CALCULATION OF PROBABILITIES OF M3 TRANSITIONS AND SECOND-FORBIDDEN BETA TRANSITIONS IN THE NILSSON MODEL

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Submitted to JETP editor, August 8, 1960

J. Exptl. Theoret. Phys. (U.S.S.R.) **40**, 801-808 (March, 1961)

Expressions are obtained for the probabilities of M3 transitions and for second-forbidden beta transitions of pure Gamow-Teller type. The specific examination of the M3 transitions which are known in deformed nuclei indicates that the use of the corrected Nilsson representation is not necessary in these cases. A study of low-probability M3 transitions and second-forbidden beta transitions shows however that sometimes, when certain definite selection rules are satisfied, the use of the corrected Nilsson representation increases the transition probability by approximately a factor of two.

1. INTRODUCTION

EARLIER[1] we have calculated nuclear quadrupole moments on the assumption that they are caused by the Z protons moving independently in the deformed nuclear field which carries out a slow rotation which does not affect the individual proton motions. Such a model, which is analogous to that used by Inglis for calculating moments of inertia, gives the following quadrupole moment operator:*

$$\mathfrak{M}''_M(E2) = e\sum_{p=1}^{Z} r''^2_p Y_{2,M}(\vartheta''_p, \varphi''_p) = \sqrt{5/16\pi}\, eQ_M . \quad (1)$$

Using the eigenfunctions of the Nilsson single-particle Hamiltonian, we showed that the use of the corrected representation, which takes account of the interaction of states with different quantum numbers N, is necessary for nuclei in the range 150 < A < 188. In a succeeding paper[2] we came to the conclusion that it is not necessary to use the corrected representation for computing the probability of E2 gamma transitions, i.e, for transitions which are accompanied by a change in the internal structure of the nucleus. The reason for this is the different way in which the operators $r'^2 Y_{2,0}(\vartheta', \varphi')$ and $r'^2 T_{2,\nu}(\vartheta', \varphi')$ (with $\nu \neq 0$) transform when we go over to dimensionless coordinates.

The presence of a term containing only ρ^2 in the expression for the operator when $\nu = 0$ has the consequence that in the expression for the nuclear quadrupole moment, the correction term (which is proportional to a parameter ϵ, related to the deformation parameter δ) has the same order of magnitude as the main term, and consequently it becomes necessary to use the corrected representation.

Let us set $Q_0 = \langle Q_{M=0} \rangle$ and let us denote by $Q_0(\epsilon)$ and $Q_0(\delta)$ the theoretical values calculated for the operator (1) using, respectively, the improved and the original Nilsson representations, and let us denote by $Q_{0h}(\epsilon)$ and $Q_{0h}(\delta)$ the same values obtained by using the operator in the hydrodynamic approximation,

$$\mathfrak{M}''_M(E2) = e\sum_{p} r''^2_p Y_{2,M}(\vartheta''_p, \varphi''_p) + \frac{3}{4\pi} ZeR_0^2 \alpha^*_{2,M} \quad (2)$$

(where R_0 is the nuclear radius). In addition, we denote by Q_{0N} the values calculated using Nilsson's formula:[3]

$$Q_{0N} \approx \tfrac{4}{5} ZR_0^2 \varepsilon \left(1 + \tfrac{1}{2}\varepsilon + \ldots\right),$$

which is derived from the Thomas-Fermi model on the assumption that $\langle r'^2 \rangle = \frac{3}{5} R_0^2$ for all the protons. The results obtained by us can then be represented by the table below (where the values of Q_0 are given in units of 10^{-24} cm^2). The experimental values of intrinsic quadrupole moments are taken from the work of Alder, A. Bohr et al.[4] The corresponding errors of the measurements vary between 10 and 20%.

We note that the values of $Q_0(\epsilon)$ are in good agreement with the experimental data, except for Lu^{175} and Hf^{177}, for which the discrepancy is somewhat greater. As expected, in both representations the hydrodynamic values are larger than the experimental values.

