EXCITATION OF NUCLEI IN HEAVY μ-MESIC ATOMS

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We consider the process where nuclei are electromagnetically excited by muons (radiationless excitation) during the 2p-1s transition in a mesonic atom. The ratio of the probability that a γ quantum is emitted by the muon to the probability of a radiationless excitation with subsequent decay of the nucleus through various nuclear channels is evaluated.

1. INTRODUCTION

ONE of the authors has shown earlier[1] that the transition of a muon from the 2p to the 1s state can in heavy mesonic atoms take place by a direct transfer of the whole of the energy of the transition to the nucleus. The probability for the excitation of the nucleus during this transition was evaluated assuming that $\rho\Gamma_{\text{nucl}} \gg 1$ (case of overlapping nuclear levels) where ρ is the density of the nuclear levels at an excitation energy equal to the energy of the transition, and Γ_{nucl} is the average width of the nuclear levels at the same energy. One can in that limiting case interpret the process where the nucleus is excited as the inverse of the internal conversion effect. Such a process was called the effect of a radiationless excitation of the nucleus. The ratio of the probability W_{nucl} of a radiationless excitation of the nucleus to the probability W_γ of the emission of a γ quantum could in that limiting case be written in the form

$$W_{\text{nucl}}/W_\gamma = \Gamma_{\text{n.r.}}/\Gamma_\gamma, \tag{1}$$

where $\Gamma_{\text{n.r.}}$ is the width of the radiationless excitation of the nucleus which is proportional to the photoexcitation cross section and Γ_γ is the width for the emission of a γ quantum by a muon.

There is also interest in the other limiting case when $\rho\Gamma_{\text{nucl}} \ll 1$ (case of non-overlapping levels). In that case the nucleus can disintegrate through one of the nuclear channels but the process of a reverse transfer of energy to the muon is also possible. This leads to the result that the yield of γ quanta for that transition is larger in comparison than the one given by Eq. (1) and that thus W_{nucl}/W_γ in the case of non-overlapping levels must depend not only on Γ_γ and $\Gamma_{\text{n.r.}}$, but also on $\rho\Gamma_{\text{nucl}}$.

We compute in the present paper W_{nucl}/W_γ for the case $\rho\Gamma_{\text{nucl}} \ll 1$ and arbitrary ratio of Γ_γ and $\Gamma_{\text{n.r.}}$ The case $\Gamma_{\text{n.r.}} \gg \Gamma_\gamma$ was considered earlier by us.[2] An estimate of $\Gamma_{\text{n.r.}}/\Gamma_\gamma$ for a number of elements[3] shows, however, that this quantity is of the order of unity. In the case of non-overlapping levels, the effect of the excitation of the nuclei by a muon is important only, if the width of the muon energy level is appreciably larger than the distance between nuclear levels. In heavy mesonic atoms such as thorium and uranium $\Gamma_{\text{n.r.}} \sim 1$ kev, and $1/\rho$ is of the order of several electron volts so that the condition

$$\rho\Gamma_{\text{n.r.}} \gg 1, \tag{2}$$

on which our calculation is based is satisfied with very good accuracy.

We assume for the sake of simplicity in this paper that the transition of the muon from a more highly excited state into the 2p-state is not accompanied by the effect of the radiationless excitation. This is a reasonable assumption, since transitions between higher states have less energy and thus also a smaller probability for a radiationless excitation. On the other hand, such an assumption is not one of principle, since one can easily generalize the calculation to the case of a radiationless excitation at any transition.

