NUCLEON-NUCLEON INTERACTIONS AT $E \approx 10^{11}$ ev

I. M. DREMIN and D. S. CHERNAVSKII

P. N. Lebedev Physics Institute, Academy of Sciences, U.S.S.R.

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Peripheral nucleon-nucleon collisions at energies around 10^{11} ev are considered. As in reference 2, the pole approximation is used. The results are compared with experimental data.[3,6] The usual form[2] of the one-meson approximation is not valid for large values of "virtuality" k^2 and needs to be improved by taking into account the dependence on k^2 of the πN interaction cross section.

1. Some time ago it was proposed[1] that inelastic nucleon-nucleon interaction processes in the high-energy region be described by using diagram techniques in the pole approximation. It was found[2] that this approximation gives a satisfactory description of the experimental data at 9 Bev. At this energy, however, only those processes in which the "virtuality" of the intermediate pion (see Fig. 1) is relatively small, $k^2 \lesssim (7\mu)^2$, contribute significantly to the cross section; here k is the four-momentum of the intermediate pion and μ is the pion mass. Therefore, of course, the conclusion that the pole approximation is applicable is limited to just the region $0 < k^2 \leq (7\mu)^2$.* In order to resolve the question of the applicability of this approximation for $k^2 > (7\mu)^2$, it is necessary to consider higher-energy processes.

Experimental data on NN interactions at energies $E_l \sim 200$ Bev[3] show that the inelasticity coefficient is small in the majority of cases (this immediately indicates that peripheral collisions play an essential role). Moreover, these data show that there are two types of interaction which differ in nature, namely: 1) asymmetric interactions in which in the center-of-mass system (c.m.s.) the secondary pions emerge predominantly in the direction of one of the primary nucleons; 2) the symmetric case in which in the c.m.s. the secondary pions are nearly isotropic although the inelasticity coefficients of both nucleons are small.

In terms of diagrams, the first type of process can be interpreted as a special case of the diagram shown in Fig. 1, namely the case in which the nucleons are not equally excited. The diagram in Fig. 2 corresponds to the symmetric case. This is also a one-meson diagram.

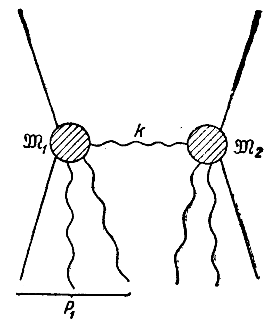

FIG. 1. General one-meson diagram for NN interaction. The numbers of pions emitted from the vertices can be different.

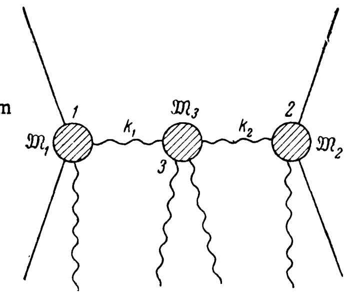

FIG. 2. One-meson diagram for NN interaction with intermediate $\pi\pi$ interaction.

It should be noted that such a division of processes and diagrams into different categories makes sense only for sufficiently high energies. In fact, the case in which three excited states are formed can be distinguished from that in which only two are formed only if their relative velocities $\overline{v}$ (more exactly, the quantities $\overline{\gamma} = (1-\overline{v^2})^{-1/3}$) are sufficiently large: $\ln\overline{\gamma} \gtrsim \Sigma$, where Σ is the half-width of the angular distribution of secondary particles in their center-of-mass system:

$$dN/d\lambda = \exp(-\lambda^2/\Sigma), \qquad \lambda = \ln \mathrm{tg}(\vartheta/2).*$$

*It should be noted that even reference 2 points out that the πN interaction cross section, $\sigma_{\pi N}(k^2)$, might decrease with increasing k^2 for $k^2 \gtrsim (6\mu - 7\mu)^2$. Here, as in reference 2, $\hbar = c = 1$; $k^2 = \mathbf{k}^2 - k_0^2$.

*tg = tan.

The smallest value of Σ is 1 (which occurs for an isotropic distribution) and consequently a necessary condition for resolving the two cases is $\overline{\gamma} \gtrsim 2.7$.

