ANGULAR CORRELATIONS IN STATISTICAL NUCLEAR REACTIONS

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The correlation between the directions of emission of particles emitted in succession by a nucleus with a large angular momentum is considered. Simple analytical expressions have been obtained for the angular correlation of two particles under the condition that the final nucleus remains in a state with a small angular momentum. The correlations between the directions of emission of a particle and of the fission fragments accompanying the particle emission are also considered.

THE dependence of the nuclear level density on the angular momentum of the nucleus leads to an anisotropy in the angular distribution of particles emitted by a compound nucleus.[1-3] A similar effect leads to an anisotropy in the fission of the nucleus. When two particles are emitted in succession or when fission takes place after the emission of a particle, an angular correlation occurs between the direction of emission of the particles or between the direction of emission of the particles and the direction of the fragments. As distinct from the particle angular correlations, usually considered in nuclear physics when one has in mind transitions between strictly defined quantum states, individual levels of the nucleus are not allowed in statistical nuclear reactions, and the angular correlation of the particles, as well as the angular distribution, arises as a result of averaging over a large number of nuclear states. The effects arising in this way can usually be given a classical interpretation (see references 1–3). In this connection, it is entirely natural to apply quasi-classical methods of calculation.

ANGULAR CORRELATION OF PARTICLES

We shall first consider the simple case of a cascade consisting of only two particles. The probability that a nucleus with an angular momentum $\mathbf{j}_1$ emits the first particle in the direction $\mathbf{n}_1$ and the second particle in the direction $\mathbf{n}_2$ is

$$W_{\mathbf{j}_1}(1, 2)\, d\Omega_1 d\Omega_2 = \iint \frac{d\Gamma^{(1)}_{p_1}(\mathbf{j}_1, \mathbf{j}_2; \mathbf{l}_1, \mathbf{n}_1)}{\Gamma^{(1)}_{tot}(j_1)} \frac{d\Gamma^{(2)}_{p_2}(\mathbf{j}_2, \mathbf{j}_3; \mathbf{l}_2, \mathbf{n}_2)}{\Gamma^{(2)}_{tot}(j_2)}, \tag{1}$$

where $d\Gamma^{(1)}_{p_1}(\mathbf{j}_1, \mathbf{j}_2, \mathbf{l}_1, \mathbf{n}_1)$ is the partial width for the emission of a particle p_1 with orbital angular momentum $\mathbf{l}_1$ in the direction $\mathbf{n}_1$ by a nucleus with an angular momentum $\mathbf{j}_1$; $d\Gamma^{(2)}_{p_2}$ is the analogous quantity for the emission of the second particle; $\mathbf{j}_1$, $\mathbf{j}_2$, and $\mathbf{j}_3$ are the angular momenta of the initial, intermediate, and final nuclei, respectively, and $\Gamma^{(1)}_{tot}$ and $\Gamma^{(2)}_{tot}$ are the total decay widths of the initial and intermediate nuclei:

$$\Gamma^{(1)}_{tot} = \Gamma^{(1)}_{p_1}(j_1) + \Gamma^{(1)}_{n}(j_1) + \ldots$$

and similarly for $\Gamma^{(2)}_{tot}$; $\Gamma^{(1)}_{p_1}(j_1)$ is the total width for the emission of particle p_1 by the initial nucleus and $\Gamma^{(2)}_{p_2}(j_2)$ is the analogous quantity for the emission of particle p_2 by the intermediate nucleus:

$$\Gamma^{(1)}_{p_1}(j_1) = \int d\Gamma^{(1)}_{p_1}(\mathbf{j}_1, \mathbf{j}_2; \mathbf{l}_1, \mathbf{n}_1). \tag{2}$$

For $d\Gamma^{(1)}_{p_1}$ we can use the quasi-classical expression[1-3]

$$d\Gamma^{(1)}_{p_1}(\mathbf{j}_1, \mathbf{j}_2; \mathbf{l}_1, \mathbf{n}_1) = C_1 \left(\rho^{(2)}(j_2)/\rho^{(1)}(j_1)\right) T_1(\mathbf{l}_1) \times \delta^3(\mathbf{j}_1 - \mathbf{j}_2 - \mathbf{l}_1)\, \delta(\mathbf{l}_1\mathbf{n})\, d^3j_2 d^3 l_1 d\Omega_1. \tag{3}$$

In formula (3), $\delta^3(\mathbf{x})$ and $\delta(x)$ are the three-dimensional and one-dimensional δ-functions, respectively; $T_1(l_1)$ and $T_2(l_2)$ are the barrier factors for particles p_1 and p_2; $\rho^{(1)}$, $\rho^{(2)}$, and $\rho^{(3)}$ are the level densities of the initial, intermediate, and final nuclei. The explicit form of the dependence of these factors and the coefficients C_1 and C_2 on the energy will not be necessary for the discussion below.

