PAIRING FORCES AND PAIR CORRELATIONS IN THE NUCLEI Tl^{206} *AND* Bi^{210}

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The residual pairing forces acting between a neutron and proton in the nucleus are determined. The energies, eigenfunctions and transition probabilities in the nuclei Tl^{206} and Bi^{210} are found and compared with experiment.

1. INTRODUCTION

THIS paper is devoted to the determination of the residual forces which act between a neutron and a proton in their motion in the self-consistent field of the nucleus. The most convenient nuclei for this purpose are those whose nucleon configurations differ from doubly closed shells by the presence of two holes or two "additional" nucleons. In the region of heavy nuclei, Tl^{206} and Bi^{210} are of this type.

At first glance it might seem that in a heavy nucleus the neutron and proton, being in different shells, interact weakly with one another. However one can convince oneself that this is not so by comparing the β decay energies of the nuclei Tl^{207} and Tl^{206} or the nuclei Pb^{209} and Pb^{210}. In the $Tl^{207} \rightarrow Pb^{207}$ β decay, an energy of 1400 kev is liberated. This energy is made up of the difference in the energy levels of the proton and neutron in the nuclei, and of the mass difference which is 780 kev. The β decay energy of Tl^{206} differs from that for Tl^{207} by the difference between the binding energies ϵ_{np} of a neutron-proton pair and ϵ_{nn} for a neutron-neutron pair. If we assume that the neutron-proton interaction is small, i.e., $\epsilon_{np} \ll \epsilon_{nn}$, then the β decay energies of Tl^{207} and Tl^{206} should differ by approximately 700 kev. The observed difference in β decay energy is only 70 kev. Consequently $\epsilon_{np} \approx \epsilon_{nn}$, and the neutron-proton interaction is of the same order as the neutron-neutron interaction. Similarly, from a consideration of the $Pb^{210} \rightarrow Bi^{210}$ β decay it follows that the binding of the n-p pair in Bi^{210} is also not less than 700 kev. For this reason an investigation of the n-p interaction in the Tl^{206} and Bi^{210} nuclei was made by the same method which was used for studying the n-n forces in Pb^{206}.[1]

The motion of the external nucleons is described by the potential

$$V = \sum_{i=1,2} V_C(i) + \sum_{i=1,2} V_S(i) + V_p(1,2). \quad (1)$$

Here

$$V_C(i) = V(r_i) - \lambda \left(\frac{\hbar}{2Mc}\right)^2 \frac{\mathbf{l}_i \mathbf{s}_i}{r_i} \frac{\partial V(r_i)}{\partial r_i},$$

$$V(r_i) = \frac{V_0}{1 + \exp\{\alpha (r_i - r_0)\}} \quad (2)^*$$

is the diffuse single particle central potential for the i-th external particle, whose parameters were determined in reference 2;

$$V_S(i) = -\varkappa(r_i) \sum_{\mu} \alpha_{2\mu} Y_{2\mu}(\vartheta_i, \varphi_i) \quad (3)$$

is the potential for the quadrupole interaction of the i-th external particle with the particles of the core;

$$V_p(1,2) = -\exp\{-(\mathbf{r}_1 - \mathbf{r}_2)^2/\rho^2\}(v_t \pi_t + v_s \pi_s) \quad (4)$$

(in which π_s and π_t are the single and triplet projection operators) is the potential of the residual pairing interaction between neutron and proton. The parameter ρ corresponds to the effective radius of interaction of the nuclear forces, while v_s and v_t determine the depth of the singlet and triplet forces.[1]

Using the potential (1), we calculated the spectrum of excited states for the Tl^{206} and Bi^{210} nuclei, in order to determine the parameters of the pairing interaction.

2. SPECTRUM OF Tl^{206}

To calculate the spectrum of Tl^{206} we used the single-particle levels and the corresponding wave functions for the nuclei Pb^{207} and Tl^{207}. A good determination of the levels of Pb^{207} was made in reference 2. For Tl^{207}, the binding energy of the proton in the ground $s_{1/2}$ state is known. The location of the first excited $d_{3/2}$ level can be determined by tracing this level in all the Tl isotopes,

*$\mathbf{l}_i\mathbf{s}_i = \mathbf{l}_i \cdot \mathbf{s}_i$.

from which one sees that for Tl^{206} the $d_{3/2}$ level should lie at ~ 150 kev. The levels which follow are determined by solving the Schrödinger equation with the potential V_C in the form (2); in particular, the $d_{5/2}$ level turns out to be at a height of 1150 kev, while the other levels are higher.

