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COMPLETE SET OF EXPERIMENTS FOR DETERMINATION OF RELATIONS BETWEEN THE AMPLITUDES FOR PION PRODUCTION BY NUCLEONS IN VARIOUS ISOTOPIC SPIN STATES

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A complete set of experiments for determination of the relations between the amplitudes for pion production by nucleons in various isotopic spin states is considered. The number of experiments so far performed with nucleons in the 600-Mev energy region is not sufficient to determine all the relations among the amplitudes. One of the discussed experiments may provide a sensitive test of the validity of the resonance theory of pion production by nucleons.

INTRODUCTION

THE attempts at constructing a theory for pion production by nucleons, carried out by a number of authors, have usually consisted of phenomenological considerations which would give quantitative explanations of available experimental data. Thus, for example, Watson and Brueckner[1] gave a partial analysis of the meson production process by nucleons. They supposed that only a small number of states with definite orbital angular momentum participate in the process. The matrix elements for the processes were taken to be constant, the principle of charge independence was taken into account as well as other conservation laws, and the attractive final state interaction of the nucleons was also included. On the basis of such a phenomenological approach threshold data were analyzed by Rosenfeld[2] and Gell-Mann and Watson.[3] In the analysis they have made use of a hypothesis proposed by Brueckner,[4] namely that at nucleon energies close to the threshold one of the secondary nucleons and the produced pion are predominantly in the resonant $(^3/_2, {}^3/_2)$ state.

Mandelstam[5] has extended this phenomenological theory to higher energies, from 400 to 600 Mev. In his considerations the matrix elements were assumed constant, the πN resonant $(^3/_2, {}^3/_2)$ interaction was introduced, all kinematic factors were taken into account, as well as the rules of composition of angular momentum and spin of the particles. According to this model, which gives the best explanation of all experimental data obtained by proton beams, there should be no significant production of pions by nucleons in isotopic spin state zero $(T_{NN} = 0)$. However, recent experimental data on pion production in neutron-proton collisions[6,7] indicate that this prediction of the resonant theory of Mandelstam is in disagreement with experiment.

Can one therefore conclude that the resonant model of Mandelstam is inconsistent? In order to answer this question it is necessary to take into account not only the resonant πN interaction in the isotopic spin $T = {}^3/_2$ state, but also in the isotopic spin $T = {}^1/_2$ state. Doubtlessly both resonances should make themselves felt in the general case. However the resonant Mandelstam model is consistent only with an overwhelming preponderance of the $T_{\pi N} = {}^3/_2$ resonance. Thus the resonant theory can be directly tested if the ratio of the probabilities of the two resonant interactions could be determined. However the experiments performed so far are not sufficient to yield the desired relation. All these reasons have stimulated us anew to discuss in more detail a complete set of experiments, necessary to establish relations among the amplitudes for pion production by nucleons in various isotopic spin states.

1. REACTION CROSS SECTIONS

As is well known, in the isotopic spin space the phenomenological description of pion production by nucleons requires the introduction of three independent amplitudes. The probability of an arbitrary pion production process by a two-nucleon system can be explicitly expressed in terms of three independent amplitudes. If the amplitude for pion production in the collision of two nucleons N_1 and N_2 be denoted by $M(N_1N_2 \rightarrow N_1'N_2'\pi)$, where N_1' and N_2' are the secondary nucleons, then this amplitude will change under the exchange of nucleons N_1, N_2 or N_1', N_2'. When charge symmetry is taken into account the number of different pion production processes reduces to seven.[8]

The differential cross sections for these processes may be written as follows:[8,9]

$$\text{I. } d\sigma(pp \rightarrow np\pi^+) = \tfrac{1}{2}|F_{10}|^2 + \tfrac{1}{4}|F_{11}|^2 + \sqrt{1/2}\,|F_{11}|\,|F_{10}|\cos\Phi_{10,11} = \tfrac{3}{4}|A_{13}|^2,$$