Using (3), we obtain for the total partial width $\Gamma^{(2)}_{p_2}(j_2)$

$$\Gamma^{(2)}_{p_2}(j_2) = 4\pi C_2 \int l_2^{-1} T_2(l_2) \frac{\rho^{(3)}(\mathbf{j}_2 - \mathbf{l}_2)}{\rho^{(2)}(j_1)} d^3 l_2. \tag{4}$$

The quantity $\Gamma^{(1)}_{p_1}(j_1)$ is expressed in a similar way. Substituting (3) and (4) into (1), we can represent $W_{j_1}(1, 2)$ in the form

$$W_{\mathbf{j}}(1, 2) = \frac{1}{(2\pi)^2} \iint d^3l_1\, d^3l_2 \gamma^{(1)}_{p_1}(j_1)\, \gamma^{(2)}_{p_2}(j_2)\, \rho^{(2)}(\mathbf{j}_1 - \mathbf{l}_1) \times \rho^{(3)}(\mathbf{j}_1 - \mathbf{l}_1 - \mathbf{l}_2) \frac{T_1(l_1)\, T_2(l_2)}{N_1(j_1)\, N_2(|\mathbf{j}_1 - \mathbf{l}_1|)} \delta(\mathbf{l}_1\mathbf{n}_1)\, \delta(\mathbf{l}_2\mathbf{n}_2), \tag{5}$$

where $\gamma_{p_1}^{(1)}(j_1)$ is the relative probability for the emission of particle p_1 by a nucleus with an angular momentum j_1:

$$\gamma_{p_1}^{(1)}(j_1) = \Gamma_{p_1}^{(1)}(j_1)/\Gamma_{tot}^{(1)}(j_1). \qquad (6)$$

$N_1(j_1)$ and $N_2(j_2)$ in (5) are the normalizing factors:

$$N_1(j_1) = \frac{1}{2\pi}\iint d^3\Omega_1 d^3 l_1 T_1(l_1)\,\rho^{(2)}(\mathbf{j}_1 - \mathbf{l}_1)\,\delta(\mathbf{l}_1\mathbf{n}_1)$$
$$= \int \frac{d^3 l_1}{l_1} T_1(\mathbf{l}_1)\,\rho^{(2)}(\mathbf{j}_1 - \mathbf{l}_1). \qquad (7)$$

$N_2(j_2)$ is expressed in a similar way.

As can be seen from (5),

$$\iint W_{j_1}(1, 2)\, d\Omega_1 d\Omega_2$$
$$= \gamma_{p_1}^{(1)}(j_1)\int \frac{d^3 l_1}{l_1}\frac{T_1(l_1)\,\rho^{(2)}(\mathbf{j}_1 - \mathbf{l}_1)}{N_1(j_1)}\,\gamma_{p_2}^{(2)}(|\mathbf{j} - \mathbf{l}|) \qquad (8)$$

represents the relative probability of emission of a pair of particles p_1 and p_2 by a nucleus with an angular momentum j_1.

Generally speaking, the dependence of the integrand of (5) on the orientation of the momenta is determined if the dependence of the level density of the nucleus on the angular momentum is known. In the statistical theory of the nucleus

$$\rho(j) = \rho_0(u)\exp(-\alpha j^2), \qquad (9)$$

where $\alpha = \hbar^2/2\mathcal{J}T$, $\mathcal{J} = 2AmR^2/5$ is the moment of inertia, and T is the temperature of the nucleus. For such a form of $\rho(j)$, we cannot integrate over $\mathbf{l}_1$ and $\mathbf{l}_2$ in (5) in general form.

