THE FARADAY EFFECT FOR EXCITONS

I. P. IPATOVA and R. F. KAZARINOV

Leningrad Physico-Technical Institute, Academy of Sciences, U.S.S.R.

Submitted to JETP editor January 31, 1961

J. Exptl. Theoret. Phys. (U.S.S.R.) **41**, 209-210 (July, 1961)

The rotation of the plane of polarization near an absorption line of a large-radius exciton is considered. The angle of rotation depends on the effective mass and the exciton radius. An order of magnitude estimate of the angle of rotation near the 2p state of an exciton in a Cu_2O crystal gives a measurable magnitude for this effect.

ONE may expect an appreciable rotation of the plane of polarization (Faraday effect) near exciton absorption lines corresponding to transitions to exciton p states.

For cubic crystals the angle of rotation φ can be expressed in terms of the component of the gyration vector **G** along the direction of the magnetic field **H** (**H** ∥ z)

$$\varphi = (\pi d/\lambda)\, G_z/\varepsilon, \tag{1}$$

where d is the thickness of the sample, ϵ the dielectric constant when there is no magnetic field, and λ the wavelength of the light.[1] The vector **G**, in turn, is defined as the antisymmetric part of the dielectric constant $\epsilon_{\mu\nu}(\mathbf{H})$:

$$G_\gamma = \delta_{\gamma\mu\nu}\,\varepsilon_{\mu\nu}. \tag{2}$$

The problem is thus reduced to evaluating the antisymmetric part of the exciton dielectric constant.

It is well known that the transitions to the s states are allowed transitions for Mott excitons.[2-4] Transitions to p states are forbidden; their intensity is $(a/r_0)^2$ times smaller than the intensity of transitions to s states (a: lattice constant, r_0: exciton radius). For Mott excitons $(a/r_0) \ll 1$.

However, the s state does not change in the approximation which is linear in the magnetic field and there is no Faraday effect. It is thus necessary to consider the "forbidden" transitions to p states.

The exciton conductivity connected with the transitions to p states can in the first perturbation-theory approximation in the magnetic field be obtained from the general Eq. (16) of [2]:

$$\sigma_{\mu\nu} = \sum_{n;m=0,\pm1} T^{nm}_{\mu\nu}/[\gamma - i(\omega - \omega_n - \Omega m)], \tag{3}$$

where ω is the light frequency, $\Omega = eH/2Mc$, M the reduced exciton mass, ω_n the hydrogen-like energy level, γ the width of the exciton line, m the magnetic quantum number, and $T^{nm}_{\mu\nu}$ a tensor the real part of which is connected with the oscillator strength of the transition and the imaginary part of which is responsible for the rotation of the polarization plane

$$T^{nm}_{\mu\nu} = \frac{1}{E_0}\sum_i \left(\frac{\partial J^{vc}_\nu}{\partial k_\alpha}\right)_{\mathbf{K}_i}\left(\frac{\partial J^{cv}_\mu}{\partial k_\beta}\right)_{\mathbf{K}_i}\left(\frac{\partial \Psi_{n1m}}{\partial x_\beta}\right)_{\mathbf{x}=0}\left(\frac{\partial \Psi^*_{n1m}}{\partial x_\alpha}\right)_{\mathbf{x}=0}. \tag{4}$$

Here E_0 is the minimum value of the frequency of the main transition, $\mathbf{K}_i$ is that point in momentum space which corresponds to that transition, Ψ_{n1m} are the hydrogen-like wave functions of the exciton p state

$$J^{vc}_\nu(\mathbf{k}) = J^{vc*}_\nu(\mathbf{k}) = e\int d^3r\, u^*_{v\mathbf{k}}(\mathbf{r})\hat{v}_\nu u_{c\mathbf{k}}(\mathbf{r})$$

are the interband matrix elements of the current, evaluated using Bloch wave functions.

Apart from the selection rules for the hydrogen-like wave functions, $T^{nm}_{\mu\nu}$ involves also the interband selection rules for $(\partial J^{vc}_\nu/\partial k_\alpha)_{\mathbf{K}_i}$, which must be established in each case from the symmetry properties of the crystal.

Using the relation $\Psi_{n1m}(\mathbf{x}) = \Psi^*_{n1-m}(\mathbf{x})$, we get from (4)

$$T^{nm}_{\mu\nu} = T^{n-m}_{\nu\mu}; \qquad T^{nm}_{\mu\nu} = (T^{nm}_{\nu\mu})^*. \tag{5}$$

The first of these equations ensures that the Onsager relations are satisfied, the second leads to the antisymmetry of Re $\sigma_{\mu\nu}$ when there is no absorption. Assuming that $|\omega_n - \omega| \gtrsim \nu > \Omega$ we get from (3) and (5)

$$-i\varepsilon^{\text{antis}}_{\mu\nu} = \frac{2\pi}{\omega}(\sigma_{\mu\nu} - \sigma_{\nu\mu}) = -i\frac{4\pi}{\omega}\sum_n \mathrm{Im}\,\{T^{n1}_{\mu\nu}\}\frac{\Omega}{\gamma^2 + (\omega - \omega_n)^2}. \tag{6}$$

Substituting (6) into (1) and using the explicit form of $(\partial\Psi_{n1m}/\partial x_\alpha)_{\mathbf{x}=0}$ we get for the angle of rotation near the line n = 2

$$\varphi = \frac{\pi}{2}\frac{1}{\varepsilon}\left(\frac{d}{\lambda}\right)\left(\frac{a}{2r_0}\right)^5 \frac{\Omega\omega}{\gamma^2 + (\omega - \omega_n)^2}\,\beta_{xy}, \tag{7}$$

where $\beta_{xy} \sim 1$ is a component of an antisymmetric tensor.

Obtaining an order of magnitude estimate of φ for the yellow exciton series in a Cu_2O crystal ($\epsilon = 10$) we get $\varphi \gtrsim 0.5°$ for $H = 10^3$ G, $|\omega - \omega_n| \sim \gamma \sim 10\,\Omega$, $r_0 \lesssim 30a$, $d = 500\,\mu$. The absorption coefficient in the yellow series is small ($\sim 10^2$ cm^{-1})[5] and the intensity of the incident light will thus only decrease by a factor of a hundred for a thickness of 500 μ. The use of photomultipliers makes it possible to observe intensities decreased by a factor of a thousand.

One can thus estimate from the magnitude of the angle of rotation the exciton radius, if its reduced mass is known. Moreover, the selection rules entering into $T_{\mu\nu}^{nm}$ may turn out to be an important aid for interpreting exciton spectra.

The authors express their deep gratitude to L. É. Gurevich, O. V. Konstantinov, and G. M. Éliashberg for valuable discussions of this paper.

[1] L. D. Landau and E. M. Lifshitz, Elektrodinamika sploshnykh sred (Electrodynamics of Continuous Media) Gostekhizdat, 1957, part II, Sec. 82.

[2] R. F. Kazarinov and O. V. Konstantinov, JETP **40**, 936 (1961), Soviet Phys. JETP **13**, 654 (1961).

[3] R. J. Elliott, Phys. Rev. **108**, 1384 (1957).

[4] D. S. Bulyanitsa, Vestnik LGU, No. 4 (1960).

[5] I. Pastrnyak, Opticheskie svoĭstva zakisi medi (Optical Properties of Cuprous Oxide) Thesis, Physico-Technical Inst., 1959.

Translated by D. ter Haar
42