ANALYSIS OF THE ANGULAR DISTRIBUTION OF REACTION PRODUCTS IN LIGHT NUCLEI

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Measurement of the angular distributions at energies $E_{\pm} = E_r \pm \frac{1}{2}\Gamma_r$ and $E_0 = E_r$, where E_r and Γ_r are the isolated resonance energy and width, respectively, permits one to separate the contribution of the direct mechanism to the reaction cross section.

In the (dp), (dn), (pn) and other reactions involving light nuclei and incident particles of medium energy, isolated resonances corresponding to quasi-discrete levels of a compound nucleus are observed in a number of cases.[1] However, the angular distributions of the reaction products are not usually symmetric with respect to 90°, and have maxima which are characteristic for direct interaction processes. This gives a basis for the assumption that direct and resonance mechanisms for the reaction are both present simultaneously. Then, owing to the interference between the two mechanisms, an analysis of experimental data on the angular distributions is extremely difficult. In the present article, it is shown that if the energy of the incident particles is sufficiently large in comparison with the height of the Coulomb barrier, then the measurement of the angular distribution at energies E_r and $E_r \pm \frac{1}{2}\Gamma_r$ (where E_r and Γ_r are the energy and width of the isolated resonance) permits us to separate the contribution of the direct mechanism to the differential cross section for the reaction.

The reaction amplitude in the vicinity of an isolated resonance of a compound nucleus can be written in the form

$$f = f_d + f_r,$$

$$f_d = -\frac{m_f}{2\pi\hbar^2}\langle\Phi_f|\hat{V}_d|\Phi_i\rangle,$$

$$f_r = -\frac{m_f}{2\pi\hbar^2}\frac{\langle\Phi_f|\hat{V}_f|X_r\rangle\langle X_r|\hat{V}_i|\Phi_i\rangle}{E - E_r + i\Gamma_r/2}, \qquad (1)$$

where Φ_i and Φ_f are the wave functions of the initial and final states of the system, $\hat{V}_d$ is the direct-interaction operator, $\hat{V}_i$ and $\hat{V}_f$ are the interactions in the initial and final states, X_r is the wave function of the compound nucleus, and E_r and Γ_r are the energy and width of the resonance.

For incident particle energies greater than the Coulomb barrier (in the region of carbon $E_c = Ze^2/R \sim 3$ Mev), the angular distributions of the direct reactions are satisfactorily described in the plane-wave approximation.[2] In this approximation, in the simpler case of inelastic scattering of spin-zero particles, the direct mechanism amplitude is

$$f_d = -\frac{m}{2\pi\hbar^2}\sum_L i^L\sqrt{4\pi(2L+1)}\,C^{j_f m_f}_{L0;\,j_i m_i}F^L_{j_i j_f}(K_{if}),$$

$$K_{if} = \sqrt{k_i^2 + k_f^2 - 2k_i k_f\cos\theta}, \qquad k_{i,f} = \sqrt{2m\varepsilon_{if}}/\hbar, \qquad (2)$$

where j_i and j_f are the spins of the initial and final nuclei, L is the transferred orbital angular momentum, and θ is the c.m.s. scattering angle. The nuclear matrix elements $F^L_{j_i j_f}(K_{if})$ of $\hat{V}_d$ between Φ_f and Φ_i determine the differential cross section for the direct mechanism of the reaction:

$$\frac{d\sigma_d}{d\Omega} = \frac{2j_f+1}{2j_i+1}\frac{m^2}{\pi\hbar^4}\frac{k_f}{k_i}\sum_L|F^L_{j_i j_f}(K_{if})|^2.$$

The resonance mechanism amplitude is equal to

$$f_r = -\frac{2m}{\hbar^2}\frac{1}{E - E_r + i\Gamma_r/2}\sum_{l\lambda\mu} i^{l-\lambda}\sqrt{4\pi(2l+1)}\,Y_{\lambda\mu}(\theta,\varphi)$$

$$\times\, v^l_{j_i I}v^\lambda_{j_f I}C^{IM}_{l0;\,j_i m_i}C^{IM}_{\lambda\mu;\,j_f m_f}, \qquad (3)$$

where l and λ are the orbital angular momenta of the partial waves in the incoming and outgoing channels, I is the spin of the resonance level of the compound nucleus. The nuclear matrix elements $v^l_{j_i I}$ and $v^\lambda_{j_f I}$ of $\hat{V}_i$ and $\hat{V}_f$ between Φ_i and Φ_f, and X_r determine the partial widths of the incoming and outgoing channels:

$$\Gamma^l_{j_i I} = 4m_i k_i\hbar^{-2}|v^l_{j_i I}|^2, \qquad \Gamma^\lambda_{j_f I} = 4m_f k_f\hbar^{-2}|v^\lambda_{j_f I}|^2.$$

The direct and resonance mechanism amplitudes in stripping reactions are of similar form (see, for example, reference 3).