We shall consider the important special case in which the nucleus remains in a state with small angular momentum after the emission of the second particle. Setting in (5)

$$\rho^{(3)}(\mathbf{j}_1 - \mathbf{l}_1 - \mathbf{l}_2) = \rho_0\delta^3(\mathbf{j}_1 - \mathbf{l}_1 - \mathbf{l}_2), \qquad (10)$$

we obtain

$$W_j(1, 2) = \frac{1}{2\pi^2}\gamma_{p_1}^{(1)}(j_1)\,N_1^{-1}(j_1)\int d^3 l_1 f(|\mathbf{j}_1 - \mathbf{l}_1|)\,T_1(l_1)$$
$$\times\,\delta(\mathbf{l}_1\mathbf{n}_1)\,\delta(\mathbf{j}_1\mathbf{n}_2 - \mathbf{l}_1\mathbf{n}_2), \qquad (11)$$

where

$$f(|j_2|) = \gamma_{p_2}^{(2)}(j_2)\,j_2\,\rho^{(2)}(j_2), \qquad \mathbf{j}_2 = \mathbf{j}_1 - \mathbf{l}_1.$$

Assuming that $j \gg l_1$, we expand the function $f(j_2)$ into a series in the small quantity $x = (\mathbf{l}_1\mathbf{j}_1)/j_1^2$:

$$f(j_2) \approx f(j_1)(1 + a_1 x + a_2 x^2 + \ldots). \qquad (12)$$

The coefficients a_1 and a_2 are

$$a_1 = 2\alpha_2 j_1^2 - 1 + q_1,$$

$$a_2 = 2\alpha_2^2 j_1^4 - \tfrac{1}{2} - 2\alpha_2 j_1^2 + q_2 - q_1 + 2\alpha_2 j_1^2 q_1, \qquad (13)$$

where q_1 and q_2 are determined from the equality

$$\gamma_{p_2}^{(2)}(j_2) = \gamma_{p_2}^{(2)}(j_2)(1 + q_1 x + q_2 x^2 + \ldots). \qquad (14)$$

We shall consider the important special case in which one counter is located at a right angle to the beam and another is moved either in the plane of the beam or in a plane perpendicular to it. We denote by θ_1, θ_2, Φ_1, Φ_2 the angular coordinates of the directions $\mathbf{n}_1$ and $\mathbf{n}_2$, and by ϑ_l, φ_l, ϑ_f, φ_f the angular coordinates of the vectors l_1 and j_1. Of course, $\vartheta_j = \frac{1}{2}\pi$. We shall consider these special cases separately.

1. Both counters are in the perpendicular plane ($\theta_1 = \theta_2 = \frac{1}{2}\pi$). After integrating in (5) over the directions of the vector $\mathbf{l}_1$, we obtain

$$W_j(1, 2) = \frac{2}{(2\pi)^2}\gamma^{(1)}(j_1)\frac{f(j_1)}{N_1(j_1)}\int_0^\infty \frac{l_1\,dl_1\,T_1(l_1)(1 + a_1\bar{x} + a_2\bar{x}^2 + \ldots)}{[l_1^2\sin^2\omega - (\mathbf{j}_1\mathbf{n}_2)^2]^{1/2}},$$

$$\bar{x} = \cos(\varphi_j - \omega)\sin\varphi_j/\sin\omega, \qquad (15)$$

where ω is the angle between the counters ($\omega = \Phi_1 - \Phi_2$). Expression (15) should be integrated over all values of the angle φ_j for which the expression in the denominator of the integrand is positive. The result is expressed in terms of elliptic integrals. In the limiting case of practical interest $j \gg l$, we obtain

$$W_j = \frac{1}{16\pi^2}\gamma_{p_1}^{(1)}(j)\,\gamma_{p_2}^{(2)}(j)\left(1 + \frac{a_1 - a_2}{2}\frac{\overline{l^2}}{j^2}\sin^2\omega\right),$$

$$\overline{l^2} = \int_0^\infty l^3 T_1(l)\,dl \Big/ \int_0^\infty l T_1(l)\,dl. \qquad (16)$$

In the derivation of (16) we took for $N_1(j)$ the approximate expression

$$N_1(j) \approx 4\pi\rho^{(2)}(j)\int_0^\infty l T_1(l)\,dl. \qquad (17)$$

We have discarded terms at least as small as l^4/j^4.