$$\text{II. } d\sigma(pp \rightarrow pn\pi^+) = \tfrac{1}{2}|F_{10}|^2 + \tfrac{1}{4}|F_{11}|^2 - \sqrt{1/2}\,|F_{11}|\,|F_{10}|\cos\Phi_{10,11} = \tfrac{1}{3}\left[\tfrac{1}{4}|A_{13}|^2 + 2|A_{11}|^2 - \sqrt{2}|A_{13}|\,|A_{11}|\cos\varphi_{13}\right],$$

$$\text{III. } d\sigma(pp \rightarrow pp\pi^0) = \tfrac{1}{2}|F_{11}|^2 = \tfrac{1}{3}\left[\tfrac{1}{2}|A_{13}|^2 + |A_{11}|^2 + \sqrt{2}|A_{13}|\,|A_{11}|\cos\varphi_{13}\right],$$

$$\text{IV. } d\sigma(pn \rightarrow nn\pi^+) = \tfrac{1}{6}|F_{01}|^2 + \tfrac{1}{4}|F_{11}|^2 - \sqrt{1/6}\,|F_{01}|\,|F_{11}|\cos\Phi_{01,11} = \tfrac{1}{6}\left[\tfrac{1}{2}|A_{13}|^2 + |A_{11}|^2 + |A_{01}|^2 + \sqrt{2}|A_{13}|\,|A_{11}|\cos\varphi_{13} - 2|A_{11}|\,|A_{01}|\cos\varphi_{01} - \sqrt{2}|A_{13}|\,|A_{01}|\cos\varphi_{03}\right],$$

$$\text{V. } d\sigma(np \rightarrow nn\pi^+) = \tfrac{1}{6}|F_{01}|^2 + \tfrac{1}{4}|F_{11}|^2 + \sqrt{1/6}\,|F_{01}|\,|F_{11}|\cos\Phi_{01,11} = \tfrac{1}{6}\left[\tfrac{1}{2}|A_{13}|^2 + |A_{11}|^2 + A_{01}|^2 + \sqrt{2}|A_{13}|\,|A_{11}|\cos\varphi_{13} + 2|A_{11}|\,|A_{01}|\cos\varphi_{01} + \sqrt{2}|A_{13}|\,|A_{01}|\cos\varphi_{03}\right],$$

$$\text{VI. } d\sigma(np \rightarrow np\pi^0) = \tfrac{1}{12}|F_{01}|^2 + \tfrac{1}{4}|F_{10}|^2 - 2\sqrt{1/3}\,|F_{01}|\,|F_{10}|\cos\Phi_{01,10} = \tfrac{1}{6}\left[|A_{13}|^2 + \tfrac{1}{2}|A_{11}|^2 + \tfrac{1}{2}|A_{01}|^2 - \sqrt{2}|A_{13}|\,|A_{11}|\cos\varphi_{13} - \sqrt{2}|A_{13}|\,|A_{01}|\cos\varphi_{03} + |A_{11}|\,|A_{01}|\cos\varphi_{01}\right],$$

$$\text{VII. } d\sigma(np \rightarrow pn\pi^0) = \tfrac{1}{12}|F_{01}|^2 + \tfrac{1}{4}|F_{10}|^2 + 2\sqrt{1/3}\,|F_{01}|\,|F_{10}|\cos\Phi_{01,10} = \tfrac{1}{6}\left[|A_{13}|^2 + \tfrac{1}{2}|A_{11}|^2 + \tfrac{1}{2}|A_{01}|^2 + \sqrt{2}|A_{13}|\,|A_{11}|\cos\varphi_{13} + \sqrt{2}|A_{13}|\,|A_{01}|\cos\varphi_{03} - |A_{11}|\,|A_{01}|\cos\varphi_{01}\right]. \qquad (1)$$

Here F_{ij} stand for the three reaction amplitudes in the representation in which in the final state the subsystem is formed out of the two nucleons; i stands for the isotopic spin of the two nucleons in the initial state, j – in the final state; A_{il} stand for the reaction amplitudes in the representation in which in the final state the subsystem is formed out of the pion and a nucleon, $l = 2T_{\pi N}$, where $T_{\pi N}$ refers to the isotopic spin of the πN subsystem.