The experimental spectrum for Tl^{206} has been studied in detail by Rusinov et al.[3] The spectrum is characterized by the occurrence of narrow doublets.

Turning to the theoretical interpretation of the spectrum of Tl^{206}, we should remark that if we omit the n-p interaction it is impossible to explain the sequence of levels, even forgetting about the binding energy of pairs. It is therefore necessary to solve the Schrödinger equation with the complete potential (1). The method of solution was described earlier.[1] Briefly, it reduces to the diagonalization of the energy matrices calculated for definite total angular momentum and parity. As basis functions we use linear combinations of products of single particle functions for the neutron and proton, and a wave function describing the quadrupole oscillations of the core,

$$|(l_1j_1,\ l_2j_2)J,\ NR;\ IM\rangle. \qquad (5)$$

From the diagonalization of the energy matrix, we get the sequence of the energy levels of the nucleus which have a given spin and parity, and also the coefficients $c^I\ (l_1j_1,\ l_2j_2,\ J;\ NR)$ of the linear combinations of the basis functions (5), which determine the corresponding wave functions Ψ_{IM}:

$$\Psi_{IM} = \sum c^I(l_1j_1,\ l_2j_2,\ J;\ NR)|(l_1j_1,\ l_2j_2)J;\ NR;\ IM\rangle \qquad (6)$$

[The summation in (6) extends over all variables except I and M.] The degree of the energy matrix is limited by the requirement that the energies and wave functions of the low-lying levels be determined to sufficient accuracy. Comparison of the results of the calculation with the experimental spectrum enables us to determine the optimum values of the parameters in the potential.

As it turns out, the doublets in the Tl^{206} spectrum are especially sensitive to the parameters v_t and v_s, which makes the determination of these parameters easier. For the parameters $\hbar\omega$ and C, on which the $\alpha_{2\mu}$ depend, and also for ρ, we took the values obtained from analysis of the Pb^{206} spectrum, since these parameters cannot be very different for neighboring nuclei.

In Table I we give the energy levels calculated for fixed values of the parameters v_t and v_s, omitting the interaction of the nucleons with the nuclear surface. From this table we see that there are three doublets in the Tl^{206} spectrum. As we see from Table II, variation of the parameters v_t and v_s, and including the interaction of the nucleons with the core, leads to different doublet separations and different splittings of the doublets themselves, but does not mix them. Thus there is no question that there is an excited level with $I = 1^-$ near the ground state.

Figure 1 shows the level scheme of Tl^{206} as obtained with the parameters $v_t = 22$ Mev and $v_s = 27$ Mev, values which were determined from the energy splitting of the second doublet (40 kev) and the binding energy of the pair in the ground state. For these same values of the parameters v_t and v_s, we calculated the wave functions of the doublets, i.e., the coefficients $c^I\ (l_1j_1,\ l_2j_2,\ J;\ NR)$, which are given in Table III. The theoretical level scheme is in quite good agreement with the experimental data, and one could obtain even better agreement with experiment, for example, by reducing the parameter ρ by 3 — 5%. But since the energies of the single particle levels are not known exactly, the calculation was not pursued to obtain better agreement with the experimental doublet separations.