Expression (16) should still be integrated over $|\mathbf{j}|$, where we must take into account the probability of capture of an incident particle with an angular momentum j:

$$\overline{W}_j(1, 2) = \int_0^\infty j^3 T_i(j)\,dj \Big/ \int_0^\infty j T_i(j)\,dj, \qquad (18)$$

where $T_i(j)$ is the coefficient of absorption of an incident particle with angular momentum j. If the emission of the second recorded particle is a single or basic process, then $\gamma_{p_2}^{(2)} \equiv 1$, and $q_1 = q_2 = 0$. In this case we obtain for $\overline{W}$

$$\overline{W}(1, 2) \approx 1 + \tfrac{1}{2}\alpha_2\overline{l^2}\left[4 - \tfrac{1}{2}(\alpha_2 j_0^2)^{-1}\ln(j_0^2/\overline{l^2})\right.$$
$$\left. - \alpha_2 j_0^2\right]\sin^2\omega. \qquad (19)$$

In the derivation of (19), $T_i(j)$ was taken in the form

$$T_i(j) = T_i^{(\mathrm{cl})}(j) = \begin{cases}1, & j \leqslant j_0\\ 0, & j > j_0\end{cases}. \qquad (20)$$

For an idea as to the size of the correlation effect, we shall consider the specific reaction Cu(α, 2p) studied by Lassen and Sidorov.[4] For an α particle energy of ~ 20 Mev and an energy of 6 – 9 Mev for the first proton, the intermediate nucleus can decay only by two modes: the emission of a sub-barrier proton with an energy of ~ 1 – 2 Mev or the radiation of a γ quantum. For $\alpha_2 = 0.05$, which corresponds to the rigid-body moment of inertia of the intermediate nucleus, and for $\overline{l^2} = 3 - 4$ and $j_0^2 = 50$, we obtain the value +0.1 for the coefficient of $\sin^2 \omega$ in (19). The magnitude and sign of the coefficient strongly depend on the relation between the parameters $\alpha_2\overline{l^2}$ and $\alpha_2\overline{j^2}$. For a larger value of j_0^2, for example, in the case of high-energy α particles, the coefficient drops to zero and then becomes negative.

The measurement of the angular correlation between two protons in the perpendicular plane in the reaction $Ni^{58}(\alpha, 2p)$ ($E_\alpha = 32$ Mev) was carried out by Bodansky et al.[5] Their result corresponds to a value of approximately −0.15 for the coefficient of $\sin^2 \omega$. This value is in agreement with (19). It is assumed here that the reaction can be regarded as a transition of an excited nucleus produced by the capture of an α particle to the ground state or close to the ground state through the emission of two protons, which is not an obvious assumption.

2. Counters and beam in the same plane. Two cases should be distinguished here: a) $\theta_2 = \frac{1}{2}\pi$, $\Phi_1 = \Phi_2 = 0$ or π and b) $\theta_1 = \frac{1}{2}\pi$. We shall first consider case a. Inserting (12) into the integral (11) and integrating over l and over the directions of the vector $\mathbf{j}$, we obtain, in the same approximation as in (19),

$$W_j(1, 2) = (1/16\pi^3)\, \gamma_{p_1}^{(1)}(j)\, \gamma_{p_2}^{(2)}(j)\, (1 + \overline{(l^2/2j^2)}) \times (\tfrac{1}{2} + a_1)\cos^2\theta_1). \qquad (21)$$

Apart from terms of order higher than l^4/j^4, the angular distribution of the first particle for a fixed direction of emission of the second is determined in this case by the single coefficient a_1.

Inserting in (21) the expression (13) for a_1 and setting $q_1 = q_2 = 0$ in the latter, we obtain, after integration over j,

$$\overline{W}(1, 2) \sim 1 + [\alpha_2\overline{l^2} - (\overline{l^2}/2j_0^2)\ln(j_0/l_0)]\cos^2\theta_1. \qquad (22)$$

As in the case of (19), the coefficient of $\cos^2\theta_1$ can change sign if the angular momentum of the compound nucleus changes. If the angular momentum is large, so that $\alpha_2 j^2 \gg 1$, then the angular distribution of the first particle does not depend on j if the direction of emission of the second is fixed at the angle $\theta_2 = \frac{1}{2}\pi$. This is in contrast to the usual angular distribution of particles in which the anisotropy equals $\frac{1}{2}\alpha_2^2\overline{l^2}\overline{j^2}$.[1,2]

We shall consider case b, in which $\theta_1 = \pi/2$. Here, after integration over the directions of the vectors $\mathbf{l}$ and $\mathbf{j}$, we obtain from (11)

$$W_j(1, 2) = \frac{1}{4\pi^2}\frac{\gamma_{p_1}^{(1)}(j)\, f(j)}{jN(j)} \int_0^\infty W_{l,j}\, lT_1(l)\, dl,$$

$$W_{l,j}(1, 2) = \sin^{-1}\theta_2\, [A(k) + (a_2 l^2/3j^2)\, B(k)];$$

$$A(k) = \frac{2}{\pi} \times \begin{cases} K(k), & k = l \operatorname{tg}\theta_2/j \leqslant 1, \\ k^{-1}K(k^{-1}), & k \geqslant 1, \end{cases}$$