2. ANALYSIS OF THE BOUND MUON-NUCLEUS SYSTEM

The Hamiltonian of the muon-nucleus system is of the form

$$H = H_0 + V, \tag{3a}$$

$$H_0 = H_{\text{nucl}} + T_\mu - \left\langle \psi_0 \left| \sum_{i=1}^{Z} \frac{e^2}{|\mathbf{r}_i - \mathbf{r}_\mu|} \right| \psi_0 \right\rangle, \tag{3b}$$

where H_{nucl} is the Hamiltonian of the nucleus, T_μ the kinetic energy operator of the muon, and the third term in (3b) is the potential in which the muon in the mesonic atom moves ($\mathbf{r}_\mu$ is the muon coordinate, $\mathbf{r}_i$ the proton coordinate, and the sum is taken over all protons), ψ_0 is the wave function of the ground state of the nucleus. V is the dipole part of the operator

$$\left\langle \psi_0 \right| \sum_{i=1}^{Z} \frac{e^2}{|\mathbf{r}_i - \mathbf{r}_\mu|} \left| \psi_0 \right\rangle - \sum_{i=1}^{Z} \frac{e^2}{|\mathbf{r}_i - \mathbf{r}_\mu|}. \quad (4)$$

The eigenfunctions of the Hamiltonian H_0 are clearly all possible products of wave functions ψ_i of the nucleus with muon wave functions φ_k. The wave functions of the total Hamiltonian H which satisfy the Schrödinger equation $H\Psi_\lambda = E_\lambda\Psi_\lambda$ will be expanded in terms of the complete set of wave functions of the Hamiltonian H_0:

$$\Psi_\lambda = \sum_{i,k} C_{\lambda;i,k} \psi_i \varphi_k, \quad (5)$$

where the $C_{\lambda;i,k}$ are the expansion coefficients.

Since $\Gamma_{n.r.}$ is much smaller than the distance between the muon levels, but much larger than the distance between the nuclear levels which correspond to the energy of excitation of the nucleus during the 2p-1s transition, one can for energies E_λ of the system which are close to the energy E_p of the 2p-1s transition write the sum (5) in the form

$$\Psi_\lambda = C_{\lambda;0,2p} \psi_0 \varphi_p + \sum_c C_{\lambda;c,1s} \psi_c \varphi_s, \quad (6)$$

where φ_p and φ_s are the muon wave functions for the 2p and the 1s state, respectively; in the following we shall for the sake of simplicity write $C_{\lambda,p}$ instead of $C_{\lambda;0,2p}$ and $C_{\lambda,c}$ instead of $C_{\lambda;c,1s}$. The other terms in (5) are negligibly small and can be omitted.

Substituting (6) into the Schrödinger equation with the Hamiltonian H and using the normalization of the Ψ_λ we get the following set of equations for the coefficients C_λ and the eigenvalues E_λ:

$$C_{\lambda,p} V_c = (E_\lambda - E_c)\, C_{\lambda,c}, \quad (7)$$

$$\sum_c C_{\lambda,c} V_c^* = (E_\lambda - E_p)\, C_{\lambda,p}; \quad (8)$$

$$|C_{\lambda,p}|^2 + \sum_c |C_{\lambda,c}|^2 = 1, \quad (9)$$

where $V_c = \langle \psi_c \varphi_s | V | \psi_0 \varphi_p \rangle$. Substituting (7) into (9) we get

$$|C_{\lambda,p}|^2 = \left\{1 + \sum_c |V_c|^2 / (E_\lambda - E_c)^2\right\}^{-1}. \quad (10)$$

From (7) and (8) we find an equation for the eigenvalues:

$$E_\lambda - E_p = \sum_c [\,|V_c|^2 / (E_\lambda - E_c)]. \quad (11)$$

From (10) we see that a characteristic interval for the sum over c is $|E_\lambda - E_c| \lesssim \Gamma_{n.r.}$. One may assume that $|V_c|^2$ which is proportional to the cross section for the photoexcitation of the nucleus[1] is constant in an interval of $\sim \Gamma_{n.r.}$ and that the levels E_c are distributed equidistantly with a density $\rho = 1/\bar{D}$ where $\bar{D}$ is the average distance between the levels of the nucleus at the given energy E_p. Taking this into account we find

$$\sum_c [\,|V_c|^2/(E_\lambda - E_c)] = \pi \overline{|V_c|^2}\, \rho \operatorname{ctg} \pi\rho\Delta, \quad (12)^*$$

$$\sum_c [\,|V_c|^2/(E_\lambda - E_c)^2] = \pi^2 \overline{|V_c|^2}\, \rho^2 / \sin^2 \pi\rho\Delta, \quad (13)$$

where Δ is the distance from E_λ to the nearest level E_c. The details of the calculations leading to (12) and (13) were given in [2].

