SCATTERING OF K MESONS ON NUCLEONS AT LARGE ORBITAL MOMENTA

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Peripheral KN scattering is investigated by taking into account only two-pion exchange. The KN-scattering phase shifts are expressed in terms of the $K\pi$ coupling constant. A phase-shift analysis of experimental data on KN scattering, in which high phase shifts $(l \gtrsim 2)$ are taken into account, can yield in principle information on the magnitude of the $K\pi$ interaction. The contribution of the single-hyperon diagram to the KN-scattering phase shift is discussed.

OKUN' and Pomeranchuk[1] have shown that in KN scattering with large orbital momenta the principal role is played by the two-pion diagram shown in Fig. 1. The scattering phase shifts at large orbital momenta are determined by the absorptive part of the amplitude in the vicinity of the nearest singularity of the transferred momentum q^2 (cf. reference 2). In this case the nearest singularity is located at $q^2 = 4\mu^2$, and the next singularity is only at $q^2 = 16\mu^2$. The large distance between these nearest singularities gives grounds for hoping that the two-pion diagram will make the main contribution to the phase shift of the KN scattering even when the orbital momenta are not very large.

According to the unitarity relation, the absorptive part of the diagram of Fig. 1 is determined by the amplitudes of the πN and $K\pi$ scattering. The general expression for the amplitude of the $K\pi$ scattering is

$$F_{\alpha\beta}(k_1, k_2, k_3, k_4) = F^{(1)}\delta_{\alpha\beta} - i\varepsilon_{\alpha\beta\gamma}\tau_\gamma^K F^{(2)}, \qquad (1)$$

where $F^{(1)}$ and $F^{(2)}$ are invariant functions of the scalar products of the momenta k_1, k_2, k_3, and k_4; α and β are the isotopic variables of the pion; τ^K is the isotopic spin of the kaon. The following symmetry conditions apply: as $k_3 \to k_4$ we have $F^{(1)} \to F^{(1)}$ and $F^{(2)} \to -F^{(2)}$; k_3 and k_4 are the pion momenta before and after scattering.

The amplitude of the πN scattering near the point $Q^2 = 4\mu^2$ has been calculated by Galanin et al.[2] and has the form

$$f_{\alpha\beta}\left(p, p'; k+\frac{q}{2}; k-\frac{q}{2}\right) = g^2\,\bar{u}(p')\left\{\tau_\beta^N\tau_\alpha^N\frac{\hat{k}}{\mu^2 - q^2/2 - 2kp} - \tau_\alpha^N\tau_\beta^N\frac{\hat{k}}{\mu^2 - q^2/2 + 2kp} + \frac{\alpha}{m}\delta_{\alpha\beta}\right\}u(p). \qquad (2)$$

The notation in (2) is as follows: $k_3 = k + q/2$; $k_4 = k_{q/2} - k$; p and p′ are the nucleon momenta before and after scattering; τ^N is the isotopic spin of the nucleus; $\bar{u}(p')$ and $u(p)$ are four-component spinors describing the initial and final spin states of the nucleus, respectively; $g^2 = 14.5$ and $\alpha \approx 1.2$.

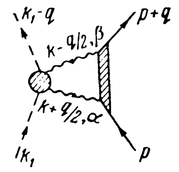

FIG. 1. Principal diagram of KN scattering at large orbital momenta.

The absorptive part of the amplitude of the KN scattering in the two-pion approximation, with allowance for Eqs. (1) and (2), can be readily calculated:

$$A(E, x) \approx \frac{3\lambda g^2\,\bar{u}(p')\,u(p)}{4(E+\omega)}\left[(\alpha - 1)\,x + \frac{\varepsilon}{2}\tan^{-1}\frac{2x}{\varepsilon}\right], \qquad (3)$$

where $x^2 = q^2/4\mu^2 - 1$, $\epsilon = \mu/m$ (μ is the pion mass and m the nucleon mass), ω is the kaon energy in the c.m.s., and E is the nucleon energy.

In the derivation of (3) it was assumed that the amplitudes of the $K\pi$ scattering are smooth functions of k in the region $|k^2_{\text{eff}}| \lesssim \mu^2/L$ (cf. references 2 and 3), and terms which are small near $q^2 = 4\mu^2$ were disregarded. The amplitude $F^{(2)} = \lambda$ was taken out at the point $q^2 = 4\mu^2$ and $k^2 = 0$ (see also reference 3), and the amplitude $F^{(2)}$ was discarded in view of the oddness in k. If we use for the $K\pi$ interaction the Lagrangian $\mathcal{L} = -4\pi\lambda\varphi_K^+\varphi_K\varphi_\alpha^2$, then λ is the constant of the πK interaction. If the functions $F^{(1)}$ and $F^{(2)}$ are not sufficiently smooth near this point, the expression (3) changes appreciably.

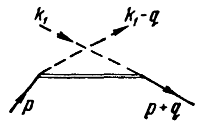

FIG. 2. Single-hyperon diagram of KN scattering.