$$B(k) = \frac{2}{\pi} \times \begin{cases} K(k) + E(k) - k^{-2}[K(k) - E(k)], & k \leqslant 1, \\ k^{-1}[K(k^{-1}) + E(k^{-1}) - k^2(K(k^{-1}) - E(k^{-1}))], & k \geqslant 1, \end{cases} \qquad (23)^*$$

where K(k) and E(k) are the complete elliptic integrals of the first and second kinds, respectively:

$$K(k) = \int_0^{\pi/2} (1 - k^2\sin^2\varphi)^{-1/2}\, d\varphi,$$

$$E(k) = \int_0^{\pi/2} (1 - k^2\sin^2\varphi)^{1/2}\, d\varphi.$$

The function $W_{j,l}(1, 2)$ describes the angular correlation of the particles for fixed j and l. The dependence of $W_{j,l}$ on the angle θ_2 is determined mainly by the factor $1/\sin\theta_2$, which results from the fact that for $l_1 \ll j$, the orbital angular momentum of the second particle has a direction close to $\mathbf{j}$ and, consequently, lies in the plane perpendicular to the beam. The deviations of $\mathbf{l}_2$ from this plane, which are connected with the fact that the first particle carries away a certain angular momentum, lead to a deviation of the angular distribution from $1/\sin\theta_2$. These deviations are described by the factor in the brackets in the expression for $W_{j,l}$. The function $W_{j,l}$ strongly depends on the angle in the small-angle region $\theta_2 \lesssim l_1/j$. At large angles, the deviations from $1/\sin\theta_2$ are small ($\sim l_1^2/j^2$):

$$\overline{W} \sim \sin^{-1}\theta_2\, \{1 + \overline{(l_1/4j^2)}\operatorname{ctg}^2\theta_2 + \ldots\}. \qquad (24)^\dagger$$

In (24), we have discarded terms of the order $l_1^4 \times \cot^4\theta_2/j^4$.

It is natural to compare the angular distribution (23) of the second particle for a fixed direction of emission of the first particle with the angular distribution of the second particle for the free emission of the first. Normalized to unity for $\theta_2 = \theta/2$, this angular distribution is given by the expression

*tg = tan.

†ctg = cot.

$$[\sin^{-1}(l/j\sin\theta_2)]/\sin^{-1}(l/j), \tag{25}$$

which is readily obtained from (11). Comparing (25) and (24), we find that, for $\theta_2 \gtrsim l/j$, the angular distribution of the second particle, when the counter registering the first particle is connected, changes by the quantity

$$\Delta W_{l,j} = \tfrac{1}{12}\sin^{-1}\theta_2\,\overline{(l^2/j^2)}\,\mathrm{ctg}^2\,\theta_2. \tag{26}$$

It should be borne in mind that the distribution $\sim 1/\sin\theta_2$ is valid only with quasi-classical accuracy in the region $\theta_2 \gtrsim l_1/j$ and is not at all valid for $\theta_2 \lesssim l_1/j$. In this case, the effect of the angular correlation can therefore be established only when the angular distribution of the second particle, for a fixed direction of emission of the first particle (close to $\theta_1 = \frac{1}{2}\pi$), is compared with the angular distribution for free emission. This is the essential difference between this case and the two preceding cases, for which the result of the zero approximation—isotropy—is known exactly and the inexactness of the quasi-classical approximation leads to an unimportant error in the value of the coefficients.

For $a_1 = a_2 = 0$, one can obtain the following general expression for the angular correlation function $W_{j,l}(1, 2)$, valid for any orientation of the counters relative to the beam and relative to each other $[\rho^{(3)}(j_3) = \rho_0\delta^3(j_3)]$:

$$W_{j,l} = \frac{2}{\pi}\sin^{-1}\theta_2 \times \begin{cases} K(\varkappa), & \varkappa = l\sin\gamma/j\sin\theta_2 \leqslant 1, \\ \varkappa^{-1}K(\varkappa^{-1}), & \varkappa \geqslant 1, \end{cases} \tag{27}$$

where γ is the angle between the counters.

