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MAGNETIC DIPOLE TRANSITIONS IN EVEN-EVEN NUCLEI WITH QUADRUPOLE COLLECTIVE EXCITATIONS

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It is shown that for collective models with quadrupole excitations the branching ratio and interference phase shift of M1 and E2 radiation in a mixed M1 + E2 nuclear transition do not depend on the specific structure of the collective nuclear models. The ratio of the quantities $\delta^2(M1/E2) = W(M1; I_1 \to I_2)/W(E2; I_1 \to I_2)$ for two different M1 + E2 transitions of a given nucleus is a function only of the transition energies and nuclear spin states involved in the radiation process. By comparing this quantity with the experimental data one can estimate to what extent the collective degrees of freedom are smeared out with increasing excitation energy of the nucleus.

1. It has been variously proposed to describe the spectra of even-even nuclei in the intervals $60 \le A \le 196$ and $A > 210$ by means of the vibration model,[1,2] the axial-rotator model,[1,2] and the non-axial rotator model,[5] in which quadrupole collective excitations of the nucleus are considered. Common to all these models is the assumption that the excitation spectrum and the radiation properties of the nucleus can be described in terms of the parameters $\alpha_{2\mu}$ of the deformation of the nuclear surface, and their derivatives $\dot{\alpha}_{2\mu}$, regarded as dynamic variables ($\alpha_{2\mu}$ and $\dot{\alpha}_{2\mu}$ are defined in the laboratory system throughout). It is assumed here that the coordinates $\alpha_{2\mu}$ and the generalized momenta $\pi_{2\mu} = B_{2\mu}\dot{\alpha}^*_{2\mu}$, where B_2 is the inertia parameter of the collective motion of the nucleus, obey the commutation rule

$$[\pi_{2\mu}\alpha_{2\mu'}]_- = -i\hbar\delta_{\mu\mu'}$$

and the corresponding excitations of the nucleus are of the boson type.

This naturally raises the question of how accurately a boson-type collective excitation can be built up from the nucleons of the nucleus. It is of interest to estimate experimentally the accuracy of separation of such excitations. As will be shown below, a study of the angular correlations in a cascade of gamma quanta, one of which is a mixed M1 + E2 radiation of the nucleus, enables us to make this estimate.

2. The angular correlation functions (with and without detection of the quantum polarization) in a cascade that includes the nuclear transition $I_1 \to I_2$ with mixed M1 + E2 radiation depend essentially on the branching ratios of the M1 and E2 transitions

$$\delta^2(M1/E2,\ I_1 \to I_2) = W(M1;\ I_1 \to I_2)/W(E2;\ I_1 \to I_2) \tag{1}$$

and the relative phase ξ of the reduced matrix elements of the E2 and M1 transitions, which we define through

$$\delta e^{i\xi} = i\sqrt{\frac{50}{3}}\frac{1}{\omega}\frac{\langle I_2 \| M1 \| I_1\rangle}{\langle I_2 \| E2 \| I_1\rangle}, \tag{2}$$

it being assumed here that δ is always positive; ω is the energy of the nuclear radiation transition $I_1 \to I_2$, expressed in $m_e c^2$ units (multiples of 0.511 Mev).

The reduced matrix elements of the M1 and E2 transitions are defined by the equations

$$\langle I_2 \| M1 \| I_1\rangle C^{I_1M_1}_{I_2M_21M} = \left\langle I_2 M_2 \left| \sum_{i=1}^{A} r_i \mathbf{Y}^{0*}_{1M}\mathbf{j}_i \right| I_1 M_1\right\rangle, \tag{3}$$

$$\langle I_2 \| E2 \| I_1\rangle C^{I_1M_1}_{I_2M_22M} = \left\langle I_2 M_2 \left| \sum_{p=1}^{Z} e r_p^2 Y^*_{2M} \right| I_1 M_1\right\rangle. \tag{4}$$

All the quantities are best defined in units for which $\hbar = m_e = c = 1$ and $e^2 = 1/137$; the nuclear radius is $R_0 = 0.43A^{1/3}e^2$, corresponding to $R_0 = 1.2 \times 10^{-13}A^{1/3}$ cm. $C^{a\alpha}_{b\beta c\gamma}$ is the Clebsch-Gordan coefficient and $\mathbf{Y}_{LM}$ are spherical vector harmonics (see reference 4).

