THE MEAN FREE PATH OF MOLECULES IN A MOLECULAR BEAM

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It is shown that the mean free path of molecules at any point in a molecular beam with a Maxwellian velocity distribution is almost exactly three times greater than the path length in a gas of the same density.

THE question of the mean free path of molecules in the beam, which limits the possible increase in beam power, arises at the present time in connection with the construction of molecular generators using relatively dense beams of molecules. A determination of the path length enables an estimate of the possible beam density to be made for a given length, and its destruction by diffusion to be calculated.

For the calculation we shall assume that the molecular motion at the point considered is unidirectional. This is clearly justifiable at distances from the beam source greater than its diameter, i.e., in the region usually used in practice. We will neglect impacts with molecules scattered diffusely at collisions, which limits the applicability of the expression derived below to cases where the fraction of diffused molecules is small.

In view of what has been said, the calculation of the path length of a molecule in the beam reduces to the calculation of the path length in a gas with a Maxwellian law for the distribution of velocities c:

$$F(c) = 4\pi(\beta^3/\pi^3)^{1/2} e^{-\beta c^2} c^2, \qquad \beta = m/2kT.$$

Considering two groups of molecules in the beam with velocities c and c_1, lying in the ranges dc and dc_1, we can write for the total number of collisions between them in unit volume per second $d\nu(c, c_1) = n^2 F(c) F(c_1)\pi\sigma^2 g\, dc\, dc_1$,[1] where n is the molecular density at the point of the beam considered, $g = c - c_1$ is the relative velocity of the molecules in the beam, and σ is the effective molecular radius.

The number of collisions in unit volume per second of any molecule of the first group, with velocity c, with all the other molecules equals

$$d\nu_1(c) = n\pi\sigma^2 \int_0^\infty |c - c_1| F(c_1)\, dc_1.$$

The number of collisions, averaged for all possible molecular velocities, will be

$$\nu = n\pi\sigma^2 \int_0^\infty F(c)\, dc \int_0^\infty |c - c_1| F(c_1)\, dc_1.$$

Breaking down the inner integral into two with limits $0, c$ and c, ∞, integrating, and expressing β in terms of the mean thermal velocity $\bar{c} = 2/\sqrt{\pi\beta}$, we obtain

$$\nu = n\pi\sigma^2\bar{c}\,(7 - 4\sqrt{2})/2\sqrt{2} \approx n\pi\sigma^2\bar{c} \cdot 0.475.$$

The mean free path $\bar{c}/\nu$ at the point in the beam with the given density n is

$$\lambda_{\text{beam}} = \frac{4}{7 - 4\sqrt{2}} \frac{1}{n\pi\sigma^2\sqrt{2}} = \frac{4}{7 - 4\sqrt{2}}\lambda_{\text{gas}} \approx 3\lambda_{\text{gas}},$$

where λ_{gas} is the mean free path of molecules in the gas. The path length of molecules in the beam is thus three times greater than the path length in a gas of the same density.

It is of interest to determine the path length in a quasi monokinetic beam, when the molecular velocity is distributed uniformly in some interval Δc around the mean velocity c. The distribution law in this case is $F(c) = 1/\Delta c$. Substituting this into the expression for ν and integrating, we obtain $\nu = n\pi\sigma^2\Delta c/3$, whence the path length $\lambda_{\text{beam}} = \lambda_{\text{gas}} \times c3\sqrt{2}/\Delta c$.

[1] L. Boltzmann, Vorlesungen über Gastheorie, Barth, Leipzig, 1910.

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