For the diagram in Fig. 2, the best conditions for the resolution occur when the excited state at vertex 3 (with mass $\mathfrak{M}_3$) is at rest in the c.m.s. and the nucleons are equally excited (their masses are $\mathfrak{M}_1$ and $\mathfrak{M}_2$) and move in opposite directions. $\mathfrak{M}_3$ is of the order of magnitude $\mathfrak{M}_3 \sim 2\gamma_C\mu$ with $\gamma_C = (1-v_C^2)^{-1/2}$ and v_C the velocity of the c.m.s. with respect to the laboratory system. The conservation laws give $\overline{\gamma}_{max} = \gamma_C(\mathfrak{M}_3 - \mu)/\mathfrak{M}_{1,2}$. The minimum value for the mass of the excited nucleons is $\mathfrak{M}_{1,2} = 1.3\,m$, where m is the nucleon mass (this is the mass of the "usual" isobar which appears in the work of Tamm, Gol'fand, and Faĭnberg[4]). Then the condition $\overline{\gamma} \gtrsim 2.7$ leads to the inequality $\gamma_C > 4$. From this it follows that such a process can be observed at an energy $E_{lab} \sim 200$ Bev ($\gamma_C = 10$).

2. In order to compute the total cross section for peripheral one-meson collisions at the energy $E_{lab} = 200$ Bev, we used expression (5) of reference 1:*

$$\sigma_{NN}(E_0) = \frac{2}{(2\pi)^3 p_0 E_0^2} \int dz \int dy \int d(\cos\vartheta)$$

$$\times \frac{\sqrt{z^2 - m^2\mu^2}\,\sqrt{y^2 - m^2\mu^2}\,P_1}{[\mu^2 + \varkappa^2 + 2p_0P_1(1-\cos\vartheta)]^2}\,\sigma_{\pi N}(z)\sigma_{\pi N}(y),$$

where

$$z = (\mathfrak{M}_1^2 - m^2 - \mu^2)/2, \qquad y = (\mathfrak{M}_2^2 - m^2 - \mu^2)/2, \quad (1)$$

E_0 and p_0 are the energy and momentum of the primary nucleon in the c.m.s. E_1 and P_1 are the energy and momentum of the isobar in the c.m.s.

$$\varkappa^2 = 2(E_0E_1 - p_0P_1) - \mathfrak{M}_1^2 - m^2,$$

$$\varkappa^2 + 2p_0P_1(1-\cos\vartheta) = k^2,$$

ϑ is the angle between $\mathbf{p}_0$ and $\mathbf{P}_1$; $\sigma_{\pi N}(z)$ is the total πN interaction cross section at an energy (in the pion-nucleon c.m.s.) equal to $\mathfrak{M}_1$. The energy of the pion in the lab system is $\omega_{lab} = z/m$.

We note that the main contribution to the integral comes from $\sigma_{\pi N}(\omega_{lab})$ with $\omega_{lab} \sim 7-10$ Bev. On the basis of the data of Belyakov et al.[5] the magnitude of the cross section $\sigma_{\pi N}$ was taken to be constant and equal to 30 mb.

Integration of (1) over all values of y and z allowed by the conservation laws gave the result $\sigma_{NN} = 1400$ mb. The values of k^2 which contributed most were $k^2 \sim (50\mu - 100\mu)^2$. Such a large cross section is absurd and shows that the pole approximation is not valid for such large values of k^2. It should be emphasized that giving up any one of all the assumptions on which the pole method is based* (especially, giving up the equality $\sigma_{\pi N}(\mathfrak{M}, k^2) = \sigma_{\pi N}(\mathfrak{M}, k^2 = -\mu^2)$) can lead to the necessary decrease of the calculated cross section σ_{NN} by almost two orders of magnitude (see reference 2).

Thus, the result obtained above can be interpreted to mean that the cross section $\sigma_{\pi N}(\mathfrak{M}, k^2)$ is not a constant independent of k^2, but decreases strongly with increasing k^2. In order to find out the values of k^2 at which this occurs, expression (1) was integrated with the supplementary limitation that the integral was taken only over the region $k^2 \le \delta^2$.†

We give the result of the integration for various values of δ:

δ/μ:	3	4	5	7	10	14	(no limit on k^2)‡
σ_{NN}, mb:	8	16	28	80	190	420	1400

If δ is determined by the requirement that the calculated cross section not exceed the value obtained experimentally (for $E_{lab} \sim 10^{11}$ ev, $\sigma_{exp} \approx 30$ mb[3,6]), then we find $\delta \sim 4\mu$, corresponding to a value of $\mathfrak{M}$ of about 4 Bev. Previously,[1] in studying the NN interaction at $E_{lab} = 9$ Bev, we concluded that the pole method is valid clear up to $k^2 \sim (7\mu)^2$ (equivalent to $\delta \gtrsim 7\mu$), which corresponds to $\mathfrak{M} \sim 2$ Bev. By comparing the results obtained here and in reference 1, one can conclude that the function $\sigma(\mathfrak{M}, k^2)$ is a complicated nonmultiplicative function of its variables such that the decrease of σ with increasing k^2 begins sooner for larger values of $\mathfrak{M}$.**