2. METHODS FOR MEASUREMENT OF INTERFERENCE EFFECTS

In Eq. (1) the processes I and II, IV and V, as well as VI and VII differ from each other because of interference between two amplitudes. The establishment of a difference between two such processes will provide us with valuable additional data. Let us consider various methods by which this difference can be observed experimentally.

A. If the proton and neutron are exchanged in the initial state (processes IV and V), then the difference between these two processes may be determined, because of charge symmetry, by comparison of the processes $pn \rightarrow nn\pi^+$ and $pn \rightarrow pp\pi^-$ at the same pion emission angle.

B. If instead we exchange the neutron and proton in the final state (processes I and II or VI and VII), then it is necessary to observe simultaneously two particles, for example the pion and one of the nucleons, in order to detect the difference between the probabilities for the two processes. Here it becomes necessary to agree which nucleon will be referred to as the first one. We shall use the definition given by Fermi,[10] according to which the first nucleon is the one whose momentum, when projected along the direction of the pion momentum in the c.m.s. of the two colliding nucleons, is algebraically larger.

Since at this point we are interested only in relations among the amplitudes F (or A) and not in their spin and angular dependence, it is sufficient to observe the total cross sections for the appropriate processes.

If only the pion is detected in the experiment, then processes I and II, and also VI and VII, cannot be distinguished since only the sums of the probabilities of each pair of processes is measured. The corresponding total cross sections are given by

$$\sigma(pp \rightarrow \pi^+) = \int \frac{d\sigma}{d\Omega}(pp \rightarrow np\pi^+)\,d\Omega + \int \frac{d\sigma}{d\Omega}(pp \rightarrow pn\pi^+)\,d\Omega,$$
$$\sigma(np \rightarrow \pi^0) = \int \frac{d\sigma}{d\Omega}(np \rightarrow np\pi^0)\,d\Omega + \int \frac{d\sigma}{d\Omega}(np \rightarrow pn\pi^0)\,d\Omega. \qquad (2)$$

In order to distinguish between process I and II, and also VI and VII, it is necessary to measure the total cross section of each of the two processes separately: processes, when the first nucleon is a proton and when the first nucleon is a neutron.

In the general case each of the spin components of the wave functions describing the final state of the process $NN \rightarrow NN'\pi$ is a function of the pion momentum, the pion angle of emission $(\theta_\pi, \varphi_\pi)$, and also the nucleon angle of emission $(\theta_{12}, \varphi_{12})$. The angles θ_{12}, φ_{12} are here measured from the direction of the momentum of the pion in the barycentric frame of the two final state nucleons. When discussing the total cross sections it is sufficient to consider the differential cross sections averaged over the azimuth angles φ_π and φ_{12}. If we consider only contributions due to s and p waves of the pion and nucleons then the averaged cross sections may be written

$$d\sigma^{pp}_{np\pi^+}(\theta_\pi, \theta_{12}; p_\pi) \sim a_0(p_\pi) + a_1(p_\pi)\cos\theta_{12}\cos\theta_\pi + a_2(p_\pi)\cos^2\theta_{12} + a_3(p_\pi)\cos^2\theta_\pi + a_4(p_\pi)\cos^2\theta_{12}\cos^2\theta_\pi,$$

$$d\sigma^{pp}_{pn\pi^+}(\theta_\pi, \theta_{12}; p_\pi) \sim a_0(p_\pi) - a_1(p_\pi)\cos\theta_{12}\cos\theta_\pi + a_2(p_\pi)\cos^2\theta_{12} + a_3(p_\pi)\cos^2\theta_\pi + a_4(p_\pi)\cos^2\theta_{12}\cos^2\theta_\pi, \quad (3)$$

where $a_i(p_\pi)$ are some functions of the pion momentum.

