POLARIZATION RESULTING FROM SCATTERING OF NEUTRONS BY FERROMAGNETIC SUBSTANCES

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An expression is obtained for the polarization of slow neutrons scattered by a ferromagnet. It is shown that the polarization vector consists of two mutually perpendicular components, one of which is due to the interference of nuclear and magnetic scattering, and the other to inelastic magnetic scattering with spin flip of the neutron.

In the present paper it is shown that an experimental study of the polarization resulting from the scattering of unpolarized slow neutrons in ferromagnets can, in certain cases, enable one to determine the fraction of those neutrons scattered in a given direction which are magnetically scattered.

As was shown by Halpern and Johnson,[1] the amplitude for scattering of a neutron by a system of N magnetic atoms has the form

$$f = \frac{1}{\sqrt{N}} \sum_l e^{i\mathbf{q}\mathbf{R}_l} \left\{ A_l + \frac{1}{2} B_l \mathbf{J}_l \boldsymbol{\sigma} - \gamma r_0 F(q) (\mathbf{S}_l, \boldsymbol{\sigma} - (\mathbf{e}\boldsymbol{\sigma})\mathbf{e}) \right\}. \quad (1)$$

Here $\mathbf{p}$ and $\mathbf{p}'$ are the neutron momentum before and after the scattering, $\mathbf{q} = \mathbf{p} - \mathbf{p}'$; $\mathbf{R}_l$ and $\mathbf{S}_l$ are the coordinates and spin of the l-th atom; $A_l + \frac{1}{2} B_l \mathbf{J}_l \cdot \boldsymbol{\sigma}$ is the amplitude for scattering of the neutron by the nucleus, where $\mathbf{J}_l$ is the spin of the nucleus and $\boldsymbol{\sigma}/2$ is the neutron spin. γ is the absolute value of the magnetic moment of the neutron in nuclear magnetons; $r_0 = e^2/mc^2$ is the classical radius of the electron; $F(q)$ is the magnetic form factor of the atom; $\mathbf{e} = \mathbf{q}q^{-1}$.

From (1) we see that the magnetic scattering amplitude is proportional to the matrix elements of the vector

$$\mathbf{K} = \sum_l \exp(i\mathbf{q}\mathbf{R}_l)\, \mathbf{S}_l.$$

The vector $\mathbf{K}$ consists of two parts: a longitudinal component K_z along the magnetization vector of the system, and two perpendicular components $K^{\pm} = K_x \pm iK_y$. As shown by Holstein and Primakoff,[2] the component K_z is responsible for transitions in which the total number of spin waves is unchanged (in our case, for elastic scattering and for scattering with absorption of one spin wave and emission of another), or is changed by two (simultaneous emission or absorption of two spin waves by the neutron). The components $K^{\pm}$ are responsible for transitions with a change of of the total number of spin waves by unity.*

Thus for scattering processes for which the matrix elements of the operator K_z are different from zero, the polarization of the scattered neutrons can arise only from interference with that part of the nuclear scattering which is independent of the nuclear spins (where we assume that the nuclei are unpolarized). Such interference is obviously possible only for elastic and for magneto-vibrational scattering.

Now let us consider those scattering processes in which the matrix elements of the operators $K^{\pm}$ are different from zero. We shall neglect the small relativistic corrections to the interaction of the atomic spins (dipole-dipole interaction and anisotropy energy). Then the matrix elements of K^+ (K^-) are different from zero only for transitions in which the total number of spin waves increases (decreases) by unity. Using this fact and the well known formula for the polarization in the scattering of an unpolarized beam

$$\mathbf{P} = \tfrac{1}{2}\,\mathrm{Sp}\,\langle f^+ \boldsymbol{\sigma} f \rangle / \sigma(\mathbf{n}) \quad (2)$$

(where Sp is the trace over the spin indices of the neutron, and $\langle \ldots \rangle$ denotes an average over the initial state of the scatterer and an integration over all energies of the neutrons scattered in the direction $\mathbf{n}$; $\sigma(\mathbf{n})$ is the total cross section for scattering in the direction $\mathbf{n}$), we get the expression for the polarization:

*If we neglect relativistic corrections to the interaction of the atomic spins, we may say that the matrix elements of K_z are different from zero only for transitions in which the total spin of the system is unchanged, and $K^{\pm}$ for transitions with a change of the spin of the system by ± 1.

$$\mathbf{P} = \sigma^{-1}(\mathbf{n})\{-2\,\mathrm{Re}\,\bar{A}Sr_0\gamma F(q)(1-G(T)) \times \sigma_{coh}(\mathbf{n})|A|^{2}(\mathbf{m}-(\mathbf{em})\mathbf{e}) + 2(\mathbf{em})\mathbf{e}\,[1+(\mathbf{em})^2]^{-1}[\sigma_-(\mathbf{n})-\sigma_+(\mathbf{n})]\}. \quad (3)$$

Here $\mathbf{m}$ is a unit vector along the direction of magnetization; $\bar{A}$ is the average of the amplitude A over the distribution of isotopes in the lattice; S is the spin of the atom; G(T) is the percentage deviation of the spontaneous magnetization at temperature T from its value at T = 0; $\sigma_{coh}(\mathbf{n})$ is the cross section for coherent scattering by the crystal into the direction $\mathbf{n}$; $\sigma_+(\mathbf{n})$ and $\sigma_-(\mathbf{n})$ are the respective cross sections for magnetic scattering with increase or decrease by unity in the number of spin waves.