Using these wave functions for the Tl^{206} levels, we calculated the reduced probabilities $B(\lambda)$ for γ transitions; the results are shown in Table V. For the electric transitions $2^- \to 0^-$ and $2^- \to 1^-$ the theoretical values of B(E2) are in good agreement with the experimental data. From this it fol-

Table I. Energy levels of Tl^{206} (in Mev) calculated for $v_t = 20$ and $v_s = 20$, including only pairing interaction

Number of level	$I = 0^-$		$I = 1^-$		$I = 2^-$		$I = 3^-$	
	energy of single-particle levels	final energy of level	energy of single-particle levels	final energy of level	energy of single-particle levels	final energy of level	energy of single-particle levels	final energy of level
1	0.00	—0.520	0.00	—0.500	0.150	—0.280	0.570	0.210
2	1.070	0.460	0.150	—0.340	0.570	0.180	0.720	0.510
3	1.720	1.040	0.920	0.200	0.720	0.370	1.070	0.590

Table II. Energy levels of Tl^{206} and Bi^{210} for different values of the parameters in the interaction potential*

Tl^{206}					Bi^{210}				
Number of level	$I=0^-$	$I=1^-$	$I=2^-$	$I=3^-$	Number of level	$I=0^-$	$I=1^-$	$I=2^-$	$I=9^-$
$v_t=20,\ v_s=20,\ \hbar\omega=3,\ C=1000$					$v_t=20,\ v_s=20,\ \hbar\omega=3,\ C=1000$				
1	—0.770	—0.810	—0.462	0.010	1	—0.970	—0.690	—0.590	—0.620
2	0.350	—0.490	—0.060	0.300	2	2.600	—0.790	—0.540	—0.270
3	0.980	0.100	0.190	0.510	3	2.890	0.310	0.520	0.570
$v_t=25,\ v_s=25,\ \hbar\omega=3,\ C=1000$					$v_t=20,\ v_s=35,\ \hbar\omega=3,\ C=1000$				
1	—0.900	—0.950	—0.580	—0.100	1	—0.970	—1.010	—0.750	—0.760
2	0.190	—0.630	—0.040	0.340	2	2.400	—0.810	—0.560	—0.290
3	0.820	0.240	0.050	0.400	3	2.900	0.050	0.400	0.450
$v_t=25,\ v_s=35,\ \hbar\omega=3,\ C=1000$					$v_t=20,\ v_s=20,\ \hbar\omega=3,\ C=2000$				
1	—0.900	—0.880	—0.570	—0.010	1	—0.840	—0.560	—0.500	—0.510
2	0.190	—0.490	—0.025	0.370	2	2.600	—0.670	—0.460	—0.200
3	0.820	0.390	0.060	0.430	3	2.760	0.320	0.520	0.580
$v_t=20,\ v_s=35,\ \hbar\omega=3,\ C=1000$					$v_t=20,\ v_s=35,\ \hbar\omega=3,\ C=2000$				
1	—0.770	—0.970	—0.490	—0.140	1	—0.840	—0.910	—0.680	—0.650
2	0.350	—0.600	—0.031	0.330	2	2.400	—0.680	—0.470	—0.230
3	0.980	—0.110	0.170	0.370	3	2.760	0.060	0.410	0.450
$v_t=20,\ v_s=20,\ \hbar\omega=3,\ C=2000$									
1	—0.650	—0.660	—0.380	0.110					
2	0.310	—0.410	0.120	0.450					
3	1.000	0.120	0.280	0.550					

*The energy is measured from the ground single particle 0^- level (all values are in Mev).