In order to define δ and ξ uniquely, we give the angular correlation function for two gamma-quanta in the cascade $I_1(M1 + E2)I_2(L)I_3$, where L is the multipolarity of the second quantum

$$W(\theta_{12}) = \sum_{\varkappa=0,2,4} C_\varkappa P_\varkappa(\cos\theta_{12}); \tag{5}$$

$$C_\varkappa = C^{L1}_{L1\varkappa 0}(2\varkappa+1)\, u(\varkappa L I_2 I_3;\ L I_2)$$
$$\times \{C^{21}_{21\varkappa 0}\, u\,(2 I_1 \varkappa I_2; I_2 2) + \delta^2 C^{11}_{11\varkappa 0}\, u\,(1 I_1 \varkappa I_2;\ I_2 1)$$
$$+ 2\delta \cos \xi\, C^{10}_{10\varkappa 0}\, u(21\varkappa 1;\ 11)\, u\,(1 I_1 \varkappa I_2;\ I_2 2)\}. \qquad (6)$$

Here u(abcd, ef) is the normalized Racah function, tables for which are found in the paper by Yang.[5]

For the inverse cascade $I_3(L)I_2(M1 + E2)I_1$, the gamma-quantum correlation function is also given by (5) and (6), but it is necessary to reverse in (6) the sign of the interference term proportional to $\delta \cos \xi$. The angular correlation functions with and without detection of quantum polarizations are described in greater detail in the review of Biederharn and Rose.[6]

3. A sufficiently accurate correlation experiment will thus enable us to find two physical parameters of the radiative transition of the nucleus $I_1 \to I_2$, namely $\delta^2(M1/E2, I_1 \to I_2)$ and $\cos \xi$. Let us consider these quantities in the collective models of the nucleus.

In all the previously mentioned collective models of the nucleus[1-3] the operators of the E2 and M1 transitions in terms of the variables $\alpha_{2\mu}$ and $\dot{\alpha}_{2\mu}$ have the following form:

the E2 transition operator

$$e \sum_{p=1}^{Z} r_p^2 Y^*_{2M} \to \int \rho_p(\mathbf{r})\, r^2 Y^*_{2M}(\mathbf{r})\, dv \approx \frac{3}{4\pi} Z R_0^2 e \alpha_{2M} + \ldots, \qquad (7)$$

the M1 transition operator

$$\sum_{i=1}^{A} r_i \mathbf{Y}^{0*}_{1M} \mathbf{j}_i \to \int \mathbf{j}_N\, r \mathbf{Y}^{0*}_{1M}\, dv = \hat{J}^{(0)}_M + \hat{J}^{(1)}_M + \ldots \qquad (8)$$

Here

$$\hat{J}^{(0)}_M = -\sqrt{\frac{15}{4\pi}}\, g_R \frac{e}{M} \sum_{m\nu} C^{1M}_{2m2\nu}\, \alpha_{2\nu} B_2 \dot{\alpha}_{2m}, \qquad (9)$$

$$\hat{J}^{(1)}_M = -\frac{5\sqrt{3}}{2\pi}\, g_R \frac{e}{M} C^{20}_{2020} \sum_{m\nu\mu_1\mu_2} C^{1M}_{2m2\nu} C^{2\nu}_{2\mu_1 2\mu_2}\, \alpha_{2\mu_1} \alpha_{2\mu_2} B_2 \dot{\alpha}_{2m}, \qquad (10)$$

M is the mass of the nucleon in m_e units ($M = 1840$) and g_R is the gyromagnetic ratio for the collective motion of the nucleus in the hydrodynamic model, $g_R = Z/A$.

In the derivation of (7) — (9) it is usually assumed that the charge density of the nuclear transition, $\rho_p(\mathbf{r})$, is uniformly spread over the volume of the nucleus, and the collective current of the nuclear transition, $\mathbf{j}_N$ is determined in terms of the rate of flow of the nuclear liquid

$$\mathbf{V} = \tfrac{1}{2} \sum_m \dot{\alpha}_{2m} \nabla r^2 Y_{2m}. \qquad (11)$$

We note, however, that these model assumptions are essential only for the values of the coefficients of the operators of the E2 and M1 transitions of the nucleus, whereas the functional dependence of E2 and M1 operators on the variables α_{2m} and $\pi^*_{2m} = B_2 \dot{\alpha}_{2m}$ does not involve the models. In order for relations (7) and (8) to hold, it is sufficient to assume that the expansion terms containing the higher powers of α_{2m} and π_{2m} are small and can be neglected. We note also that the operators $B_2 \dot{\alpha}_{2m}$ and $\alpha_{2\mu}$ freely commute in (10).