The phase shifts of the KN scattering are expressed in terms of the absorptive part (3) as follows:[3]

$$\frac{(l+1)\,\delta_{l+}-l\delta_{l-}}{2l+1}\approx\frac{2\mu\xi}{\pi}\,Q_l\,(1+2\xi^2)\int\limits_0^\infty dx^2\,e^{-Lx^2}\,A_0\,(x^2),$$

$$\frac{l\,(\delta_{l+}-\delta_{l-})}{2l+1}\approx\frac{4\mu^3\sqrt{1+\xi^2}}{\pi}\,Q_l\,(1+2\xi^2)\int\limits_0^\infty dx^2\,e^{-Lx^2}\,A_1\,(x^2), \tag{4}$$

where $\xi=\mu/p$, $L=(l+1)\,\xi/\sqrt{1+\xi^2}$, $Q_l(1+2\xi^2)$ is the Legendre function of the second kind, A_0 and A_1 are the absorptive parts of the KN scattering amplitude without and with spin flip, respectively, δ_l^+ and δ_l^- are the KN-scattering phase shifts, corresponding to a total momentum $j=l\pm 1/2$. Formulas (4) are valid for $L\gg 1$. We note that by virtue of the assumed smooth behavior of the amplitude of the $K\pi$ scattering, the phase shifts are independent of the isotopic variables: $\delta_{l\pm}^{T=1}\approx\delta_{l\pm}^{T=0}$.

If we are interested in orbital momenta that are not very large, $L\ll 4m^2/\mu^2=180$, we readily obtain from (3) and (4)

$$\frac{(l+1)\,\delta_{l+}+l\delta_{l-}}{2l+1}\approx\frac{3g^2\,\lambda\mu\xi Q_l\,(1+2\xi^2)}{4\sqrt{\pi}L^{3/2}\,(E+\omega)}\left[\alpha-1+\sqrt{\pi}\,\zeta\left(1-\frac{2\zeta}{\sqrt{\pi}}\right)\right],$$

$$\frac{l\,(\delta_{l+}-\delta_{l-})}{2l+1}\approx\frac{3g^2\,\lambda\mu^2\epsilon Q_l\,(1+2\xi^2)}{4\sqrt{\pi}L^{3/2}\,(E+m)\,(E+\omega)}\sqrt{1+\xi^2}\left[\alpha-1+\sqrt{\pi}\zeta\left(1-\frac{2\zeta}{\sqrt{\pi}}\right)\right], \tag{5}$$

where $\zeta=\epsilon L^{1/2}/2\ll 1$ and $1\ll L\ll 180$.

It follows from (5) that in the approximation which is nonrelativistic with respect to the nucleon, $p\ll m$, but $p\sim\mu$ and $l\gg 1$, the ratio is

$$\frac{\delta_{l+}-\delta_{l-}}{\delta_{l+}+\delta_{l-}}\approx\frac{1}{\sqrt{2}}\left(\frac{\mu}{m}\right)^2,\ \text{i.e.}\ \delta_{l+}\approx\delta_{l-}. \tag{6}$$

This conclusion is also the consequence of the assumed smooth behavior of the amplitude of the $K\pi$ scattering. We note that by virtue of this assumption the peripheral phases of the elastic $\tilde{K}N$ scattering are also calculated by formula (5) ($\tilde{K}$ is the symbol for the anti-kaon).

In the analysis of the πN scattering[3] it was found that at small orbital momenta, $l\sim 2$, an important role may be played by the single-nucleon pole diagram, and therefore the two-pion approximation is not suitable for such small orbital momenta.

In the case of KN scattering, even a rough estimate* of the contribution of the single-hyperon states (Fig. 2) shows that regardless of the type of the KNY coupling (Y stands for the hyperon), the two-pion D phases predominate (accurate to 10 — 15%) over the single-hyperon phases in the kaon energy region $\omega_{\text{kin}}^{\text{lab}}\lesssim 180$ Mev, while the F phases predominate in the region $\omega_{\text{kin}}^{\text{lab}}\lesssim 400$ Mev. The calculated values of the phase shifts (for $\lambda<5$) do not exceed the experimental errors.[5]

Thus, the phase-shift analysis of the KN scattering, with allowance for the large phases ($l\gtrsim 2$), can in principle yield information on the magnitude of the $K\pi$ interaction, and also throw light on the limits of applicability of the two-pion approximation and its practical significance.

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[4] R. T. Matthews and A. Salam, Phys. Rev. **110**, 569 (1958).

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*For a rough estimate we used the KNY coupling constants given by Matthews and Salam,[4] and the $K\pi$ interaction constant was estimated at $\lambda\sim(m_K+\mu)\sqrt{\sigma_{K\pi}/4\pi}$, where $\sigma_{K\pi}$ is the $K\pi$-scattering cross section at low energies (it was assumed that $\sigma_{K\pi}\sim 1/\mu^2$).