Eliminating Δ from (12) and (13) we find a connection between the sums over c in (12) and (13)

$$\sum_c \frac{|V_c|^2}{(E_\lambda - E_c)^2} = \frac{1}{\overline{|V_c|^2}} \left[\pi^2\rho^2 \overline{|V_c|^4} + \left(\sum_c \frac{|V_c|^2}{E_\lambda - E_c}\right)^2\right]. \quad (14)$$

Substituting (14) into (10) and using (11) we find

$$|C_{\lambda,p}|^2 = \overline{|V_c|^2} / [(E_\lambda - E_p)^2 + \pi^2\rho^2 \overline{|V_c|^4} + \overline{|V_c|^2}]. \quad (15)$$

Using the condition (2) we can neglect the last term in the denominator in (15) and find finally

$$|C_{\lambda,p}|^2 = (\Gamma_{n.r.}/2\pi\rho)\, [(E_\lambda - E_p)^2 + (\Gamma_{n.r.}/2)^2]^{-1}, \quad (16)$$

where $\Gamma_{n.r.} = 2\pi \overline{|V_c|^2} \rho$.

In the following we shall need to evaluate different sums over the levels E_λ of the bound muon-nucleus system. To find the eigenvalues E_λ it is in principle necessary to solve Eq. (11). However, condition (2) has as a consequence that the levels E_λ are distributed nearly equidistantly up to terms of the order $1/\Gamma_{n.r.} \ll 1$. Indeed, one gets easily from (11) the following expression for the distance between two consecutive levels E_λ and $E_{\lambda'}$:

$$E_\lambda - E_{\lambda'} = \frac{1}{\rho}\left\{1 + \frac{\pi}{2\rho\Gamma_{n.r.}}\left[1 + 4\left(\frac{E_\lambda - E_p}{\Gamma_{n.r.}}\right)^2\right]\right\}. \quad (17)$$

The second term in (17) is small compared to unity.

3. ACCOUNT OF THE DAMPING OF THE MUON-NUCLEON SYSTEM

We neglect according to our scheme the dipole interaction between the muon and the nucleus in more highly excited muon states so that the wave function of the system is $\psi_0\varphi_{exc}$, where φ_{exc} is an arbitrary excited muon state from which it can make the transition to the 2p state. During the transition from $\psi_0\varphi_{exc}$ the level Ψ_λ of the muon-nucleus system is excited. The probability ω_λ for the excitation of level λ is proportional to the square of the matrix element for the transition, i.e.,

*ctg = cot.

$$\omega_\lambda \sim |\langle\Psi_\lambda| H_\gamma |\psi_0\varphi_{exc}\rangle|^2 = |C_{\lambda,p}|^2 |\langle\varphi_0| H_\gamma |\varphi_{exc}\rangle|^2,$$

where H_γ is the interaction between the muon and the electromagnetic field which leads to this transition. Taking the normalization $\Sigma\omega_\lambda = 1$ into account we get $\omega_\lambda = |C_{\lambda,p}|^2$.

Without loss of generality we can assume that the $C_{\lambda,p}$ are real. Since the probability of finding the system in the state λ after the muon has made the transition from the higher state is equal to $C^2_{\lambda,p}$ the total wave function of the system at $t = 0$ must be written in the form

$$\Psi|_{t=0} = \sum_\lambda C_{\lambda,p}\Psi_\lambda = \sum_\lambda C^2_{\lambda,p}\psi_0\varphi_p + \sum_{\lambda,c} C_{\lambda,c}C_{\lambda,p}\psi_c\varphi_s = \psi_0\varphi_p. \quad (18)$$

The sum over λ, c in (18) vanishes because of the orthonormality of the coefficients C_λ.