It should be mentioned that Berestetskii and Pomeranchuk[7] were the first to point out that the cross section σ_{NN} calculated in the pole approximation, with the assumption that $\sigma_{\pi N}$ is constant, increases without bound as the energy increases. However, in further investigating the problem they still in essence assumed that σ is muliplicative, $\sigma(\mathfrak{M}, k^2) = \sigma(\mathfrak{M})f(k^2)$. It seems to us that the example cited shows that it is unlikely that σ is

*In formula (2) of reference 1, P_1 was set equal to p_0, since there we considered the region in which k^2 is small. Here the exact expression is given.

*These assumptions are considered in more detail in reference 2.

†This is equivalent to approximating the function $\sigma(\mathfrak{M}, k^2)$ by the step function

$$\sigma(\mathfrak{M}, k^2) = \sigma(\mathfrak{M}, k^2 = -\mu^2) \text{ for } k^2 \le \delta^2, \quad \sigma(\mathfrak{M}, k^2) = 0 \text{ for } k^2 > \delta^2.$$

‡The maximum value of k^2 is determined by the conservation laws.

**In the approximation used above, this means that δ decreases with increasing $\mathfrak{M}$.

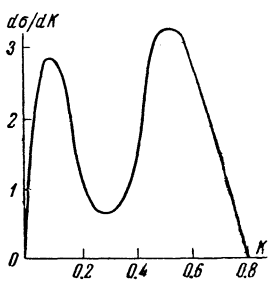

FIG. 3 Distribution of inelasticity coefficient K.

multiplicative, but rather that a more complicated dependence actually occurs.

In a comparison with the experimental data,[3,6] it is necessary to consider other features of the process besides the total cross section. However, this gives rise to some difficulties connected with the necessity of taking into account the dependence of $\sigma(\mathfrak{M}, k^2)$ on k^2. The step approximation to this function seems rather crude and, moreover, the choice of δ is not unique. Therefore, we limit our further considerations to those features which depend weakly on δ and on the form of $\sigma(\mathfrak{M}, k^2)$.

We have calculated the distribution of inelasticity coefficients K for these events. The results of the computation are shown in Fig. 3* (with $\delta = 4\mu$). The peak at $K = 0.0 - 0.2$ stands out. This peak comes from the cases in which an elastic πN interaction occurs at one of the vertices. At high energies, the elastic part of the πN interaction is basically diffractive and amounts to one-third of the total. Therefore, the fraction of such cases is about 33 per cent. In this case, the inelasticity coefficient of the nucleon at one of the vertices is $K_1 = \mathfrak{M}_2^2/E_0^2$, where $\mathfrak{M}_2$ is the "mass" of the excitation at the other vertex. In our case, $E_0 = 10$ Bev, $\mathfrak{M}_2 \sim 4$ Bev, $K \approx 0.16$. The other peak in the curve is due to the cases in which the πN interaction is inelastic. There the inelasticity coefficient is $K \gtrsim 0.5$.

These features of the distribution depend weakly on the assumed value of δ and are essentially due to the presence of diffractive πN interaction at high energies. These results are in qualitative agreement with the experimental data. It should be noted that according to this curve the charge-exchange probability should be small when the energy loss of the particle is small.

3. In order to obtain theoretical results for the so-called symmetric cases which have been observed experimentally,[3,6] the process represented in Fig. 2 was considered. An expression for the cross section for such a process can be obtained by a method analogous to that used in reference 1:

$$\sigma = \frac{32}{(2\pi)^8 E_0^2 I} \int \frac{d^4k_1\, d^4k_2}{(k_1^2+\mu^2)^2 (k_2^2+\mu^2)^2} \omega_1 E' I_1 \sigma_1 \cdot \omega_1\omega_2 I_3 \sigma_3 \cdot \omega_2 E'' I_2 \sigma_2. \tag{2}$$

Here σ is the desired cross section, σ_i is the cross section for the process corresponding to vertex i $(i = 1, 2, 3)$,* I_i are the corresponding particle current densities, k_1 and k_2 are the four-momenta of the virtual pions, and E' and E'' are the energies of the primary nucleons in the rest system of the excited nucleons.