From a comparison of the values of $Q_0(\epsilon)$ and $Q_0(\delta)$, we see that the use of the corrected repre-

*We use x″, y″, z″ for the space-fixed coordinate system, and x′, y′, z′ for the coordinate system fixed on the nucleus.

Nucleus	Eu^{153}	Gd^{155}	Gd^{157}	Tb^{159}	Ho^{165}	Lu^{175}	Hf^{177}	Hf^{179}
ε_e	0.33	0.31	0.31	0.34	0.30	0.26	0.25	0.24
$Q_{0\,\mathrm{exp}}$, $10^{-24}\,\mathrm{cm}^2$	7.7	8.0	7.7	6.9	7.8	8.2	7.5	7.0
$Q_0(\varepsilon)$	7.3	7.5	7.5	7.6	7.1	6.6	6.5	6.4
Q_{0N}	7.9	7.6	7.7	8.8	8.0	7.5	7.3	7.1
$Q_0(\delta)$	3.7	3.9	3.9	4.0	3.5	3.2	3.7	3.3
$Q_{0h}(\varepsilon)$	12	11.9	12	12.5	11.4	11.5	11.2	10.5
$Q_{0h}(\delta)$	10.2	10.1	10.2	10.3	9.6	9.5	9.5	9.3

sentation is necessary for computing nuclear quadrupole moments using the operator (1) ($Q_0(\epsilon) \approx 2Q_0(\delta)$). In the case where one applies operator (2), the difference which appears when one uses the two representations is much smaller, because the collective term does not change when we go over from one representation to the other.

In the present paper we consider other nuclear phenomena in whose mathematical description the operator $r'^2 Y_{2,0}(\vartheta', \varphi')$ appears, and where the use of the corrected representation may be necessary. Phenomena of this type are the M3 gamma ray transitions and second-forbidden beta transitions.

2. M3 TRANSITIONS

The reduced probability for an M3 transition is given by the expression

$$B(M3;\ J_i \to J_f) = \sum_{M,\,M_f} |\langle \Omega_f;\ J_f K_f M_f | \mathfrak{M}_M(M3) | \Omega_i;\ J_i K_i M_i\rangle|^2. \tag{3}$$

In the space-fixed coordinate system x″, y″, z″, we write the transition operator in the form

$$\mathfrak{M}_M(M3) = \frac{e\hbar}{2Mc}\sum_p \left(g_s \mathbf{s} + \frac{1}{2} g_l \mathbf{l}\right)\nabla_p\,[r_p''^3 Y_{3,M}(\vartheta_i'', \varphi_i'')]. \tag{4}$$

In the coordinate system fixed in the nucleus (x′, y′, z′), we consider the expression

$$\mathbf{l}\,(\nabla\, r'^3 Y_{3,\nu}) = \sqrt{7/5}\,\Big[l_z r'^2 \sqrt{9-\nu^2}\,Y_{2,\nu} + \tfrac{1}{2}\sqrt{(3-\nu)(2-\nu)}\,l_- r'^2 Y_{2,\nu+1} - \tfrac{1}{2}\sqrt{(3+\nu)(2+\nu)}\,l_+ r'^2 Y_{2,\nu-1}\Big]$$

(where $l_\pm = l_x \pm i l_y$) and an analogous expression for the operator $\mathbf{s}\ (\nabla r'^3 Y_{3,\nu})$. Transforming to the new variables

$$\xi = x'\sqrt{M\omega_x/\hbar}, \qquad \eta = y'\sqrt{M\omega_y/\hbar}, \qquad \zeta = z'\sqrt{M\omega_z/\hbar}, \tag{5}$$

where

$$\omega_x = \omega_y = \omega_0(\varepsilon)\left(1 + \tfrac{1}{3}\varepsilon\right),\quad \omega_z = \omega_0(\varepsilon)\left(1 - \tfrac{2}{3}\varepsilon\right),\quad \omega_0(\varepsilon) \approx \omega_0^0\left(1 + \tfrac{1}{9}\varepsilon^2\right),$$