If for the barrier factor $T_1(l)$ we use the classical expression similar to (2), then expression (27) for $W_{j,l}$ can be integrated over l. As a result, we obtain

$$W_j(1, 2)|_{\mathrm{av}\ l} = \sin^{-1}\theta_2 \times \begin{cases} \varkappa_0^{-1}E(\varkappa_0^{-1}), & \varkappa_0 = l_0\sin\gamma/j\sin\theta_2 \leqslant 1, \\ \varkappa_0^{-2}[E(\varkappa_0) - (1-\varkappa_0^2)K(\varkappa_0)], & \varkappa_0 \geqslant 1. \end{cases} \tag{28}$$

Besides the correlation of two particles when the final nucleus remains in a state with a small angular momentum, as was considered above, other cases may also be of interest. In particular, with the aid of (11), it is not difficult to calculate the angular correlation between two particles when, for example, the level density of the final nucleus as well as the initial and intermediate nuclei weakly depend on the angular momentum. This can happen if the excitation energy of the final nucleus is comparatively high and the angular momentum of the initial nucleus is not very large. It is also easy to obtain the generalization of formula (5) to the case of a large number of particles in cascade. The probability of registering m particles in the directions $n_1, n_2, \ldots, n_m$ is

$$W_{j_1}(1,\ldots,m) = \frac{1}{(2\pi)^2}\int d^3l_1\ldots d^2l_m \prod_{i=1}^{m} \gamma_{p_i}^{(i)}(j_i)\,\rho^{(i+1)}(j_{i+1}) \times T_i(l_i)N_i^{-1}(j_i)\delta(\mathbf{l}_i\mathbf{n}_i),$$

$$\mathbf{j}_1 = \mathbf{j}_1 - \sum_{k=1}^{i-1}\mathbf{l}_k. \tag{29}$$

If the direction of emission of one of the particles in the experiment is not registered, expression (29) should be integrated over the angular variables of the corresponding particle. The angular correlation in this case depends also on the parameters characterizing the emission of the unregistered particle. This circumstance causes the results of the angular correlation experiment to be less definite. If, however, the unobserved particle is light and the registered particles are heavy, for example, α particles, the correlation will be approximately the same as if only two particles were emitted.

If identical particles are registered, the expression for the angular correlation should be symmetrized in a suitable way. This may not be necessary, however, if the energy of the particles is also registered in the experiment. Since the probability of emission of a particle strongly depends on the particle energy and the excitation energy of the nucleus, it is often possible to establish which particle is first and which is second. This applies, in particular, to the above-mentioned reaction[4] Cu (α, 2p), where the second proton is a sub-barrier particle and consequently the probability that a particle of such energy is emitted from the nucleus first is negligibly small.

ANGULAR CORRELATION BETWEEN A PARTICLE AND FISSION FRAGMENTS

Formally, this case of angular correlation can be regarded as a special case of the angular correlation of two particles. In formula (5), instead of the level density of the final nucleus $\rho^{(3)}$, we should insert the product of the level densities of the fragments and introduce the corresponding summation over the fragment spins. It will be more convenient, however, to consider first the distribution of the number of total spin states of the fragments $\mathbf{S} = \mathbf{j}' + \mathbf{j}''$. This distribution coincides formally with (9), where for the constant α we have

$$\alpha \equiv 1/2K_0^2 = (1/\mathcal{J}' + 1/\mathcal{J}'')T^*,$$

here $\mathcal{J}'$, $\mathcal{J}''$, and T* are the moments of inertia and temperature of the fragments.[6] Hence, to calculate the angular correlation between the particle and the fragments, one can use directly formula (5), where $\rho^{(3)}(j_3)$, $\mathbf{j}_3$, and $\mathbf{l}_2$ are replaced by the distribution function $\rho(\mathbf{S})$, the vector $\mathbf{S}$, and the orbital angular momentum of the fragments $\mathbf{l}'$, respectively. The barrier factor $T_f(l')$ can be set equal to unity, since the mass of the fragments is large and the classical limitation on the size of the angular momentum is not important, even when the energy of the nucleus is only slightly above the fission threshold. The integration over l' can then be carried out in general form. After integrating over l', we obtain

$$W_{\mathbf{j}}(p, f) = \frac{1}{(2\pi)^2}\frac{\gamma_p^{(1)}(j_1)}{N_1(j_1)}\int d^3 l_1 \gamma_f^{(2)}(j_2)\, \rho^{(2)}(j_2)\, \delta(\mathbf{l}_1\mathbf{n}_1)\, \mathfrak{R}(\mathbf{j}_2\mathbf{n}_f), \tag{30}$$