Table III. Expansion coefficients of eigenfunctions of Tl^{206} levels

I	$c^I(l_1j_1,\ l_2j_2,\ J;\ NR)$					
	$p_{1/2},\ s_{1/2},\ 0;\ 00$	$p_{3/2},\ d_{3/2},\ 0;\ 00$	$f_{5/2},\ d_{5/2},\ 0;\ 00$	$p_{1/2},\ d_{3/2},\ 2;\ 12$	$f_{5/2},\ s_{1/2},\ 2;\ 12$	
0_1	0.953	—0.102	0.134	0.159	0.194	
	$p_{1/2},\ s_{1/2},\ 1;\ 00$	$p_{1/2},\ d_{3/2},\ 1;\ 00$	$p_{3/2},\ s_{1/2},\ 1;\ 00$	$f_{5/2},\ d_{3/2},\ 1;\ 00$	$p_{3/2},\ d_{3/2},\ 1;\ 00$	$f_{5/2},\ d_{5/2},\ 1;\ 00$
1_1	0.914	0.034	0.008	—0.260	0.093	0.103
1_2	—0.030	0.899	0.247	0.110	0.251	0.019
	$p_{1/2},\ s_{1/2},\ 1;\ 12$	$p_{1/2},\ d_{3/2},\ 1;\ 12$	$p_{1/2},\ d_{3/2},\ 2;\ 12$	$f_{5/2},\ s_{1/2},\ 2;\ 12$	$f_{5/2},\ s_{1/2},\ 3;\ 12$	
1_1	—0.006	—0.135	0.137	0.086	0.177	
1_2	—0.166	0.114	0.109	0.021	—0.026	
	$p_{1/2},\ d_{3/2},\ 2;\ 00$	$f_{5/2},\ s_{1/2},\ 2;\ 00$	$f_{5/2},\ d_{3/2},\ 2;\ 00$	$p_{3/2},\ d_{3/2},\ 2;\ 00$	$f_{5/2},\ d_{5/2},\ 2;\ 00$	$p_{1/2},\ s_{1/2},\ 0;\ 12$
2_1	0.898	0.279	0.209	—0.101	0.039	0.112
2_2	0.379	0.749	0.504	0.013	0.072	0.061
	$p_{1/2},\ s_{1/2},\ 1;\ 12$	$p_{1/2},\ d_{3/2},\ 1;\ 12$	$p_{1/2},\ d_{3/2},\ 2;\ 12$	$f_{5/2},\ s_{1/2},\ 2;\ 12$	$f_{5/2},\ s_{1/2},\ 3;\ 12$	
2_1	—0.132	—0.083	0.131	0.047	—0.038	
2_2	—0.033	—0.035	—0.004	0.136	—0.107	
	$f_{5/2},\ s_{1/2},\ 3;\ 00$	$f_{5/2},\ d_{3/2},\ 3;\ 00$	$p_{3/2},\ d_{3/2},\ 3;\ 00$	$p_{1/2},\ d_{5/2},\ 3;\ 00$	$f_{5/2},\ d_{5/2},\ 3;\ 00$	$p_{1/2},\ s_{1/2},\ 1;\ 12$
3_1	0.855	—0.317	0.031	0.267	0.166	0.199
3_2	0.314	0.877	0.289	0	0.015	0.068
	$p_{1/2},\ d_{3/2},\ 1;\ 12$	$p_{1/2},\ d_{3/2},\ 2;\ 12$	$f_{5/2},\ s_{1/2},\ 2;\ 12$	$f_{5/2},\ s_{1/2},\ 3;\ 12$		
3_1	—0.031	0.065	0.038	0.160		
3_2	0.148	—0.091	0.104	—0.055		

lows that the values of the parameters $\hbar\omega$ and C were chosen correctly.

The magnetic transitions $1^- \to 0^-$ and $1^- \to 1^-$ are forbidden on the single particle model ($d_{3/2} \to s_{1/2}$), and the reduced probabilities for M1 transitions differ from zero only because of the admixture of other states. But nevertheless the theoretical value of B(M1) is found to be 10^3 times as great as the experimental value. Variation of the parameters v_t and v_s within reasonable limits enables us to reduce the value of B(M1), but by no more than an order of magnitude. With the same values of v_t and v_s with which the theoretical values of B(M1) agree with experiment, one cannot obtain either the observed spectrum or the binding energy of a pair.

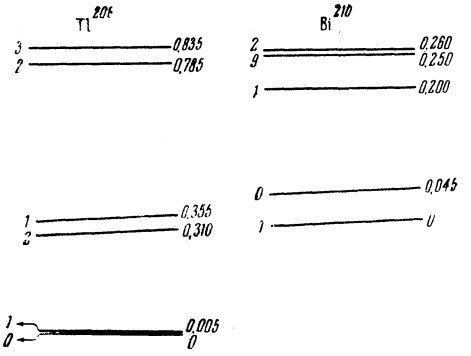

FIG. 1. Level schemes of Tl^{206} and Bi^{210} for the following parameter values: Tl^{206} for $v_t = 22$, $v_s = 27$, $\hbar\omega = 3$, $C = 1000$; Bi^{210} for $v_t = 20$, $v_s = 35$, $\hbar\omega = 3$, $C = 1000$ (all values in Mev).