and using the same nuclear wave function as previously,[1,2] we obtain, in place of (3), the formula*

$$B(M3;\ J \to J_f) = \left(\frac{e\hbar}{2Mc}\right)^2 \left(\frac{\hbar}{M\omega_0(\varepsilon)}\right)^2 \frac{7}{16\pi}\langle J_i 3 K_i k | J_f K_f\rangle^2 G_{M3}^2(\varepsilon), \tag{6}$$

$$k = K_f - K_i, \qquad G_{M3}(\varepsilon) \approx G_{M3}^t + \varepsilon G_{M3}^{t'}. \tag{7}$$

G_{M3}^t in Eq. (7) coincides with the term $G_{M3}(\delta)$ found by Nilsson[3] (his formula 36c), with the one difference that the quantum numbers N, l and Λ should be replaced by N_t, l_t and Λ_t, determined in the corrected representation.†

We found the following expression for the correction term $\epsilon G_{M3}^{t'}$:

$$\begin{aligned}
G_{M3}^{t'} = {} & \sum_{l_f,\,l_i} \langle N_f l_f | \rho^2 | N_i l_i\rangle \sum_{\Lambda_f,\,\Lambda_i,\,\Sigma_f,\,\Sigma_i} a_{l_f,\Lambda_f}\, a_{l_i,\Lambda_i} \\
& \times \Big\{ g_s \Big[\langle l_i 200 | l_f 0\rangle \sqrt{\frac{2l_i+1}{2l_f+1}}\,\big(A(k)\,\delta_{\Sigma_f,\Sigma_i} \\
& \times (-1)^{\Sigma_i - 1/2} u(k) \langle l_i 2\Lambda_i k | l_f \Lambda_f\rangle \\
& + B(k)\,\delta_{\Sigma_f,-1/2}\delta_{\Sigma_i,1/2}\, v(k) \langle l_i 2 \Lambda_i k+1 | l_f\Lambda_f\rangle \\
& - C(k)\,\delta_{\Sigma_f,1/2}\delta_{\Sigma_i,-1/2}\, w(k)\langle l_i 2\Lambda_i k-1 | l_f \Lambda_f\rangle\big) \\
& + \tfrac{1}{3}\left(1 + \tfrac{1}{3}\varepsilon\right)\sqrt{5/4\pi}\,\big(A(k)\,\delta_{k,0}\delta_{\Sigma_f,\Sigma_i}(-1)^{\Sigma_i - 1/2} \\
& + B(k)\,\delta_{k,-1}\delta_{\Sigma_f,-1/2}\delta_{\Sigma_i,1/2} \\
& - C(k)\,\delta_{k,1}\delta_{\Sigma_f,1/2}\delta_{\Sigma_i,-1/2}\big)\,\delta_{l_i l_f}\delta_{\Lambda_i,\Lambda_f}\Big] \\
& + \tfrac{1}{2} g_l \delta_{\Sigma_f,\Sigma_i}\Big[\langle l_i 200 | l_f 0\rangle\sqrt{\frac{2l_i+1}{2l_f+1}}\,\big[A(k)\,2\Lambda_f u(k) \\
& \times \langle l_i 2\Lambda_i k | l_f\Lambda_f\rangle + B(k)\big(\sqrt{(l_f - \Lambda_f)(l_f + \Lambda_f + 1)}\, v(k) \\
& \times \langle l_i 2\Lambda_i k+1 | l_f \Lambda_f + 1\rangle + p(k)\langle l_i 2\Lambda_i k | l_f \Lambda_f\rangle\big) \\
& - C(k)\big(\sqrt{(l_f + \Lambda_f)(l_f - \Lambda_f + 1)}\, w(k) \\
& \times \langle l_i 2\Lambda_i k - 1 | l_f \Lambda_f - 1\rangle + q(k)\langle l_i 2\Lambda_i k | l_f\Lambda_f\rangle\big)\big] \\
& + \tfrac{1}{3}\sqrt{5/4\pi}\,\big[\left(1 + \tfrac{1}{3}\varepsilon\right) A(k)\cdot 2\Lambda_f \delta_{k,0}\delta_{l_f,l_i}\delta_{\Lambda_f,\Lambda_i}
\end{aligned}$$