It is seen from Eq. (3) that a measure of the difference between the probabilities for the process $pp \to np\pi^+$ and the process $pp \to pn\pi^+$ is provided by the coefficient a_1. However the total cross section for either of these processes is independent of a_1 since the difference of the differential cross sections changes sign under the transformations $\theta_\pi \to (\pi - \theta_\pi)$ and $\theta_{12} \to (\pi - \theta_{12})$. Hence it follows that the difference in the differential cross sections for these processes must be measured only for angles lying in the regions $0 < \theta_\pi < \pi/2$, $0 < \theta_{12} < \pi/2$; furthermore, one may average over relative azimuthal angles which enter into the cross section in the form[11]

$$\cos(\varphi_\pi - \varphi_{12}) \text{ and } \cos 2(\varphi_\pi - \varphi_{12}).$$

In this way the defined below difference of cross sections $\Delta\sigma$ corresponds to the difference of probabilities of the processes I and II:

$$\Delta\sigma_{10,11} = 4\int_0^{\pi/2}\int_0^{\pi/2}[d\sigma^{pp}_{np\pi^+}(\theta_\pi,\theta_{12}) - d\sigma^{pp}_{pn\pi^+}(\theta_\pi,\theta_{12})]\,d\Omega(\theta_\pi)\,d\Omega(\theta_{12}), \quad (4)$$

and, analogously, for processes VI and VII:

$$\Delta\sigma_{01,10} = 4\int_0^{\pi/2}\int_0^{\pi/2}[d\sigma^{np}_{np\pi^0}(\theta_\pi,\theta_{12}) - d\sigma^{np}_{pn\pi^0}(\theta_\pi,\theta_{12})]\,d\Omega(\theta_\pi)\,d\Omega(\theta_{12}). \quad (5)$$

The difference of cross sections for processes IV and V may be written in a simpler form

$$\Delta\sigma_{01,11} = 2\int_0^{\pi/2}[d\sigma^{pn}_{nn\pi^+}(\theta_\pi) - d\sigma^{np}_{nn\pi^+}(\theta_\pi)]\,d\Omega(\theta_\pi) = 2\int_0^{\pi/2}[d\sigma^{np}_{pp\pi^-}(\theta_\pi) - d\sigma^{np}_{nn\pi^+}(\theta_\pi)]\,d\Omega(\theta_\pi). \quad (6)$$

As in the previous two cases, the cross section (6) is a measure of the coefficient b in the angular distribution of the pions

$$d\sigma(np \to \pi^-) = a + b\cos\theta_\pi + c\cos^2\theta_\pi,$$

which, naturally, does not contribute to the usual total cross section.

3. RELATIONS AMONG AMPLITUDES WITH VARIOUS ISOTOPIC SPIN

Any six independent equations out of the following seven possible ones

$$\sigma(pp \to \pi^+) = |F_{10}|^2 + \tfrac{1}{2}|F_{11}|^2,$$
$$\sigma(np \to \pi^0) = \tfrac{1}{6}|F_{01}|^2 + \tfrac{1}{2}|F_{10}|^2,$$
$$\sigma(pp \sim \pi^0) = \tfrac{1}{2}|F_{11}|^2, \qquad \sigma(np \to \pi^+) = \tfrac{1}{3}|F_{01}|^2 + \tfrac{1}{2}|F_{11}|^2,$$
$$\Delta\sigma_{10,11} = \sqrt{2}\,\Omega_{10,11}, \quad \Delta\sigma_{01,11} = \sqrt{2/3}\,\Omega_{01,11},$$
$$\Delta\sigma_{01,10} = \sqrt{1/3}\,\Omega_{01,10}, \quad (7)$$

may be used to determine the three amplitudes F_{ij} or A_{ij} — i.e., the three absolute magnitudes and three relative phases (three-dimensional case!). Here

$$\Omega_{ij,lk} = |F_{ij}|\,|F_{kl}|\cos\Phi_{ij,kl}.$$

One of the equations relating the usual total cross sections is not independent in view of the existence of the following relation among total cross sections:

$$\sigma(pp \to \pi^+) + \sigma(np \to \pi^+) + \sigma(np \to \pi^-) = 2[\sigma(pp \to \pi^0) + \sigma(np \to \pi^0)].$$

The amplitudes F_{ij} are obtained from experimentally observable quantities with the help of the following relations

$$|F_{10}|^2 = \sigma(pp \to \pi^+) - \sigma(pp \to \pi^0),$$
$$\Omega_{10,11} = \sqrt{1/2}\,\Delta\sigma_{10,11},$$
$$|F_{11}|^2 = 2\sigma(pp \to \pi^0), \qquad \Omega_{11,01} = \sqrt{3/2}\,\Delta\sigma_{11,01},$$
$$|F_{01}|^2 = 3[\sigma(np \to \pi^+) + \sigma(np \to \pi^-) - \sigma(pp \to \pi^0)],$$
$$\Omega_{10,11} = \sqrt{3}\,\Delta\sigma_{10,01}. \quad (8)$$