Strictly speaking, in Eq. (3) we should include the dependence of the vector $\mathbf{e}$ on the energy of the scattered neutrons, write the expression $\mathbf{P}$ for a given energy of scattering of the neutron and then integrate over all energies. But usually the intensity of the scattered neutrons is significantly differently from zero only when $\mathbf{q} \approx \boldsymbol{\tau}$, where $\boldsymbol{\tau}$ is a vector of the reciprocal lattice multiplied by 2κ, and consequently the vector $\mathbf{e} \approx \boldsymbol{\tau}\tau^{-1}$ can be regarded as independent of energy to a high degree of accuracy.

The most interesting consequence of formula (3) is that the polarization vector consists of two mutually perpendicular components. One of them, directed along the vector $\mathbf{m} - \mathbf{e}(\mathbf{e}\cdot\mathbf{m})$, is the result of interference between nuclear and magnetic scattering. As we have already remarked, this interference occurs for elastic and magneto-vibrational scattering. The other component, which is along the vector $\mathbf{e}$, is proportional to the difference of the cross sections $\sigma_-(\mathbf{n}) - \sigma_+(\mathbf{n})$. By separately studying these two components of the polarization vector, one can draw various conclusions concerning elastic magnetic and magneto-vibrational scattering one the one hand, and concerning inelastic magnetic scattering with emission and absorption of spin waves on the other. Since the scatterer is magnetized, the polarization vector will rotate around the field. As a result of such rotation, at sufficiently large distances from the scatterer the component of $\mathbf{P}$ which is perpendicular to the field will effectively be averaged to zero (since the neutrons arriving at the detector will have traversed different paths in the magnetic field). Thus we practically observe only the component of $\mathbf{P}$ which is parallel to the field.

Having magnetized the crystal perpendicular to the vector $\mathbf{e}$ (where $\mathbf{e} \approx \boldsymbol{\tau}/\tau$, with $\boldsymbol{\tau}$ a vector of the reciprocal lattice, in whose neighborhood we are studying the scattering), we separate out the part of the polarization vector which is due to interference of magnetic and nuclear scattering:

$$\mathbf{P}_{int} = -\frac{2\,\mathrm{Re}\,\bar{A}Sr_0\gamma F(q)(1-G(T))\,\sigma_{coh}(\mathbf{n})\,|\bar{A}|^{-2}}{\sigma(\mathbf{n})}\mathbf{m}. \quad (4)$$

This part of the polarization vector was studied experimentally by Nathans, Shull et al.[3] If we magnetize the scatterer along $\mathbf{e}$, we get the part of the vector $\mathbf{P}$ due to inelastic magnetic scattering:

$$\mathbf{P}_{mag} = [(\sigma_-(\mathbf{n}) - \sigma_+(\mathbf{n}))/\sigma(\mathbf{n})]\,\mathbf{e}. \quad (5)$$

The numerator of (5) contains the difference $\sigma_-(\mathbf{n}) - \sigma_+(\mathbf{n})$. It is known[4,5] that at low temperatures (when one can speak of spin waves*) the main scattering processes are those with absorption or emission of one spin wave. The kinematics of these scattering processes are such that there are cases where there is a considerable intensity of inelastic magnetic scattering and, at the same time, there is scattering accompanied only by emission or only by absorption of a spin wave. Then Eq. (5) becomes

$$\mathbf{P}_{mag} = \mp(\sigma_\pm(\mathbf{n})/\sigma(\mathbf{n}))\,\mathbf{e}, \qquad \mathbf{m} \parallel \mathbf{e}. \quad (6)$$

From (6) it follows that if we know $\mathbf{P}_{mag}$ and the total scattered intensity we can easily determine $\sigma_\pm(\mathbf{n})$.

Let us consider the two most important cases.

1. Scattering when the Bragg condition $|\mathbf{p} + \boldsymbol{\tau}| \approx p$ is almost satisfied. If $\Delta > (2\alpha)^{-1}$, then only scattering with absorption of a spin wave can occur; but if $\Delta < (2\alpha)^{-1}$, then one can have only emission of a spin wave. Here Δ is defined by the formula

$$\Delta = (|\mathbf{p} + \boldsymbol{\tau}| - p)/p \approx 2\sin 2\psi_B\,\delta\psi$$

(where ψ_B is the angle between $\mathbf{p}$ and $\boldsymbol{\tau}$ satisfying the Bragg condition:

$$|\mathbf{p} + \boldsymbol{\tau}| = p \qquad (\cos\psi_B = -\tau/2p),$$

$\delta\psi$ is the deviation of the angle ψ between $\mathbf{p}$ and $\boldsymbol{\tau}$ from the angle ψ_B), and

$$\alpha = 2JMS\gamma_0\delta^2/3\hbar^2 \gg 1$$

(J is the exchange integral, γ_0 the number of nearest neighbors, δ the distance between them, and M the mass of the neutron).

2. Scattering of slow neutrons, $p < (\tau_{min}/2) \times (1 - 1/4\alpha)$. In this case scattering at large angles

*Recently Engert[6] and Ginzburg and Fain[7] have attempted to extend the spin wave picture to the whole range of temperature below the Curie point. In this note we shall not discuss the consequences of their theory for the scattering of neutrons.

can occur only with absorption of a spin wave. A detailed discussion of the kinematics of the scattering as well as formulas for the cross section for these cases can be found in reference 4.

We note that for both cases the wave vector of the spin wave cannot be very small. But we know[2] that relativistic corrections to the interaction of the atomic spins affect the dispersion law only for very long spin waves. Thus they need not be included in the cases considered here.

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