We consider now transitions from the state Ψ_λ which are accompanied by the emission of a γ quantum by the muon (H_γ is the interaction between the muon and the electromagnetic field) and by the decay of the nucleus through different nuclear channels (the neutron channel, nuclear quanta, fission). We do not know the explicit form of the interaction operator H'_{nucl} corresponding to the nuclear processes, but only the square of the matrix element of this operator enters in the final result and is proportional to the nuclear width. We assume for the sake of simplicity that only one nuclear channel is open. If the decay of the excited nucleus proceeds independently through the different channels (decay of the compound nucleus) we can replace in the final result the partial nuclear width by the total width.

The wave function of the system when the nucleus is in its ground state, the muon in its ground state, and the γ quantum emitted by the muon has an energy E_ν is denoted by Φ_ν; the wave function of the system when the muon is in the ground state while the nucleus after decay and the decay products of the nucleus have a total energy E_k is denoted by Φ_k. We can write the wave function Ψ of the Hamiltonian $\mathcal{H} = H_0 + V + H_\gamma + H'_{nucl}$ which satisfies the initial condition (18) in the form

$$\Psi = \sum_\lambda b_\lambda(t)\Psi_\lambda e^{-iE_\lambda t/\hbar} + \sum_\nu b_\nu(t)\Phi_\nu e^{-iE_\nu t/\hbar} + \sum_k b_k(t)\Phi_k e^{-iE_k t/\hbar}, \quad (19)$$

where $b_\lambda(0) = C_{\lambda,p}$, $b_\nu(0) = b_k(0) = 0$. Substituting (19) into the Schrödinger equation we get an equation for the coefficients b, which we shall assume to be equal to zero for $t < 0$ (see, for instance,[4]):

$$i\hbar\dot{b}_n(t) = \sum_m \mathcal{H}_{nm} e^{i(E_n-E_m)t/\hbar} b_m(t) + i\hbar\delta(t)\sum_\lambda C_{\lambda p}\delta_{n\lambda}, \quad (20)$$

where n and m stand for all indices λ, ν, and k.

We Fourier-transform the coefficients b:

$$b_\lambda = -\frac{1}{2\pi i}\int_{-\infty}^{+\infty} G_\lambda(E)\, e^{-i(E_\lambda-E)t/\hbar}\, dE,$$

$$b_{\nu,k} = -\frac{1}{2\pi i}\int_{-\infty}^{+\infty} G_{\nu,k}(E)\,\zeta(E-E_{\nu,k})\, e^{i(E_{\nu,k}-E)t/\hbar}\, dE, \quad (21)$$

where

$$\zeta(x) = \lim_{\sigma\to+0}[1/(x+i\sigma)].$$

We are interested in the total probability W_γ that the muon has emitted a γ quantum; it is clearly equal to

$$W_\gamma = \sum_\nu |b_\nu(\infty)|^2,$$

and similarly

$$W_{nucl} = \sum_k |b_k(\infty)|^2.$$

On the other hand[4]

$$b_\nu(\infty) = G_\nu(E)|_{E=E_\nu}, \qquad b_k(\infty) = G_k(E)|_{E=E_k}.$$

Substituting (21) into (20) we get equations for G_λ, G_ν, and G_k:

$$(E-E_\lambda)G_\lambda = \sum_\nu H^{(\gamma)}_{\lambda\nu}\zeta(E-E_\nu)G_\nu + \sum_k H^{(nucl)}_{\lambda k}\zeta(E-E_k)G_k + C_{\lambda p},$$

$$G_\nu = \sum_\lambda H^{(\gamma)}_{\nu\lambda}G_\lambda, \qquad G_k = \sum_\lambda H^{(nucl)}_{k\lambda}G_\lambda, \quad (22)$$

where

$$H^{(\gamma)}_{\lambda\nu} = \langle\Psi_\lambda|H_\gamma|\Phi_\nu\rangle = C_{\lambda p}\langle\varphi_p|H_\gamma|\varphi_s\rangle = C_{\lambda p}H_\nu,$$

$$H^{(nucl)}_{\lambda k} = \langle\Psi_\lambda|H_{nucl}|\Phi_k\rangle = \sum_c C_{\lambda c}\langle\psi_c|H_{nucl}|\Phi_k\rangle = \sum_c C_{\lambda c}H_{ck}.$$