*The details of the calculation will be published elsewhere.[5]

†For simplicity, in the following we omit the subscript t in writing the quantum numbers.

$$+ B(k)\left((1+\tfrac{1}{3}\varepsilon)\sqrt{(l_f-\Lambda_f)(l_f+\Lambda_f+1)}\right.$$
$$\times\, \delta_{k,-1}\delta_{l_f,l_i}\delta_{\Lambda_i,\Lambda_f+1} + \sqrt{3}\,(2+\tfrac{1}{3}\varepsilon)\,\delta_{k,0}\delta_{l_f,l_i}\delta_{\Lambda_i,\Lambda_f}\Big)$$
$$- C(k)\left((1+\tfrac{1}{3}\varepsilon)\sqrt{(l_f+\Lambda_f)(l_f-\Lambda_f+1)}\right.$$
$$\times\, \delta_{k,1}\delta_{l_f,l_i}\delta_{\Lambda_i,\Lambda_{f-1}} + \sqrt{3}\,(2+\tfrac{1}{3}\varepsilon)\,\delta_{k,0}\delta_{l_f,l_i}\delta_{\Lambda_i,\Lambda_f}\Big)\Big]\Big]\Big\}. \tag{7a}$$

Here

$$\rho^2 = \xi^2+\eta^2+\zeta^2; \qquad A(k) = \sqrt{9-k^2},$$

$$B(k) = \sqrt{(3-k)(2-k)}, \qquad C(k) = \sqrt{(3+k)(2+k)},$$

$$p(k) = \begin{cases} 1 & \text{for } k=1 \\ -1 & \text{for } k=-2 \\ \sqrt{1/6} & \text{for } k=0, \\ -\sqrt{1/6} & \text{for } k=-1 \end{cases} \qquad q(k) = \begin{cases} 1 & \text{for } k=-1 \\ -1 & \text{for } k=2 \\ \sqrt{1/6} & \text{for } k=0 \\ -\sqrt{1/6} & \text{for } k=1 \end{cases},$$

$$u(k) = \begin{cases} 0 & \text{for } k=\pm 3 \\ -1/3 & \text{for } k=\pm 2 \\ 1/6 & \text{for } k=\pm 1 \\ 1/3 & \text{for } k=0 \end{cases}, \qquad v(k) = \begin{cases} -1/3 & \text{for } k=-3.1 \\ 1/6 & \text{for } k=-2.0 \\ 1/3 & \text{for } k=-1 \\ 0 & \text{for } k=-2,3 \end{cases},$$

$$w(k) = \begin{cases} 0 & \text{for } k=-3,\,-2 \\ -1/3 & \text{for } k=-1.3 \\ 1/6 & \text{for } k=0.2 \\ 1/3 & \text{for } k=1 \end{cases}.$$

3. SECOND-FORBIDDEN BETA TRANSITIONS

The set of nuclear matrix elements which appear in the form factors of second-forbidden β spectra are written, following Konopinski,[6] as follows:*

$$S, V:\ \int(\beta)\,\mathcal{Y}_{2,M}(\mathbf{r}), \quad \Delta J = 2;$$

$$T, A:\ \int(\beta)\,\mathcal{Y}_{2,M}([\boldsymbol{\sigma}\mathbf{r}]), \quad \Delta J = 2;$$

$$T, V:\ \int(\beta)\,\mathcal{Y}_{2,M}(\boldsymbol{\alpha}), \quad \Delta J = 2;$$

$$T, A:\ \int(\beta)\,\mathcal{Y}_{3,M}(\boldsymbol{\sigma}), \quad \Delta J = 3,$$

