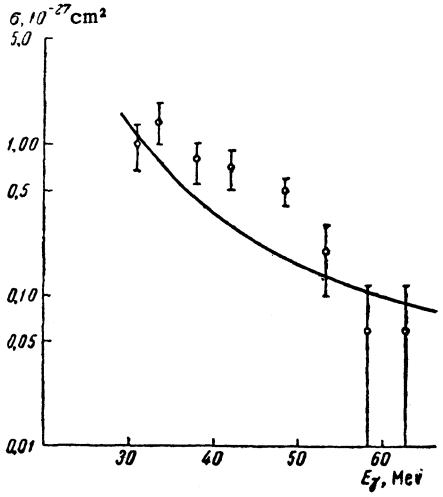

FIG. 3. Overall total cross section for the $C^{12}(\gamma, p)B^{11*}$ reaction with transition to the states ^2D [43], ^2F [43], and ^4P [421]. The curve is calculated for $\beta = 0.55 \times 10^{26}$. The experimental data are those of reference 7. In addition to the statistical errors shown, there is an uncertainty of ± 50%.

the $C^{12}(\gamma, n)$ reaction which has been observed by Kul'chitskii and Presperin[8] and others.

In fact, all the E1 transitions shown in Fig. 1 are also possible for neutrons, and, in the region of p-shell nuclei, for example, they give even partial waves in the continuous spectrum. Furthermore even though the neutron does not possess an "effective charge" with respect to EL transitions for $L \geq 2$, nevertheless the transitions δ_1 and δ_2 shown in Fig. 1 are possible. It is easy to see that for such transitions the neutron is in the continuous spectrum with an odd orbital angular momentum. The interference of these transitions with those previously mentioned leads to an asymmetry in the angular distribution of the neutrons.

5. The results obtained lead one to think that, despite the somewhat arbitrary method of introduction of the correlation, the model used correctly reflects the properties of atomic nuclei which are important for reactions at high energy. This applies especially to the assumption that the correlators are small, which is a self-consistent assumption in the sense that a comparison of the computations with experiment leads to this same conclusion. A check of the whole model is the satisfactory result of the calculation of transitions with excitation of the product nucleus.

Another important question is the choice of the specific form of the correlators. In connection with wellknown properties of photonuclear reactions (the smallness of the momentum of a photon even when its energy is large) it is especially significant that the introduction of correlators leads to large gradients of the two-particle function $\psi^S_{\alpha\beta}(ij)$ at small distances. Thus any function which falls off sufficiently rapidly with increasing r_{ij} gives the same results as the function used in the present paper. This also leads to the result that the choice (within reasonable limits) of the specific form and the parameters of the function Ψ_{IPM} has little effect on the final results.

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Translated by M. Hamermesh
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