We shall assume that H_ν is independent of the energy E_ν in an interval $\Gamma_\gamma + \Gamma_{n.r.}$. This assumption is satisfied, since $\Gamma_\gamma + \Gamma_{n.r.} \ll E_p$. Similarly, $H_{c,k}$ is independent of E_k. G_ν and G_k are then also independent of E_ν and E_k and the indices ν and k are merely indicators of the channel through which the muon-nucleus system has decayed. Using this assumption one can easily evaluate the sums over ν and k in (22). Eliminating G_λ we get a set of equations for G_k and G_ν

$$G_\nu\left[1 + i\frac{\Gamma_\gamma}{2}\sum_\lambda\frac{C^2_{\lambda p}}{E-E_\lambda}\right] + G_k\left[i\pi H^*_\nu\rho_k\sum_{\lambda,c}\frac{C_{\lambda p}C_{\lambda c}H_{ck}}{E-E_\lambda}\right] = H^*_\nu\sum_\lambda\frac{C^2_{\lambda p}}{E-E_\lambda},$$

$$G_\nu\left[i\pi H_\nu\rho_\nu\sum_{\lambda,c}\frac{C_{\lambda p}C^*_{\lambda c}H^*_{ck}}{E-E_\lambda}\right] + G_k\left[1 + i\frac{\Gamma_{nucl}}{2}\sum_{\lambda,c}\frac{C^2_{\lambda,c}}{E-E_\lambda}\right] = \sum_{\lambda c}\frac{C_{\lambda p}C^*_{\lambda c}H^*_{ck}}{E-E_\lambda}, \quad (23)$$

where $\Gamma_\lambda = 2\pi\,|H_\nu|^2\rho_\nu$, $\Gamma_{\rm nucl} = 2\pi\,\overline{|H_{ck}|^2}\rho_k$; ρ_ν and ρ_k are respectively the densities of the final states ν and k.

When solving Eqs. (23) we meet with double sums over c; to evaluate them we assume that the phases of the matrix elements are random. We get then, for instance,

$$\Big|\sum_c C_{\lambda c}H_{ck}\Big|^2 = \sum_c |C_{\lambda c}|^2\,|H_{ck}|^2 = \overline{|H_{ck}|^2}\sum_c |C_{\lambda c}|^2,$$

$$\sum_{cc'} C_{\lambda c}C^*_{\lambda' c'}\,H_{ck}\,H_{c'k} = \sum_c |H_{ck}|^2\,C_{\lambda c}C_{\lambda' c} = \overline{|H_{ck}|^2}\,(\delta_{\lambda\lambda'} - C_{\lambda p}C_{\lambda' p}). \tag{24}$$

Indeed, as

$$C_{\lambda c}H_{ck} = V_c H_{ck} C_{\lambda p}/(E_\lambda - E_c),$$

the phase of this product is determined by the phase of the product of the matrix elements $\langle\psi_0\,|\,V\,|\,\psi_c\rangle$ $\langle\psi_0\,|\,H_{\rm nucl}\,|\,\Phi_k\rangle$. On the other hand, the number of effective terms in the sum over c is of the order $\rho\Gamma_{\rm n.r.} \gg 1$ so that already a small change in phase from one level to the next of the order of $\Delta\varphi \sim 1/\rho\Gamma_{\rm n.r.} \ll 1$ reduces the sum of the cross terms to zero. In that approximation we get, using (24)

