MARSHAK INVARIANCE AND FOUR-FERMION INTERACTIONS

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A classification of baryons and leptons which is useful for describing weak interactions is proposed. Marshak invariance is generalized to all baryons and leptons.

It has been shown[1] that every known baryon can be characterized by a triplet of numbers t_1, t_2, t_3, which are determined as follows. We imagine that the baryons are located at the vertices of a unit cube (see Fig. 1). Then the numbers t_1, t_2, t_3, are the coordinates of the corresponding vertices in a Cartesian coordinate system.

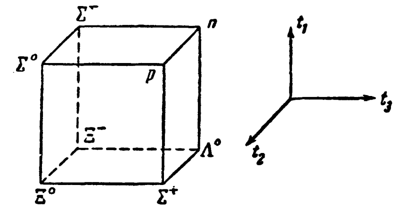

FIG. 1

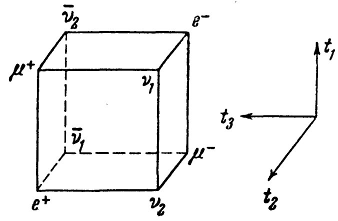

FIG. 2

Marshak[2] pointed out the interesting fact that weak interaction processes are invariant to the interchanges $p \longleftrightarrow \nu$, $n \longleftrightarrow e^-$, $\Lambda^0 \longleftrightarrow \mu^-$. One may conjecture that this invariance extends to all baryons.

We represent the set of leptons in the form of a unit cube like the baryon cube (Fig. 2). In order to keep the expression for electric charge, $q = e(t_2 + t_3)$, the same for baryons and leptons, one must assume that the cube is described in a left-handed coordinate system. In the case shown, we have two neutrinos, ν_1 and ν_2, which differ in the values of their leptonic charges $l = 2t_1$ (see also reference 3).

Marshak invariance can now be formulated in the statement that the set of weak interaction processes is invariant to interchange of the particles which are at the same cube vertex, if only those interchanges which conserve the number of baryons are allowed. Comparison of the lepton cube with the baryon cube gives the transformation

$$\begin{array}{llll} p \leftrightarrow \nu_1, & \Lambda^0 \leftrightarrow \mu^-, & \Sigma^0 \leftrightarrow \mu^+, & \Xi^- \leftrightarrow \bar{\nu}_1, \\ n \leftrightarrow e^-, & \Sigma^+ \leftrightarrow \nu_2, & \Sigma^- \leftrightarrow \bar{\nu}_2, & \Xi^0 \leftrightarrow e^+. \end{array} \quad (1)$$

The requirement of baryon number conservation arises from $[\mathcal{H}, N] = 0$, where N is the baryon number operator and $\mathcal{H}$ is the Hamiltonian describing the weak interactions of baryons with leptons and baryons. In the case of baryon-lepton interactions, this requirement is automatically taken into account when the transformation (1) is applied to the equations written in the form $B + L \rightarrow B' + L'$, where B, B′ are baryons and L, L′ are leptons. For example, according to the rule, the equation $n \rightarrow p + e^- + \bar{\nu}_1$ should be written in the form $n + e^+ \rightarrow p + \bar{\nu}_1$, or $n + \nu_1 \rightarrow p + e^-$. The transformation (1) transforms the latter two equations into $e^- + \Xi^0 \rightarrow \nu_1 + \Xi^-$ and $e^- + p \rightarrow \nu_1 + n$ which can be rewritten as $n \rightarrow p + e^- + \bar{\nu}_1$ and $\Xi^- \rightarrow \Xi^0 + e^- + \bar{\nu}_1$.

For weak interactions involving antibaryons, it is necessary to apply the transformation (1) in its charge-conjugate form, i.e., $\bar{p} \longleftrightarrow \bar{\nu}_1$, etc.

The selection rules for weak interactions are now expressed as conservation of the three quantum numbers t_1, t_2, t_3 in weak interactions. With this rule, the process $\mu^+ \rightarrow e^+ + \bar{\nu}_1 + \nu_2$, for example, is allowed; the process $\mu \rightarrow 3e$ is forbidden.

The four-Fermion interaction Hamiltonian then has the isospin invariant form

$$H = \sum_k g_k \,|\, (\bar{\psi}_B T^{(k)} O \psi_B) + (\bar{\psi}_L T^{(k)} O \psi_L) \,|^2. \quad (2)$$

Here ψ_B is the potential of the baryon field. (32-component spinor) and ψ_L is the potential of the lepton field. The matrices $T^{(k)}$ are the isotopic spin vectors[1], the numbers t_k are eigenvalues of the third components of these vectors. The 32-rowed matrix O determines the type of interaction. We note that the additional requirement that the Hamiltonian, H, be invariant to Fierz transformations gives $O = \Gamma^{\nu}(1 - \Gamma^{5})$, where $\Gamma^{\nu} = 1 \times \gamma^{\nu}$, $\Gamma^{5} = 1 \times \gamma^{5}$.

[1] H. Oiglane, JETP **37**, 558 (1959), Soviet Phys. JETP **10**, 394 (1960).

[2] R. E. Marshak, Theoretical Status of Weak Interactions, Report to the 9th International Conference on High-Energy Physics, Kiev, 1959.

[3] É. M. Lipmanov, JETP **37**, 1054 (1959), Soviet Phys. JETP **10**, 750 (1960).

[4] M. Fierz, Z. Physik **104**, 553 (1937).

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