Consequently, on this point there is disagreement with the results of reference 3.

3. SPECTRUM OF Bi^{210}

The nucleon configurations in Bi^{210} are a combination of the single particle levels of Pb^{209} and Bi^{209}. The multiplets found are characterized by a high degree of degeneracy, which is lifted when the interactions V_S and V_p are included. When this is done, there is a splitting and mixing of levels. Our problem consists in determining parameters which give agreement with the experimental spectrum.

First one must make sure that the 1^- level lies lower than the 0^- level. Under the influence of both V_S and V_p, the 0^- level is pushed farthest down of all the levels in the multiplet $(g_{9/2}, h_{9/2})$. This is a general result, valid for any values of the parameters of the potential. From this it follows that the 1^- level belongs to some other multiplet, namely the multiplet $(i_{11/2}, h_{9/2})$. But in order to become the ground state, the 1^- level must be pushed down 800 kev more than the 0^- level, which is impossible for any reasonable values of the parameters v_t and v_S.

The 1^- level may be lowest, if the single particle level $i_{11/2}$ is lower in Bi^{210} than in Pb^{209}. Here we should recall one property of the $i_{11/2}$ level.[2] It appears that the $i_{11/2}$ level is very sensitive to changes in the diffuseness parameter α in the potential V_C. Increasing the parameter α by 0.1 pushes the $i_{11/2}$ level down by 0.5 Mev, whereas the other levels are practically not shifted. Therefore one can assume that when a proton is added to Pb^{209} to form the Bi^{210} nucleus, there is a slight change in the diffuseness of the boundary of the potential for the neutron, as a result of which the $i_{11/2}$ level approaches the $g_{9/2}$ level. In these computations the $i_{11/2}$ single particle level of Pb^{209} was pushed down by 0.500 Mev.

Table IV. Expansion coefficients of eigenfunctions of Bi^{210} levels

I	$c^I(l_1j_1, l_2j_2, J; NR)$				
0_1	$g_{9/2}, h_{9/2}$, 0; 00 0.966	$g_{9/2}, h_{9/2}$, 2; 12 —0.259	$i_{11/2}, h_{9/2}$, 2; 12 0.032		
1_1	$g_{9/2}, h_{9/2}$, 1; 00 —0.237	$i_{11/2}, h_{9/2}$, 1; 00 0.934	$g_{9/2}, f_{7/2}$, 1; 00 0.019	$g_{9/2}, h_{9/2}$, 1; 12 —0.062	$g_{9/2}, h_{9/2}$, 2; 12 0.020
1_2	0.934	0.238	0.043	0.166	0.006
1_1	$g_{9/2}, h_{9/2}$, 3; 12 0.038	$i_{11/2}, h_{9/2}$, 1; 12 —0.083	$i_{11/2}, h_{9/2}$, 2; 12 0.178	$i_{11/2}, h_{9/2}$, 3; 12 —0.163	
1_2	—0.193	—0.044	0.053	—0.017	
2_1	$g_{9/2}, h_{9/2}$, 2; 00 —0.490	$i_{11/2}, h_{9/2}$, 2; 00 0.840	$g_{9/2}, f_{7/2}$, 2; 00 —0.017	$g_{9/2}, h_{9/2}$, 2; 12 —0.082	$g_{9/2}, h_{9/2}$, 3; 12 0.016
2_1	$g_{9/2}, h_{9/2}$, 4; 12 0.083	$i_{11/2}, h_{9/2}$, 2; 12 0.070	$i_{11/2}, h_{9/2}$, 3; 12 0.094	$i_{11/2}, h_{9/2}$, 4; 12 —0.160	
9_1	$g_{9/2}, h_{7/2}$, 9; 00 0.967	$i_{11/2}, h_{9/2}$, 9; 00 0	$i_{11/2}, f_{7/2}$, 9; 00 0.006	$g_{9/2}, h_{9/2}$, 7; 12 —0.042	$g_{9/2}, h_{9/2}$, 8; 12 0
9_1	$g_{9/2}, h_{9/2}$, 9; 12 0.249	$i_{11/2}, h_{9/2}$, 7; 12 —0.001	$i_{11/2}, h_{9/2}$, 8; 12 —0.005	$i_{11/2}, h_{9/2}$, 9; 12 —0.014	$i_{11/2}, h_{9/2}$, 10; 12 —0.031