$$G_\nu = H_\nu\left[\sum_\lambda \frac{C^2_{\lambda p}}{E-E_\lambda} - i\frac{\Gamma_{\rm nucl}}{2}\sum_{\lambda'\lambda}\frac{C^2_{\lambda p}}{(E-E_\lambda)(E-E_{\lambda'})}(\delta_{\lambda\lambda'}-1)\right]$$
$$\times\left[1+\frac{\Gamma_\gamma\Gamma_{\rm nucl}}{4}\sum_{\lambda\lambda'}\frac{C^2_{\lambda p}}{(E-E_\lambda)(E-E_{\lambda'})}(\delta_{\lambda\lambda'}-1)\right.$$
$$\left.+\frac{i}{2}\sum_\lambda\frac{\Gamma_\gamma C^2_{\lambda p}+\Gamma_{\rm nucl}}{E-E_\lambda}\right]^{-1},$$

$$G_k = \left[\sum_{\lambda c}\frac{C_{\lambda p}C_{\lambda c}H_{ck}}{E-E_\lambda}\right]\left[1+\frac{\Gamma_\gamma\Gamma_{\rm nucl}}{4}\sum_{\lambda\lambda'}\frac{C^2_{\lambda p}}{(E-E_\lambda)(E-E_{\lambda'})}(\delta_{\lambda\lambda'}-1)\right.$$
$$\left.+\frac{i}{2}\sum_\lambda\frac{\Gamma_\gamma C^2_{\lambda p}+\Gamma_{\rm nulc}}{E-E_\lambda}\right]^{-1}. \tag{25}$$

We get thus for the probability that the muon-nucleus system decays through nuclear channels

$$W_{\rm nucl} = \frac{\Gamma_{\rm nucl}}{2\pi}\int_{-\infty}^{+\infty} dE\left[\sum_\lambda\frac{C^2_{\lambda p}}{(E-E_\lambda)^2} - \left(\sum_\lambda\frac{C^2_{\lambda p}}{E-E_\lambda}\right)^2\right]$$
$$\times\left\{\left[1+\frac{\Gamma_\gamma\Gamma_{\rm nucl}}{4}\sum_{\lambda\lambda'}\frac{C^2_{\lambda p}}{(E-E_\lambda)(E-E_{\lambda'})}(\delta_{\lambda\lambda'}-1)\right]^2\right.$$
$$\left.+\frac{1}{4}\left(\sum_\lambda\frac{\Gamma_\gamma C^2_{\lambda p}+\Gamma_{\rm nucl}}{E-E_\lambda}\right)^2\right\}^{-1}. \tag{26}$$

We obtain for W_γ a more complicated integral; we evaluate therefore only $W_{\rm nucl}$ and we find W_γ from the normalization condition $W_\gamma = 1 - W_{\rm nucl}$.

One can evaluate the sums over λ occurring in (26), but the integrand we obtain then is too complicated for immediate integration. To evaluate (26) we break up the path of integration into sections about each value E_{λ_0} as shown in Fig. 1.

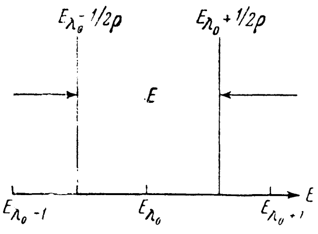

FIG. 1

The integrand in each such section is appreciably simpler than the general expression (26). After first evaluating the integral along this section as a function of E_{λ_0} we sum then over all such sections.

Using (16) we get*

$$\sum_\lambda\frac{C^2_{\lambda p}}{E-E_\lambda} = \frac{E-E_p+{}^1\!/_2\,\Gamma_{\rm n.r.}\,{\rm ctg}\,\pi\rho\,(E-E_p)}{(E-E_p)^2+({}^1\!/_2\,\Gamma_{\rm n.r.})^2}. \tag{27}$$

Along the above-mentioned interval around E_{λ_0} we write (27) in the form

$$\sum_\lambda\frac{C^2_{\lambda p}}{E-E_\lambda} = \frac{2\pi\rho}{\Gamma_{\rm n.r.}}$$
$$C^2_{\lambda_0 p}\,\frac{(E_{\lambda_0}-E_p)+(E-E_{\lambda_0})+{}^1\!/_2\,\Gamma_{\rm n.r.}\,{\rm ctg}\,\pi\rho\,(E-E_{\lambda_0})}{1+2\pi\rho C^2_{\lambda_0 p}\,[(E-E_{\lambda_0})^2+2\,(E-E_{\lambda_0})(E_{\lambda_0}-E_p)]/\Gamma_{\rm n.r.}}. \tag{28}$$