Given the F_{ij}, the A_{ij} may be determined by making use of the following relations:

$$A_{13} = \sqrt{2/3}F_{10} + \sqrt{1/3}F_{11},$$
$$A_{13}^2 = \tfrac{2}{3}F_{10}^2 + \tfrac{1}{3}F_{11}^2 + \tfrac{2}{3}\sqrt{2}\,\Omega_{10,11}$$
$$A_{11} = -1/3F_{10} + \sqrt{2/3}F_{11},$$
$$A_{11}^2 = \tfrac{1}{3}F_{10}^2 + \tfrac{2}{3}F_{11}^2 - \tfrac{2}{3}\sqrt{2}\,\Omega_{10,11} \quad A_{01} = F_{01}, \quad A_{01}^2 = F_{01}^2;$$
$$\omega_{13} = \tfrac{1}{3}\sqrt{2}F_{11}^2 - \tfrac{1}{3}\sqrt{2}F_{10}^2 + \tfrac{1}{3}\Omega_{10,11},$$
$$\omega_{01} = -\sqrt{1/3}\,\Omega_{10,01} + \sqrt{2/3}\,\Omega_{11,01},$$
$$\omega_{03} = \sqrt{2/3}\,\Omega_{10,01} + \sqrt{1/3}\,\Omega_{11,01} \quad (9)$$

or, explicitly in terms of the observable quantities

$$|A_{13}|^2 = \tfrac{2}{3}\sigma(pp\to\pi^+) + \tfrac{2}{3}\Delta\sigma_{10,11},$$
$$|A_{11}|^2 = \sigma(pp\to\pi^0) + \tfrac{1}{3}\sigma(pp\to\pi^+) - \tfrac{2}{3}\Delta\sigma_{10,11},$$
$$|A_{01}|^2 = 3[\sigma(np\to\pi^+) + \sigma(np\to\pi^-) - \sigma(pp\to\pi^0)],$$
$$\omega_{13} = \tfrac{1}{3}\sqrt{2}[3\sigma(pp\to\pi^0) - \sigma(pp\to\pi^+)] + \tfrac{1}{3}\sqrt{1/2}\,\Delta\sigma_{10,11},$$
$$\omega_{01} = \Delta\sigma_{11,01} - \Delta\sigma_{10,01}, \quad \omega_{03} = \sqrt{2}\,\Delta\sigma_{10,01} + \sqrt{1/2}\,\Delta\sigma_{11,01}, \tag{10}$$

where $\omega_{ij} = |A_{1i}||A_{j1}|\cos\varphi_{ij}$.

4. EXPERIMENTS WITH A NEUTRON BEAM

From a study of the reactions $np \to pp\pi^-$ and $np \to nn\pi^+$ induced by 600 Mev neutrons,[7] one may conclude that $|F_{01}| \neq 0$. From the same data it follows that $\Delta\sigma_{01,11}$ is very nearly zero in this energy region. This means that the amplitudes F_{01} and F_{11} are nearly orthogonal to each other. With the neutron beam it is essential to measure the cross section $\Delta\sigma_{01,10}$. This would permit a determination of the relative phase, as well as produce a substantially more reliable value for $|F_{01}|$.

5. EXPERIMENTS WITH A PROTON BEAM

The cross sections $\sigma(pp\to\pi^+)$[12] and $\sigma(pp\to\pi^0)$[13] were measured using 600 Mev protons. From these data $|F_{10}|$ and $|F_{11}|$ may be determined. Of greatest interest are the values of $|A_{13}|$ and $|A_{11}|$ for 600 Mev protons, i.e., in the $(3/2, 3/2)$ resonance region where $|A_{13}|$ should dominate. However the relation between $|A_{13}|$ and $|A_{11}|$ can be found, if $\Delta\sigma_{10,01}$ is measured.