where, in the space-fixed coordinate system,

$$\mathcal{Y}_{2,M}(\mathbf{r}) = r''^2 Y_{2,M}(\vartheta'',\varphi''),$$

$$\mathcal{Y}_{3,M}(\boldsymbol{\sigma}) = \sqrt{7/5}\Big[\sqrt{9-M^2}\, r''^2 Y_{2,M} s_z$$
$$+ \tfrac{1}{2}\sqrt{(3-M)(3-M-1)}\, r''^2 Y_{2,M+1} s$$
$$- \tfrac{1}{2}\sqrt{(3+M)(3+M-1)}\, r''^2 Y_{2,M-1} s_+\Big],$$

$$\mathcal{Y}_{2,M}(\boldsymbol{\alpha}) = \tfrac{1}{2} r'' \sqrt{5/3}\Big[\sqrt{4-M^2}\, Y_{1,M}\alpha_z$$
$$- \sqrt{(2+M)(2+M-1)}\, Y_{1,M-1}\alpha_+$$
$$+ \sqrt{(2-M)(2-M+1)}\, Y_{1,M+1}\alpha_-\Big],$$

$$\mathcal{Y}_{2,M}([\boldsymbol{\sigma}\mathbf{r}]) = \frac{i}{2\hbar}\boldsymbol{\sigma}\mathbf{L}\mathcal{Y}_{2,M}(\mathbf{r})$$
$$= \frac{i}{2\hbar^2}(j^2 - L^2 - s^2)\, r''^2 Y_{2,M}(\vartheta'',\varphi'').$$

The matrix element containing the operator $\mathcal{Y}_{3,M}(\sigma)$ has a form which is very similar to the matrix element $\mathfrak{M}_M(M3)$, which corresponds to M3 gamma radiation. The β transitions described by this matrix element, the so-called "unique" transitions, have the remarkable property that their spectra are easily determined. The change in parity for such a transition is $\Delta\pi = (-1)^{\Delta J+1}$.

The corresponding values of $f_2\tau$ have the following form (where we use Nilsson's[3] notation; x is the fraction of pure Gamow-Teller interaction):

$$f_2\tau = B_g[xD_{G.T.}(2)]^{-1}, \qquad B_g = 2\pi^3\hbar\ln 2/g^2 m_e^5 c^4, \tag{8}$$

where the reduced probability of transition is

$$D_{G.T.}(2) = S^2(2)\sum_{M,M_f}\left|\int\mathcal{Y}_{3,M}(\boldsymbol{\sigma})\right|^2,$$

$$\int\mathcal{Y}_{3,M}(\boldsymbol{\sigma})$$
$$= \langle\Omega_f; J_fK_fM_f\Big|\sum_p s_p\nabla_p[r_p''^3 Y_{3,M}(\vartheta_p'',\varphi_p'')\tau_\pm^p]\Big|\Omega_i;\ J_iK_iM_i\rangle,$$

$$S(2) = \sqrt{\frac{4\pi 2^5}{7!}\frac{(3!)^2}{3}\left(\frac{m_e c}{\hbar}\right)^2}.$$

Proceeding in the same way as for M3 transitions, we get for $D_{G.T.}(2)$ the expression

$$D_{G.T.}(2) = S^2(2)\frac{7}{16\pi}\left(\frac{\hbar}{M\omega_0(\varepsilon)}\right)^2\langle J_i3K_ik\,|\,J_fK_f\rangle^2\gamma_3^2(\varepsilon). \tag{9}$$