The second term in the denominator of (28) is less than or of the order of magnitude of $1/\rho\Gamma_{\rm n.r.} \ll 1$. Neglecting that term compared to unity and introducing for the sake of simplicity the notation $x = E_\lambda - E_p$, $y = E_{\lambda_0} - E_p$, we get

$$\sum_\lambda\frac{C^2_{\lambda p}}{E-E_\lambda} = \frac{2\pi\rho}{\Gamma_{\rm n.r.}}\,C^2_{\lambda_0 p}\left(x+y+\frac{\Gamma_{\rm n.r.}}{2}\,{\rm ctg}\,\pi\rho x\right). \tag{29}$$

We similarly evaluate the other sums near $E = E_{\lambda_0}$:

$$\sum_\lambda (E-E_\lambda)^{-1} = \pi\rho\,{\rm ctg}\,\pi\rho x,$$
$$\sum_\lambda C^2_{\lambda p}/(E-E_\lambda)^2 = \pi^2\rho^2 C^2_{\lambda_0 p}\,(1+{\rm ctg}^2\pi\rho x). \tag{30}$$

We take further into account that $2x/\Gamma_{\rm n.r.}\times\cot\pi\rho x \ll 1$ in the above mentioned interval of integration everywhere except where $x \gtrsim (1/2\rho)\times(1-2/\pi\rho\Gamma_{\rm n.r.})$, since the integrand in (26) is essentially positive and has its minimum value in the given interval for $x = \pm 1/2\rho$; we can thus neglect x in (29). The error introduced then is of the order of $1/\rho\Gamma_{\rm n.r.} \ll 1$. Substituting (29) and (30) into (26), we perform the substitution $\xi = \tan\pi\rho x$ and change from a sum over λ_0 to an integral

$$\sum_{\lambda_0}\to\int\rho\,dE_{\lambda_0} = \frac{1}{2}\rho\Gamma_{\rm n.r.}\int_{-\infty}^{+\infty} dt, \qquad t = 2\frac{E_{\lambda_0}-E_p}{\Gamma_{\rm n.r.}}. \tag{31}$$

We have then

*ctg = cot

$$W_{\mathrm{nucl}} = \frac{\pi\rho\Gamma_{\mathrm{nucl}}}{2}\frac{1}{\pi^2}\iint\limits_{-\infty}^{+\infty} dt\, d\xi\, (t^2+1)$$
$$\times\left\{\left[\left(1+t^2+\frac{\Gamma_\gamma}{\Gamma_{\mathrm{n.r.}}}\frac{\pi\rho\Gamma_{\mathrm{nucl}}}{2}\right)\xi - \frac{\Gamma_\gamma}{\Gamma_{\mathrm{n.r.}}}\frac{\pi\rho\Gamma_{\mathrm{nucl}}}{2}t\right]^2 + \left[\frac{\Gamma_\gamma}{\Gamma_{\mathrm{n.r.}}}(t\xi+1) + \frac{\pi\rho\Gamma_{\mathrm{nucl}}}{2}(t^2+1)\right]^2\right\}^{-1}. \quad (32)$$

Using the theory of residues one can easily evaluate the integrals in (32). We obtain finally

$$W_{\mathrm{n.r.}} = \left[1+\frac{\Gamma_\gamma}{\Gamma_{\mathrm{n.r.}}}\frac{1+(\pi\rho\Gamma_{\mathrm{nucl}}/2)^2}{\pi\rho\Gamma_{\mathrm{nucl}}/2} + \left(\frac{\Gamma_\gamma}{\Gamma_{\mathrm{n.r.}}}\right)^2\right]^{-1/2}, \quad (33)$$

and for the ratio (1)

$$\frac{W_\gamma}{W_{\mathrm{nucl}}} = \left[1+\frac{\Gamma_\gamma}{\Gamma_{\mathrm{n.r.}}}\frac{1+(\pi\rho\Gamma_{\mathrm{nucl}}/2)^2}{\pi\rho\Gamma_{\mathrm{nucl}}/2} + \left(\frac{\Gamma_\gamma}{\Gamma_{\mathrm{n.r.}}}\right)^2\right]^{1/2} - 1. \quad (34)$$