The specific forms of the operators $\alpha_{2\mu}$ and $\pi_{2\mu}$ may differ in the vibration and rotation models; for the vibrator, $\alpha_{2\mu}$ and $\pi_{2\mu}$ can be represented in terms of operators of creation and annihilation of quadrupole phonons $\hat{c}^+_\mu$ and $\hat{c}_\mu$, while in the rotation models $\alpha_{2\mu}$ and $\pi_{2\mu}$ act on the angles of orientation of the deformed nucleus. In both models, however, the operator $\hat{J}^{(0)}_M$ (see reference 1) is proportional to the angular momentum operator of the nucleus I, for which we have according to Bohr[1]

$$(-1)^\nu \hat{I}_{-\nu} = i\sqrt{10} \sum_{mq} C^{1\nu}_{2m2q}\, \alpha_{2q} B_2 \alpha_{2m}, \qquad (12)$$

and in the particular case of the vibration model

$$(-1)^\nu \hat{I}_{-\nu} = \frac{i\sqrt{10}}{2} \sum_{mq} C^{1\nu}_{2m2q} [(-1)^m \hat{c}^+_{-m} \hat{c}_q - (-1)^q \hat{c}^+_{-q} \hat{c}_m]. \qquad (13)$$

Formula (13) for the operator $\hat{I}_{-\nu}$ holds also for the anharmonic vibrator, inasmuch as the state function $\Psi_{IM;i}$ of the vibrator can be expanded in terms of the states χ^n_{IM} of the harmonic vibrator (n is the number of phonons):

$$\Psi_{IM;\,i} = \sum_n a_{in} \chi^n_{IM}. \qquad (14)$$

Applying the operator $(-1)^\nu \hat{I}_{-\nu}$ (13) to $\Psi_{IM;i}$, we get

$$(-1)^\nu \hat{I}_{-\nu} \Psi_{IM;\,i} = \sum_n a_{in} (-1)^\nu \hat{I}_{-\nu}\, \chi^n_{IM}$$
$$= (-1)^\nu \sqrt{I(I+1)}\, C^{IM-\nu}_{IM1-\nu} \Psi_{IM-\nu;\,i}. \qquad (15)$$

Thus, regardless of the specific structure, the operator $\hat{J}^{(0)}_M$ makes no contribution to the radiative M1 transition of the nucleus, and the probability of the M1 transition is determined by the operator $\hat{J}^{(1)}_M$.

The operator $\hat{J}^{(1)}_M$ can be expressed in terms of the operator of the E2 transition, proportional to $\alpha_{2\mu}$, and the operator of angular momentum of the nucleus $\hat{I}_{-\nu}$:

$$\hat{J}^{(1)}_M = i g_R \frac{e}{M} \frac{5\sqrt{3}}{7\pi} \sum_{\nu\mu} C^{1M}_{1\nu 2\mu}\, \alpha_{2\mu} \hat{I}_{-\nu} (-1)^\nu. \qquad (16)$$

Relation (16) for $\hat{J}^{(1)}_M$ is not self-evident. It was obtained by Davydov and Filippov and given without proof in reference 7, where relation (16) was used to calculate the probability of the M1 transition between the states $2' \to 2$ of a non-axial nucleus.

Relation (16) can be obtained from (10) by using the following equation for the Clebsch-Gordan coefficients

$$\sum_{\mu_1\mu_2} C^{1M}_{2m2\nu} C^{2\nu}_{2\mu_1 2\mu_2} \xi_{\mu_1\mu_2} = -2\sqrt{\frac{5}{7}} \sum_{\mu_1\mu_2} C^{1M}_{1n2\mu_1} C^{1n}_{2m2\mu_2} \xi_{\mu_1\mu_2}, \quad (17)$$

where $\xi_{\mu_1\mu_2} = \xi_{\mu_2\mu_1}$ is an arbitrary symmetrical function of $\mu_1\mu_2$ [a derivation of (17) is given in the Appendix]. It is not necessary to consider here the specific action of the operators $\alpha_{2\mu}$ and $\pi_{2\mu}$ on the wave function of the nucleus. Relation (16) holds for any model with collective quadrupole excitations of the nucleus.