It is convenient to use the following as independent variables:

$$z = \tfrac{1}{2}(\mathfrak{M}_1^2 - m^2 - \mu^2),\quad y = \tfrac{1}{2}(\mathfrak{M}_2^2 - m^2 - \mu^2),$$
$$\mathfrak{M}_3,\quad E_1,\quad \vartheta_{1,2},\quad \varphi_{1,2}.$$

Here $\mathfrak{M}_i$ is the mass of the excitation at vertex i, $E_{1,2}$ are the energies of the isobars formed at vertices 1 and 2, and $\vartheta_{1,2}$ and $\varphi_{1,2}$ are the polar and azimuthal angles at which the isobars are emitted in the c.m.s. Then

$$k_1^2 + \mu^2 = \mu^2 + \varkappa_1^2 + 2p_0P_1(1-\cos\vartheta_1),$$
$$k_2^2 + \mu^2 = \mu^2 + \varkappa_2^2 + 2p_0P_2(1+\cos\vartheta_2),$$
$$\varkappa_i^2 = 2(E_0E_i - p_0P_i) - \mathfrak{M}_i^2 - m^2. \tag{3}$$

The integration was carried out with the supplementary limitation $k_{1,2}^2 \le \delta^2$. In this process, the values of $\mathfrak{M}_{1,2}$ which contribute strongly are smaller than those in the process of Fig. 1 for the same value of E_0. Therefore, for this process we took the larger value of δ, $\delta = 7\mu$. We note that the integrand in (2) has sharp maxima at $\vartheta_1 = 0$, $\vartheta_2 = \pi$; this simplifies the integration.

On the basis of the expression (2), distributions of $\mathfrak{M}_1$, $\mathfrak{M}_2$, and $\mathfrak{M}_3$ were obtained.† It turned out that the distributions of $\mathfrak{M}_{1,2}$ were such that only values of $\mathfrak{M}_{1,2}$ near 1.3 m contribute significantly. The contribution of values of $\mathfrak{M}_{1,2}$ greater than 1.5 m was negligibly small. This is due to two circumstances: first, the integrand is a decreasing function of $\mathfrak{M}_{1,2}$ and second, $\sigma_{\pi N}(\mathfrak{M}_{1,2})$ has a strong maximum at $\mathfrak{M}_{1,2} = 1.3$ m and then falls rapidly.

The $\mathfrak{M}_3$ distribution is shown in Fig. 4; clearly values of $\mathfrak{M}_3$ between 2m and 4m are dominant. The maximum in the distribution occurs at $\mathfrak{M}_3 = 3$m; we note that its position is determined by the condition $\mathfrak{M}_3 = (2\mu/m)E_0$.

*In the report we sent to the High-energy Physics Conference (Rochester, 1960), results of analogous calculations with $\delta = 7\mu$ were given (see reference 8).

*We took the πN interaction cross sections (σ_1 and σ_2) from experiment. $\sigma_3 = \sigma_{\pi\pi}$ was taken to be constant.

†We do not give these calculations here because they are so long.

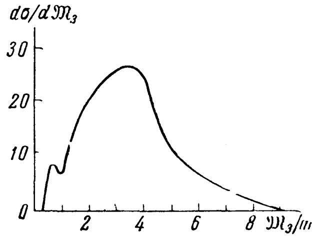

FIG. 4. Distribution of mass of excited ππ cloud ($\mathfrak{M}_3$).

The inelasticity coefficients can be estimated by using the $\mathfrak{M}_3$ distribution, the nucleon excitations $\mathfrak{M}_{1,2}$, and the conservation laws. According to this estimate, K varies between 0.2 and 0.4. All these quantities depend weakly on the choice of δ and agree qualitatively with the experimental data.[3,6]

The magnitude of the cross section for process (2), on the other hand, depends very strongly on the choice of δ; moreover, it involves the unknown quantity $\sigma_{\pi\pi}$. Therefore the estimate of the cross section can be off by orders of magnitude. For $\delta = 7\mu$, the calculation gives $\sigma_2 \approx 0.16\,\sigma_{\pi\pi}$. If $\sigma_{\pi\pi} \sim \sigma_{\text{geom}} \approx 60$ mb, then $\sigma_2 = 10$ mb; this is not in disagreement with experiment.[3,6]

It should be noted that all these results, as well as the experimental data,[3,6] are still preliminary and need to be more accurately determined. From the theoretical point of view, the main question is the best way of taking into account the dependence of the πN interaction on k^2.

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228