The interference term in the differential cross section, as seen from (2) and (3), contains the factor

$$\mathrm{Re}\,\frac{i^{L+l+\lambda}F^{L*}_{j_i j_f}v^{l}_{j_i l}v^{\lambda}_{j_f l}}{E-E_r+i\Gamma_r/2}\,\delta_{\mu 0}.$$

Owing to the law of conservation of parity, the sum $L + l + \lambda$ is always even, regardless of the spin of the particles taking part in the reaction. Therefore the interference term close to E_r has the form

$$\frac{d\sigma_{int}}{d\Omega}=J^i_1+J^i_2,\qquad J^i_1=\frac{E-E_r}{(E-E_r)^2+\Gamma_r^2/4}\sum_{Ll\lambda}g^L_{l\lambda}(\theta)\,A,$$
$$J^i_2=\frac{{}^1\!/_2\Gamma_r}{(E-E_r)^2+\Gamma_r^2/4}\sum_{Ll\lambda}g^L_{l\lambda}(\theta)\,B, \tag{4}$$

where we have introduced the notation

$$F^{L*}_{j_i j_f}v^{l}_{j_i l}v^{\lambda}_{j_f l}\equiv A+iB,$$

and $g^L_{l\lambda}(\theta)$ are real functions of the scattering angle. The term J^i_1, containing A, vanishes for $E = E_r$, its absolute value attains a maximum at $|E - E_r| = {}^1\!/_2\,\Gamma_r$, and changes sign upon passing through resonance. The term J^i_2, containing B, depends on the energy in the same way as the resonance cross section

$$\frac{d\sigma_r}{d\Omega}=\frac{G_r(\theta)}{(E-E_r)^2+\Gamma_r^2/4},\qquad G_r(\theta)=G_r(\pi-\theta). \tag{5}$$

For incident particles of medium energy, the experimentally observed resonance widths in light nuclei are of the order 10 — 100 kev. In the region far from the reaction threshold, the direct mechanism cross section $d\sigma_d/d\Omega$ varies appreciably over intervals of the order of 1 — 2 Mev and is practically constant within the limits of the resonance width. Denoting

$$J(E;\theta)=d\sigma_d/d\Omega+d\sigma_r/d\Omega+d\sigma_{int}/d\Omega,$$

we readily obtain from formulas (4) and (5)

$$d\sigma_d/d\Omega=J(E_r+\tfrac{1}{2}\Gamma_r;\ \theta)+J(E_r-\tfrac{1}{2}\Gamma_r;\ \theta)-J(E_r;\ \theta). \tag{6}$$

Hence the proposed method of measurement of the angular distributions makes it possible to separate the contribution of the direct mechanism to the reaction cross section from the compound nucleus formation mechanism.

It is also of interest to determine the sum $d\sigma_r/d\Omega + J^i_2 = \mathscr{K}_r(\theta)$, characterizing the contribution from the compound nucleus formation mechanism to the angular distribution. According to (4) and (5),

$$\mathscr{K}_r(\theta)=2J(E_r;\theta)-J(E_r+\tfrac{1}{2}\Gamma_r;\ \theta)-J(E_r-\tfrac{1}{2}\Gamma_r;\ \theta).$$

Since part of the interference term J^i_2 occurs in $\mathscr{K}_r(\theta)$, then $\mathscr{K}_r(\theta)$ is not symmetric with respect to 90° if $B \neq 0$. If $B = 0$, then $\mathscr{K}_r(\theta) = \mathscr{K}_r(\pi - \theta)$, since in this case $\mathscr{K}_r(\theta) = (d\sigma_r/d\Omega)|_{E = E_r}$. If this condition is fulfilled, analysis of the angular distribution permits one to determine the spin of the resonance level of the compound nucleus.

[1] F. Ajzenberg-Selove and T. Lauritsen, Nuclear Phys. **11**, No. 1 (1959).

[2] S. Butler, Nuclear Stripping Reactions, Horowitz Publications, Sydney, 1957.

[3] A. G. Sitenko, Usp. Fiz. Nauk **67**, 377 (1959), Soviet Phys,-Uspekhi **2**, 195 (1959).

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