4. Using (16) and taking into account the result of the action of the operator $(-1)^\nu I_{-\nu}$ on the wave function of the nucleus Ψ_{IM} [see (15)], we obtain after summation over the magnetic quantum numbers the following expression for the matrix element of the M1 transition of the nucleus

$$\begin{aligned}\langle\Psi^*_{I_2M_2}|\hat{J}^{(1)}_M|\Psi_{I_1M_1}\rangle &= \langle I_2\|M1\|I_1\rangle C^{I_1M_1}_{I_2M_21M} \\ &= -ig_R\frac{e}{M}\frac{3\sqrt{5}}{7\pi}\sqrt{I_1(I_1+1)}\,u(I_2I_1 11; 2I_1) \\ &\times \langle I_2\|\alpha_2\|I_1\rangle C^{I_1M_1}_{I_2M_21M},\end{aligned} \quad (18)$$

and for the operator of the E2 transition we get

$$\begin{aligned}\left\langle\Psi^*_{I_2M_2}\left|\frac{3}{4\pi}ZeR_0^2\alpha_{2M}\right|\Psi_{I_1M_1}\right\rangle &= \langle I_2\|E2\|I_1\rangle C^{I_1M_1}_{I_2M_22M} \\ &= \frac{3}{4\pi}ZeR_0^2\langle I_2\|\alpha_2\|I_1\rangle C^{I_1M_1}_{I_2M_22M},\end{aligned} \quad (19)$$

where we have from the definition of the reduced matrix element

$$\langle I_2M_2|\alpha_{2M}|I_1M_1\rangle = C^{I_1M_1}_{I_2M_22M}\langle I_2\|\alpha_2\|I_1\rangle. \quad (20)$$

The quantity $\langle I_2\|\alpha_2\|I_1\rangle$ depends essentially on the structure of the model, but the branching ratio of the M1 and E2 transitions does not contain this matrix element.

Using (18) and (19) for the experimentally measured quantities $\delta^2(M1/E2, I_1 \to I_2)$ and $\cos\xi$, we obtain, according to (1) and (2)

$$\cos\xi = 1, \quad (21)$$

$$\begin{aligned}\delta^2(M1/E2,\ I_1 \to I_2) &= \frac{500}{441}(I_1+I_2+3)(I_1-I_2+2)(I_2-I_1+2) \\ &\times(I_1+I_2-1)(g_R/Z\omega MR_0^2)^2;\end{aligned} \quad (22)$$

here ω is the transition energy in units of m_ec^2 (0.511 Mev). Formulas (21) and (22) are valid for all the collective models with quadrupole excitations, independently of the specific structure of the model; only these formulas are inapplicable for certain transitions of the strictly harmonic vibrator, which proceed via annihilation of two phonons.

In general, the quantities $(g_R/Z)^2$ can not be considered known, since the coefficients in the operators of the M1 and E2 transitions depend on the models assumped. It is therefore more convenient to compare the ratio of two values of $\delta^2(M1/E2)$ for different mixed M1 + E2 nuclear transitions $I_1 \to I_2$, and $I_3 \to I_4$. In this case the unknown factor $(g_R/Z)^2$ drops out of the final result

$$\frac{\delta^2(M1/E2, I_1\to I_2)}{\delta^2(M1/E2, I_3\to I_4)} = \left(\frac{\omega_{34}}{\omega_{12}}\right)^2 \frac{(I_1+I_2+3)(I_1-I_2+2)(I_2-I_1+2)(I_1+I_2-1)}{(I_3+I_4+3)(I_3-I_4+2)(I_4-I_3+2)(I_3+I_4-1)}. \quad (23)$$

5. The deviation of the experimental values from the quantities (22) and (23), predicted by the collective models of the nucleus, may be due to the inaccuracy in the separation of the collective degrees of freedom. In this case the contribution of the single-particle admixture can change the result appreciably. Actually, the estimated probability of the most intense single-particle M1 transition between the levels of one spin-orbit doublet is

$$W_{\text{s.p.}}(M1) \approx \frac{e^2\omega^3}{M^2}\frac{\mu_n^2}{3}\frac{m_ec^2}{\hbar}, \quad (24)$$

where μ_n is the magnetic moment of the nucleon in magnetons, whereas the collective M1 transitions have a probability

$$W_{\text{col}}(M1) \approx \frac{e^2\omega^3}{M^2}\frac{g_R^2}{\pi}\beta^2\frac{m_ec^2}{\hbar}, \quad (25)$$

where $\beta^2 = \langle\sum_\mu|\alpha_{2\mu}|^2\rangle$ is the nuclear deformation. Thus

$$W_{\text{s.p.}}(M1)/W_{\text{col}}(M1) \approx (\mu_n/g_R\beta)^2. \quad (26)$$

Neglecting the contribution of the single-particle transitions to the probability of the nuclear E2 transition, and taking them into account only in the M1 transition, we obtain a rough estimate for $\delta^2(M1/E2, I_1 \to I_2)$:

$$\delta^2(M1/E2, I_1\to I_2) \approx [1+a^2(\mu_n/\beta g_R)^2]\,\delta^2_{\text{col}}(M1/E2,\ I_1 \to I_2), \quad (27)$$

where δ^2_{col} is given by (22).