where $\mathbf{j}_2 = \mathbf{j}_1 - \mathbf{l}_1$, and $\mathbf{n}_1$ and $\mathbf{n}_f$ are the directions of emission of the particle and the fragments; $\gamma_f^{(2)}$ is the relative probability of fission of the intermediate nucleus; the function

$$\mathfrak{R}(\mathbf{j}_2\mathbf{n}_f) = R(j^2/2K_0^2)\exp(-K^2/2K_0^2),\quad K = \mathbf{j}_2\mathbf{n}_f \tag{31}$$

is the projection of the angular momentum of the intermediate nucleus on the direction of fission. In (31)

$$R(z) = \int_{-1}^{+1} e^{-zx^2}dx.$$

Function (31) gives the distribution of the number of cases of fission with a value K. The expression given above for K_0^2 corresponds to the assumption made in the derivation of (30) that the fission probability is determined by the statistical weight of the fragments. One may also assume that the probability of fission with a given value of $|K|$ is determined by the statistical distribution of K in the transition nucleus for a deformation approximately corresponding to a saddle point. The distribution of K in this case coincides with (31), but the constant K_0^2 is equal to $T^{*-1}(\mathcal{J}_{\|}^{-1} - \mathcal{J}_{\perp}^{-1})^{-1}$, where $\mathcal{J}_{\|}$ and $\mathcal{J}_{\perp}$ are the moments of inertia of the transition nucleus relative to the axis of symmetry and perpendicular to it, T^* is the transition nucleus temperature (see also reference 6).

It should be kept in mind that, strictly speaking, the angular distribution of the fragments is determined by the distribution of K at infinity.[7] Since the projection of the orbital angular momentum of the fragments on the direction of fission is equal to zero, the distribution of the projection K of the total angular momentum of the system $\mathbf{j} = \mathbf{l}' + \mathbf{S}$ on the fission direction coincides with the distribution of the resultant spin of the fragments on this direction. Thus, the problem is only whether the distribution of the projection of vector $\mathbf{S}$ is determined by the level density of the fragments or whether it coincides with the distribution of K in the "transition" nucleus. On the basis of the data available at the present time, one can conclude that the second assumption corresponds more closely to reality, at least for fission close to the threshold and for low excitation of the initial nucleus. From the formal point of view, the existence of these two different approaches is not important, since in both cases the distribution of K is given by the same expression, namely (31).

If the angular momentum of the compound nucleus is sufficiently large, so that the inequality

$$j^2/2K_0^2 \gg 1, \tag{32}$$

is satisfied, expression (31) for N(K) goes over into $j_2\delta(\mathbf{j}_2\mathbf{n}_f)$, and (30) then formally coincides with (11). Hence all the expressions obtained above for the angular correlation of particles also describe the correlation between the particle and the fragments for a nucleus with a sufficiently large angular momentum. An outward sign that condition (32) is fulfilled is the closeness of the fragment angular distribution to $1/\sin\theta_f$.[8] This usually occurs in the case of fission induced by the capture of heavy ions. The large angular momentum of the compound nucleus formed upon capture of heavy particles also enhances the fissionability of the nucleus and, consequently, increases the probability that the nucleus undergoes fission immediately after the emission of the registered particle.

With the aid of formula (30), one can find an angular correlation between the particle and the fission fragments also when condition (32) is not fulfilled. We shall consider, in particular, the case in which

$$lj/2K_0^2 \ll 1. \tag{33}$$

The parameter $j^2/2K_0^2$ can then have any value, but cannot be so large that inequality (33) is not fulfilled. Making the same approximation as in the derivation of (16), we write $W_j(p, f)$ in the form

$$W_{\mathbf{j}}(p, f) \approx \frac{1}{(2\pi)^3}\gamma_p^{(1)}(j)\,\gamma_f^{(2)}(j)\,\mathfrak{R}(\mathbf{j}\mathbf{n}_f)\Big(4\pi\int_0^\infty l\,dl\,T(l)\Big)^{-1} \times \int d^3 l\, T(l)\,\delta(\mathbf{l}\mathbf{n}_1)\,(1 + r(\mathbf{l}\mathbf{j}) + \ldots)\exp\{(\mathbf{l}\mathbf{n}_f)[2(\mathbf{j}\mathbf{n}_f) - \mathbf{l}\mathbf{n}_f]/2K_0^2\}, \tag{34}$$

where

$$\mathbf{j} \equiv \mathbf{j}_1,\qquad \mathbf{l} = \mathbf{l}_1;$$

$$1 + r(\mathbf{l}\mathbf{j}) + \ldots = G(\mathbf{j} - \mathbf{l})/G(j), \tag{35}$$