When the condition

$$\Gamma_{\mathrm{n.r.}} \gg \Gamma_\gamma/\rho\Gamma_{\mathrm{n.r.}} \quad (35)$$

is fulfilled we get from (34)

$$W_\gamma/W_{\mathrm{nucl}} = \Gamma_\gamma/\Gamma_{\mathrm{n.r.}}\pi\rho\Gamma_{\mathrm{nucl}}. \quad (36)$$

This result is the same as the result obtained in [2] assuming $\Gamma_{\mathrm{n.r.}} \gg \Gamma_\gamma$. From the calculations given in the foregoing it follows that (35) is a more rigorous criterion for the validity of Eq. (36).

The schematic behavior of the yield of γ quanta from the muon 2p-1s transition as a function of $\rho\Gamma_{\mathrm{nucl}}$ is given in Fig. 2. The behavior of the curve $W_\gamma(\rho\Gamma_{\mathrm{nucl}})$ for $\rho\Gamma_{\mathrm{nucl}} \approx 1$ is interpolated between the values given by (34) for $\rho\Gamma_{\mathrm{nucl}} \ll 1$ and the value $\Gamma_\gamma/(\Gamma_\gamma+\Gamma_{\mathrm{n.r.}})$ for $\rho\Gamma_{\mathrm{nucl}} \gg 1$. It is clear from Fig. 2 that the γ-quanta yield for $\rho\Gamma_{\mathrm{nucl}} \approx 1$ differs little from the case where the overlapping of the nuclear levels is complete, in agreement with reference 1. At very small $\rho\Gamma_{\mathrm{nucl}}$ the decrease in the γ-quanta yield during the 2p-1s transition is proportional to $\sqrt{\rho\Gamma_{\mathrm{nucl}}}$.

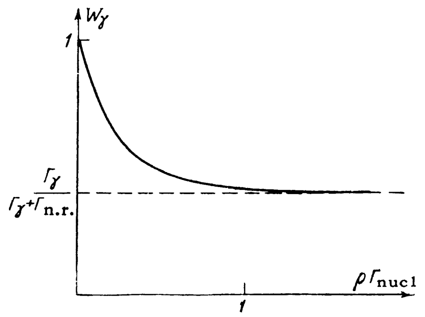

FIG. 2

4. CONCLUSION

Calculations recently performed[3] show that $\Gamma_\gamma \sim \Gamma_{\mathrm{n.r.}}$ for all nuclei in the region Th, U, and Pu. From (34) it is clear that for such a ratio of widths the dependence of the γ-quanta yield on $\rho\Gamma_{\mathrm{nucl}}$ for one muon at rest is weak. For instance, when $\Gamma_\gamma = \Gamma_{\mathrm{n.r.}}$ a change in $\pi\rho\Gamma_{\mathrm{nucl}}/2$ from 0.1 to 0.5 leads to an increase in W_γ from 0.71 to 0.79. The dependence of W_γ on $\rho\Gamma_{\mathrm{nucl}}$ can be more important only in the case when $\Gamma_{\mathrm{n.r.}} \gg \Gamma_\gamma/\rho\Gamma_{\mathrm{nucl}}$.

Experimental results obtained in reference 5 show that W_γ depends weakly upon $\rho\Gamma_{\mathrm{nucl}}$. This result is in qualitative agreement with theory. A quantitative treatment of the experiments to find $\rho\Gamma_{\mathrm{nucl}}$ will be possible once W_γ is measured more accurately for different nuclei.

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