Here $\gamma_3(\epsilon) = G_{M3}(\epsilon)$ ($g_s = 1$, $g_l = 0$), where the expression for $G_{M3}(\epsilon)$ is defined by (7). The operators $\mathcal{Y}_{2,M}(\mathbf{r})$ and $\mathcal{Y}_{2,M}(\boldsymbol{\alpha}\times\mathbf{r})$ are proportional to the operator $\mathfrak{M}_M(E2)$ of radiation theory. For this reason the square moduli of the matrix elements appearing in the expressions for the quantity $f_2\tau$ are very similar to the expression for the reduced transition probability B(E2):

$$\sum_{M,M_f}\left|\int\mathcal{Y}_{2,M}(\mathbf{r})\right|^2 = \frac{\hbar}{M\omega_0(\varepsilon)}\frac{5}{4\pi}\langle J_i2K_ik\,|\,J_fK_f\rangle^2\gamma_2^2(\varepsilon), \tag{10}$$

$$\gamma_2(\varepsilon) = G_{E2}^t + \varepsilon G_{E2}^{t'}. \tag{11}$$

In Eq. (11) G_{E2}^t is identical with the term found by Nilsson [formula (35c) in reference 3], where, however, the quantum numbers have the subscript t, and $G_{E2}^{t'}$ has the form

$$G_{E2}^{t'} = \sum_{l_i,l_f}\langle N_fl_f|\rho^2|N_il_i\rangle\sum_{\Lambda_f,\Lambda_i,\Sigma_i,\Sigma_f}a_{l_f,\Lambda_f}a_{l_i,\Lambda_i}\delta_{\Sigma_f,\Sigma_i}$$
$$\times\Big[\sqrt{(2l_i+1)/(2l_f+1)}\,\langle l_i200\,|\,l_f0\rangle\langle l_i2\Lambda_ik\,|\,f_f\Lambda_f\rangle\, p(k)$$
$$+ \tfrac{1}{3}(1+\tfrac{1}{3}\varepsilon)\,\delta_{k,0}\delta_{l_i,l_f}\delta_{\Lambda_i,\Lambda_f}\Big],$$

where

$$p(k) = \begin{cases} 1/3 & \text{for } k=0 \\ 1/6 & \text{for } k=\pm 1 \\ -1/3 & \text{for } k=\pm 2 \end{cases}.$$

*$[\sigma\mathbf{r}] = \sigma\times\mathbf{r}$.

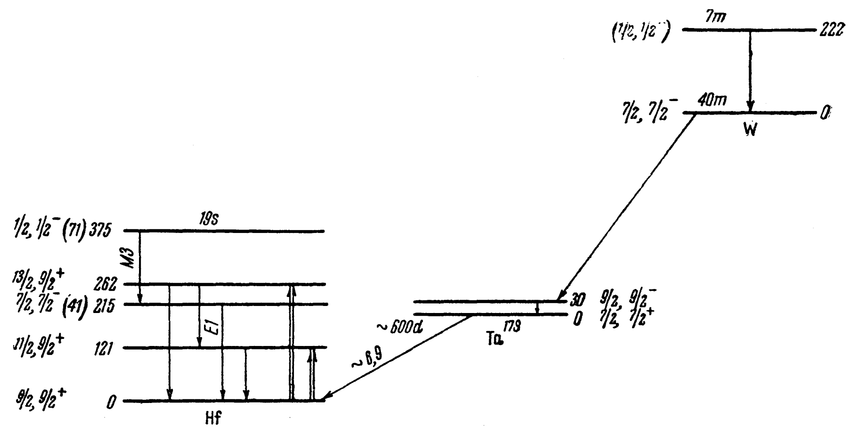

FIG. 1

4. SOME SPECIFIC CASES

One may anticipate that, just as in the calculation of quadrupole moments, the use of the corrected representation will be important in those cases where a contribution proportional to ρ^2 appears in the expression for the operator in the correction term.

For deformed nuclei, using the data of Mottelson and Nilsson,[7] we found three M3 transitions. These are shown in Figs. 1 and 2 (where we indicate for each level, in addition to its energy, the values of J, K and the parity).