$$G(j) = \gamma_f^{(2)}(j)\,\rho^{(2)}(j)\,R^{-1}(j^2/2K_0^2). \tag{36}$$

We shall consider the case in which the direction of emission of the particle and the direction of the fission fragments lie in a plane perpendicular to the beam. We expand the exponential factor in the integrand of formula (34) into a series and integrate over $\mathbf{l}$. After averaging over the directions of the vector $\mathbf{j}$, we obtain

$$W_{\mathrm{j}}(p, f) = (1/16\pi^3)\, \gamma_p^{(1)}(j)\, \gamma_f^{(2)}(j) \exp(-z/2)\, I_0(z/2)$$
$$\times \{1 - (\overline{l^2}/4K_0^2) \sin^2 \omega_f + (l^2 j^2/8K_0^4)(1 + 2K_0^2 r)$$
$$\times [1 - I_1(z/2)/I_0(z/2)] \sin^2 \omega_f + \ldots\}, \qquad (37)$$

where $z = j^2/2K_0^2$, I_0 and I_1 are Bessel functions of imaginary argument. Expression (37) should still be averaged over the quantity $|\mathbf{j}|$ with a weight $jT_i(j)$.

The coefficient r in (37) depends, generally speaking, on j. To determine r, we must use the explicit expressions for $\rho^{(2)}(j_2)$ (9), $R(z)$, and $\gamma_f^{(2)}(j_2)$. For $\gamma_f^{(2)}$, we can take the statistical expression

$$\gamma_f^{(2)}(j_2) = \{1 + \gamma_n^{(2)}(j_2)/\gamma_f^{(2)}(j_2)\}^{-1},$$

$$\gamma_n^{(2)}(j_2)/\gamma_f^{(2)}(j_2) = (\Gamma_n(0)/\Gamma_f(0)) \exp[(\alpha_3 - \alpha_1) j_2^2]\, R^{-1}(j_2^2/2K_0^2). \qquad (38)$$

Setting

$$\rho^{(2)}(j_2) \approx \rho^{(2)}(j)(1 + 2\alpha_2(\mathbf{lj}) + \ldots),$$

$$R(j_2^2/2K_0^2) \approx R(j^2/2K_0^2)(1 - s(\mathbf{lj}) + \ldots),$$

$$\gamma_f^{(2)}(j_2) \approx \gamma_f^{(2)}(j)(1 - t(\mathbf{lj}) + \ldots),$$

we obtain

$$r = 2\alpha_2 + s - t.$$

The coefficient r is a quantity of the same order of smallness as $1/2K_0^2$. The parameter $\overline{j^2}/2K_0^2$ can be determined independently with the help of the anisotropy of the fragment angular distribution.[8] Measurement of the angular correlation between the particle and the fragments permits one to obtain additional information concerning the orbital angular momentum of the particle and the dependence of the fission probability on the angular momentum.

We note that in formula (37) the dependence on the angle does not disappear for $j = 0$. This is connected with the fact that there is a known correlation between the directions of the two particles in a state with total angular momentum $j = 0$. The probability of the angle ω is proportional to the square of the Legendre polynomial

$$P_l^2(\cos\omega) \sim 1/\sin\omega \qquad (39)$$

for $l \gg 1$. This expression agrees with (16) if in the latter we set $j = 0$. The term in formula (37) not depending on j "trails along" with the correlation (39) and goes over into it for $l^2/2K_0^2 \gg 1$.

[1] V. M. Strutinskii, Сб. материалов Всесоюзн. конф. по ядерной реакции (Collection: All-Union Conference on Nuclear Reactions), Moscow, 1957.

[2] T. Erikson and V. Strutinski, Nuclear Phys. **8**, 284 (1958).

[3] T. Erikson, Nuclear Phys. **17**, 250 (1960).

[4] N. O. Lassen and V. A. Sidorov, Nuclear Phys. **19**, 579 (1960).

[5] D. Bodansky et al., Cyclotron Research, Univ. Washington Ann. Reports, 1960.

[6] V. M. Strutinskii, Атомная энергия (Atomic Energy) **2**, 508 (1957).

[7] V. M. Strutinskii, JETP **30**, 606 (1956), Soviet Phys. JETP **3**, 638 (1956).

[8] I. Halpern and V. Strutinski, Proc. of the Second Intern. Conf. on the Peaceful Uses of Atomic Energy, Geneva, 1958, Vol. 15, paper P/1513.

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