Table V. Reduced probabilities of transitions for levels of Tl^{206}

Type of transition	Transition	B			
		single particle	collective	total	experiment
$E2$	$2_1 \to 0_1$	$0.266 \cdot 10^{11}$	$0.659 \cdot 10^{11}$	$1.76 \cdot 10^{11}$	$1.6 \cdot 10^{11}$
$E2$	$2_1 \to 1_1$	$0.196 \cdot 10^{11}$	$0.981 \cdot 10^{11}$	$2.05 \cdot 10^{11}$	$2.3 \cdot 10^{11}$
$M1$	$1_2 \to 1_1$	$2.13 \cdot 10^{12}$	—	$2.13 \cdot 10^{12}$	$0.17 \cdot 11^{10}$
$M1$	$1_2 \to 0_1$	$1.89 \cdot 10^{12}$	—	$1.89 \cdot 10^{12}$	$0.33 \cdot 10^{10}$

Of the levels with larger values of the spin I, the one which is pushed farthest down is the $I = 9^-$ level; this is also a general result, independent of choice of parameters.

In Fig. 1 is shown the theoretical level scheme of Bi^{210}, calculated for the parameter values $v_t = 20$ Mev and $v_s = 35$ Mev which correspond to the experimentally measured energy difference between the levels $I = 1_1^-$ and $I = 0_1^-$, and also to the binding energy of the neutron-proton pair in the ground state as found from the $Pb^{210} \rightarrow Bi^{210}$ β decay. The existence of a second $I = 1_2^-$ level near the $I = 9^-$ level is not in contradiction with experiment, since this level would be difficult to detect because of the large spin difference and the low energy of the γ transition from the $I = 9^-$ level to the $I = 1_2^-$ level.

Using the wave functions for the various states of Bi^{210} which were found from diagonalization of the energy matrix, we calculated the probability of M1 transition from the $I = 0^-$ level to the level with $I = 1_1^-$. In Table VI we give the reduced and total probabilities of M1 transition as a function

Table VI. Probability of transition between levels $0_1^- \rightarrow 1_1^-$ for different assumptions concerning the composition of the 1_1^- level

Character of 1_1^- level	B, sec^{-1}	T, sec^{-1}
Pure $i_{11/2}\, h_{9/2}$ state	0	0
Pure $g_{9/2}\, h_{9/2}$ state	$1 \cdot 10^{14}$	$9.1 \cdot 10^9$
Mixed state, found in this work	$6.4 \cdot 10^{12}$	$5.8 \cdot 10^8$

of the character of the $I = 1_1^-$ level. The mixture of the multiplets $(i_{11/2}, h_{9/2})$ and $(g_{9/2}, h_{9/2})$ used here corresponds to the optimum parameter values. From Table VI we see that the probability of this transition is very sensitive to the composition of the mixture. An exact experimental determination of the probability of the M1 transition would make it possible to determine the character of the $I = 1_1^-$ level. It should be mentioned that if the parameter C is doubled (so that C = 2000 Mev) the energy spectrum is not changed, but the binding energy of the pair is reduced by 150 kev.

4. GENERAL TREATMENT OF THE n-p INTERACTION

From the calculations presented above it follows that the effects produced by the pairing interaction in the neutron-proton system differ essentially from the corresponding effects in a system of two identical particles. Whereas for a system of identical particles the main contribution to the pairing interaction comes from singlet forces, in the n-p system the triplet forces play the main role. Whereas in the n-n (or p-p) system the ground state (0^+) is separated from the other states by a sizable energy gap, in the n-p system the pairing energy for different values of I varies more smoothly and according to a different law. In addition, since there may be one or two levels with different spins between the $I = 0^-$ and $I = 2^-$ levels (for example, for Bi^{210}, $I = 1^-$ and $I = 9^-$), the average distance between levels is 100 — 150 kev.