In the case of Hf^{179} and W^{179}, the final state of the transition is the same (41 in Nilsson's notation[3]). In Hf^{179} the initial level is 71, whereas for W^{179} we have either 71 or 63. In both cases, $\eta \approx 4$,* and the corresponding transitions are isomeric: the half-life of the initial level corresponding to the first transition is 19 sec, while that of the initial level for the second transition is 7 min. For both transitions, k = 3, and consequently, according to the selection rule on k in Eq. (7a), the contribution of the term in ρ^2 from the expression for $\mathfrak{M}_M(M3)$ is zero. In the case of Hf^{179} we found $G^2_{M3}(\delta) = 0.553 \times 10^2$ and $G^2_{M3}(\epsilon) = 0.471 \times 10^2$. Consequently, as expected, in this case the use of the corrected representation does not essentially change the expression for the transition probability. The same conclusions apply to W^{179}.

For Ta^{181} (Fig. 2), although the levels between which the M3 transition occurs are different (43 and 25), nevertheless in this case also k = 3, and for $\eta = 4$ we found $G^2_{M3}(\delta) = 0.174 \times 10^3$ and $G^2_{M3}(\epsilon) = 0.161 \times 10^3$.

The theoretical half-life computed using the values $G^2_{M3}(\epsilon)$ is approximately 10 sec for the $(\frac{1}{2}, \frac{1}{2}^-)$ state in Hf^{179}; for the $(\frac{1}{2}, \frac{1}{2}^+)$ state

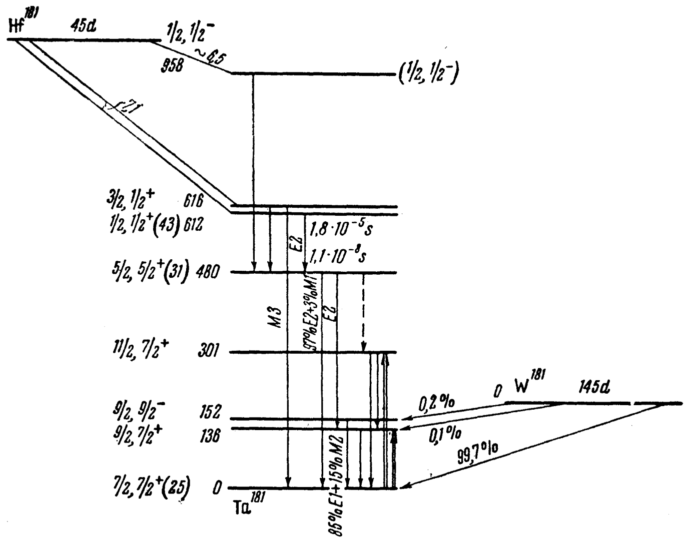

FIG. 2

in Ta^{181} it is 1.9×10^{-4} sec, when we include the E2 transition $(\frac{1}{2}, \frac{1}{2}^+) \rightarrow (\frac{5}{2}, \frac{5}{2}^+)$ which we calculated previously.[2] For the initial level of the M3 transition in W^{179} we found a value of τ of the same order as the corresponding quantity for the $(\frac{1}{2}, \frac{1}{2}^-)$ state of Hf^{179}, i.e., a value which is an order of magnitude smaller than the experimental result. The M3 transition in Hf^{179} is strongly suppressed; the experimental value of the suppression factor is $F_{exp} \approx 10^{-3}$. The corresponding theoretical factor* was $F_{th} = 2 \times 10^{-2}$.

In order to examine cases where, in the expression for $G_{M3}(\epsilon)$, the selection rules on k allow nonzero contributions of the term in ρ^2 (i.e., cases where k takes on one of the values −1, 0, or 1) we have considered transitions having low probability, which are as yet not observed, which according to the selection rules might be M3, and for which k takes on one of the values enumerated.

*η denotes the parameter introduced by Nilsson,[3] which is related to the nuclear deformation δ'.