The parameter a^2 is a measure of the admixture of the single-particle states. Its structure can be made more precise only with a specific microscopic model of the excitations of the nucleus. Since $(\mu_n/\beta g_R)^2 \approx 10^2 - 10^3$, even a small admixture of single-particle transitions changes the value of $\delta^2(M1/E2)$ appreciably. The sign of cos ξ can also change when the admixture of single-particle transitions becomes considerable.

We are thus able to estimate experimentally the purity of separation of the collective degrees of freedom for even-even nuclei, where the reduced probability of the E2 transitions is appreciably greater than the single-particle estimate. Unfortunately, there are not enough exact and complete data at present to make this analysis possible. The available experimental values of δ^2(M1/E2) apparently do not contradict the estimates of the collective models,[8] but the accuracy of these data is low. It is of exceeding interest to measure the values of δ^2(M1/E2) for several transitions of one and the same nucleus, so as to be able to trace the "smearing" of the collective degrees of freedom of the nucleus with increasing nuclear excitation energy.

APPENDIX

Relation (17) can be obtained by successive application of the expansion formula

$$C^{1M}_{Nn2\mu_2} C^{Nn}_{2m2\mu_1} = \sum_A (-1)^{1+N+A} u\,(2221;\ AN)\, C^{1M}_{Aa2\mu_1} C^{Aa}_{2m2\mu_2}. \tag{A.1}$$

We have

$$\sum_{\mu_1\mu_2} \xi_{\mu_1\mu_2} C^{1M}_{2m2\nu} C^{2\nu}_{2\mu_12\mu_2} = u\,(2221;\ 12) \sum_{\mu_1\mu_2} \xi_{\mu_1\mu_2} C^{1M}_{2\mu_21n} C^{1n}_{2m2\mu_2}$$
$$+ \sum_{\mu_1\mu_2} \xi_{\mu_1\mu_2} \{u\,(2221;\ 22)\, C^{1M}_{2n2\mu_2} C^{2n}_{2m2\mu_1}$$
$$+ u\,(2221;\ 32)\, C^{1M}_{3n2\mu_2} C^{3n}_{2m2\mu_1}\}. \tag{A.2}$$

Using the symmetry $\xi_{\mu_1\mu_2} = \xi_{\mu_2\mu_1}$ ($\xi_{\mu_1\mu_2}$ is an arbitrary function symmetrical in $\mu_1\mu_2$) and transforming, in accordance with (A.1), the terms in the curly brackets, we obtain with allowance for the numerical values of the u-functions

$$\sum_{\mu_1\mu_2} \xi_{\mu_1\mu_2} C^{1M}_{2m2\nu} C^{2\nu}_{2\mu_12\mu_2}$$
$$= \left[-\tfrac{1}{2}\sqrt{\tfrac{7}{5}} - \tfrac{13}{20}\sqrt{\tfrac{7}{5}}\right] \sum_{\mu_1\mu_2} C^{1M}_{1n2\mu_1} C^{1n}_{2m2\mu_2} \xi_{\mu_1\mu_2}$$
$$+ \sum_{\mu_1\mu_2} \xi_{\mu_1\mu_2} \left\{\tfrac{3}{20} C^{1M}_{2n2\mu_1} C^{2n}_{2m2\mu_2} + \tfrac{3}{10}\sqrt{\tfrac{2}{5}}\, C^{1M}_{3n2\mu_1} C^{3n}_{2m2\mu_2}\right\}. \tag{A.3}$$

Applying formula (A.1) successively at each stage to the terms with N ≠ 1 in the curly brackets, and separating each time the new terms with N = 1, we obtain a geometric progression

$$\sum_{\mu_1\mu_2} \xi_{\mu_1\mu_2} C^{1M}_{2m2\nu} C^{2\nu}_{2\mu_12\mu_2} = -\sqrt{\tfrac{7}{5}}\left(\tfrac{1}{2} + \tfrac{13}{20}\left[1 + \tfrac{3}{10} + \left(\tfrac{3}{10}\right)^2 + \ldots\right]\right)$$
$$\times \sum_{\mu_1\mu_2} \xi_{\mu_1\mu_2} C^{1M}_{1n2\mu_2} C^{1n}_{2m2\mu_1}, \tag{A.4}$$

and after summing this series we arrive at formula (17).

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