For a qualitative treatment of the role of singlet and triplet interactions in the n-p system, it is convenient to use δ function interactions. The operator for the pairing interaction can be written as

$$V_{12} = V_{12}^t + V_{12}^s = -\delta(\mathbf{r}_1 - \mathbf{r}_2)[v_t\pi_t + v_s\pi_s]. \quad (7)$$

The general nature of the splitting of levels of a given multiplet can be understood, starting from the treatment of the diagonal matrix elements of the n-p interaction, which are easily calculated using the algebra of irreducible tensor operators. For states of negative parity, which are most usual for heavy nuclei, the matrix elements of the triplet interaction V_{12}^t and the singlet interaction V_{12}^s are equal to:

a) for even values of the angular momentum J

$$\langle |V_{12}^t| \rangle = -\tfrac{1}{2} v_t F \langle l_1 j_1 \| T^J \| l_2 j_2 \rangle^2 \{1 + [4J(J+1)]^{-1} \times [(2j_1 + 1) + (-1)^{j_1+j_2}(2j_2 + 1)]^2\}, \quad (8a)$$

$$\langle |V_{12}^s| \rangle = 0; \quad (9a)$$

b) for odd values of the angular momentum J

$$\langle |V_{12}^t| \rangle = -\tfrac{1}{2} v_t F \langle l_1 j_1 \| T^J \| l_2 j_2 \rangle^2 [4J(J+1)]^{-1} \times [(2j_1 + 1) - (-1)^{j_1+j_2}(2j_2 + 1)]^2, \quad (8b)$$

$$\langle |V_{12}^s| \rangle = -\tfrac{1}{2} v_s F \langle l_1 j_1 \| T^J \| l_2 j_2 \rangle^2. \quad (9b)$$

Here F is the Slater integral, and

$$\langle l_1 j_1 \| T^J \| l_2 j_2 \rangle^2 = \tfrac{1}{2}[1 + (-1)^{l_1+l_2+J}] \times [(2j_1 + 1)(2j_2 + 1)/(2J + 1)]\, [C^{J0}_{j_1 1/2\, j_2 - 1/2}]^2, \quad (10)$$

where the coefficients $C^{J_0}_{j_1 \frac{1}{2}\, j_2 - \frac{1}{2}}$ for $j = 1/2, 3/2, \ldots 9/2$ are tabulated in the paper of de Shalit.[4]

Computations made for the case of $v_S = 2v_t$ show that in both case a) and case b) the dependence of the pairing energy can be described by the two curves shown in Fig. 2. For a multiplet of the type $j_1 = l_1 \pm 1/2$, $j_2 = l_2 \mp 1/2$, curve a describes the value of the pairing energy for even values of the angular momentum J, while curve b applies

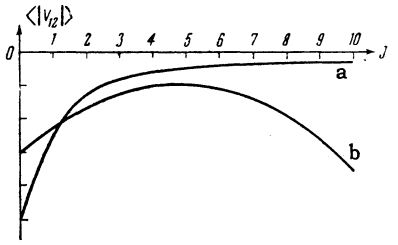

FIG. 2. Dependence of np-interaction energy on angular momentum J.

for odd J. For multiplets of the type $j_1 = l_1 \pm \frac{1}{2}$, $j_2 = l_2 \pm \frac{1}{2}$, the reverse holds: curve a is for odd J and curve b for even J. From these curves it follows that, just as in the case of identical particles, the level which is pushed deepest is the one with J_{min}, but the second level can be any other, including the level with J_{max}.

An analysis of results obtained with forces of finite range shows that the general character of the multiplet splitting remains the same as for δ forces. But one finds that the coefficient of v_t is larger, and the coefficient of v_s smaller, than the corresponding coefficients for δ forces. Consequently, for forces of finite range the triplet interaction predominates, even when the main contribution in the case of δ forces comes from the singlet interaction.

In conclusion we should mention that the values of the parameters v_t and v_s for Tl^{206} and Bi^{210} are quite close to one another, even though we are dealing in the first case with an interaction of holes and in the second with an interaction of particles. Additional data on these nuclei will make it possible to fix more precisely the values of the interaction parameters. But it is clear even now that including the n-p pairing interaction gives a complete picture of the properties of the low-lying levels of odd-odd nuclei.

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