*Our factor F is equal to H^{-1}, where H is the factor used by Mottelson and Nilsson.[7]

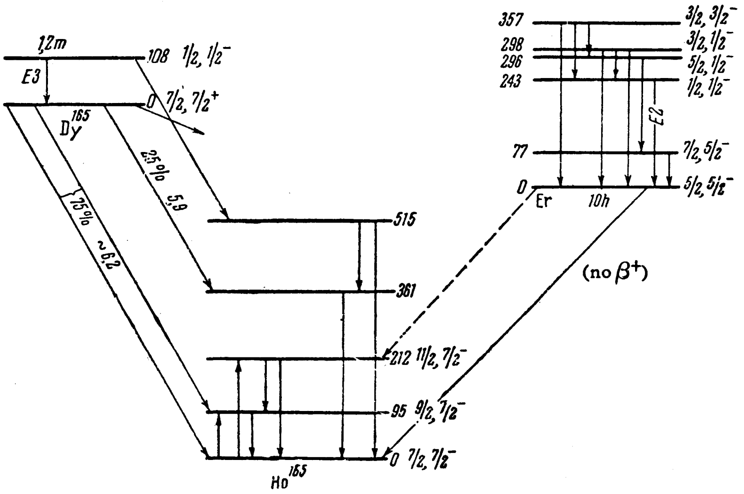

FIG. 3

In the case of Ta^{181}, a transition of this type is the one indicated by the dashed line in Fig. 2. However, it turned out in this case that when we substituted the values of the coefficients $a_{l,\Lambda}$, the term of G^t_{M3} of interest to us (which was different from zero since $k = 1$) was small compared with the others, and the use of the corrected representation gave nothing new.

Analogous results are obtained in the case of second forbidden β transitions. In deformed nuclei, not a single case of a second forbidden transition has yet been discovered, so we turned to the examination of doubtful cases. Among the nuclei with $A = 161$,[7] the transition $(3/2, 3/2^+)$ Tb $\rightarrow$ $(9/2, 5/2^+)$ Dy, with $k = 1$, might be a unique β^- transition. With $\eta = 6$, we found no significant difference between $\gamma_3^2(\delta)$ and $\gamma_3^2(\epsilon)$. However, for nuclei with $A = 165$ (cf. Fig. 3) the transition $(5/2, 5/2^-)$ Er $\rightarrow$ $(11/2, 7/2^-)$ Ho should be studied using the corrected representation, since for $\eta = 6$ we have $\gamma_3^2(\delta) = 0.13 \times 10^2$ and $\gamma_3^2(\epsilon) = 0.256 \times 10^2$. The result of this is that the difference resulting from using the two representations appears even in the $\log f_2\tau$ values:

$$\log f_2\tau\,(\varepsilon) = 15.8, \quad \log f_2\tau\,(\delta) = 16.1.$$

Similarly, in the case of nuclei with $A = 177$, for the β^- transition $(7/2, 7/2^-)$ Lu $\rightarrow$ $(11/2, 7/2^-)$ Hf[7] the absolute squares of some of the matrix elements (of type 10) are twice as great for the corrected as for the original Nilsson representation. The reason is that for such a transition $k = 0$.

5. CONCLUSION

Unlike the case of the calculation of the quadrupole moments of nuclei in the region $150 < A < 188$, where the use of the corrected Nilsson representation was always necessary, for M3 gamma transitions and second forbidden β transitions the use of the corrected representation is limited to those cases where $k = K_f - K_i = -1$, 0, or 1, and where the coefficients $a_{l,\Lambda}$ of the wave functions corresponding to the initial and final states are sufficiently large. Such cases have not yet been discovered in deformed nuclei. The reason for the difference in the two calculations is the following. Because of the normalization condition $\Sigma a^2_{l,\Lambda} = 1$, the coefficients of the wave functions do not appear in the expression for the main correction term for the quadrupole moment; however, in our case the normalization condition cannot be used in the main correction term, since the coefficients $a_{l,\Lambda}$ refer to different states. Moreover, in the quadrupole moment calculations the condition $k = 0$ is always satisfied, which guarantees the appearance of the correction term in which we are interested.

In conclusion, I express my gratitude to Prof. S. Titeica for his interest in the work.

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Translated by M. Hamermesh
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