According to the resonant model of Mandelstam, $|A_{11}| = 0$, from which follows the well known relation $\sigma_+/\sigma_0 = 5$. Experimentally one observes the value $\sigma_+/\sigma_0 = 3.4$, and this is explained theoretically by introducing a somewhat artificial assumption. In fact, the explanation is simply that $|A_{11}| \neq 0$.

In order to discuss in more detail the possible values of $|A_{13}|$ and $|A_{11}|$ it is convenient to introduce the following notation: $\alpha = \sigma_+/\sigma_0$, $k = |A_{13}|/|A_{11}|$. In Fig. 1 is shown the region of permissible values of α and k, which is filled by the family of curves

$$\alpha = (4 + 5k^2 - \sqrt{8}\,k\cos\varphi_{13})/(2 + k^2 + \sqrt{8}\,k\cos\varphi_{13}) \tag{11}$$

with $\cos\varphi_{13}$ as a parameter. This region is bounded from below by the curve

$$\alpha_{\varphi_{13}=0} = (4 + 5k^2 - \sqrt{8}k)\,/\,(2 + k^2 + \sqrt{8}k), \tag{12}$$

which has the horizontal asymptotes $\alpha = 2$ $(k\to 0)$ and $\alpha = 5$ $(k\to\infty)$ and the minimum $\alpha = 1$ at $k = 1/\sqrt{2}$.

From above the region is bounded by the two branches of the curve

$$\alpha_{\varphi_{13}=\pi} = (4 + 5k^2 + \sqrt{8}\,k)\,/(2 + k^2 - \sqrt{8}\,k), \tag{13}$$

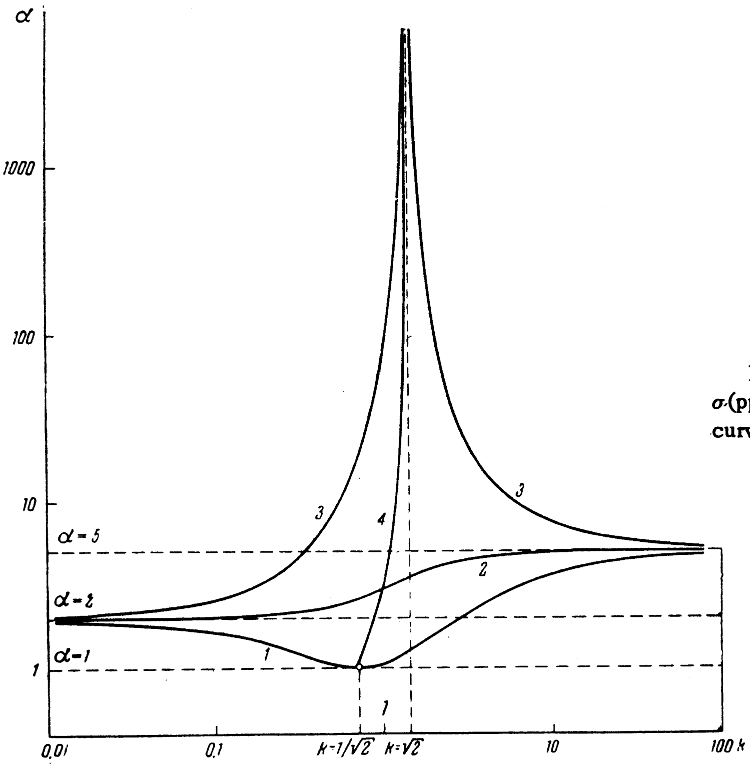

FIG. 1. Region of possible values of $\alpha = \sigma(pp\to\pi^+)/\sigma(pp\to\pi^0)$ and $k = |A_{13}|/|A_{11}|$. Curve 1 corresponds to $\varphi_{13} = 0$, curve 2 – $\varphi_{13} = \pi/2$, curve 3 – $\varphi_{13} = \pi$, curve 4 – $\Delta\sigma_{10,11} = 0$.

which has two horizontal asymptotes $\alpha = 2$ ($k \to 0$) and $\alpha = 5$ ($k \to \infty$) and one common vertical asymptote at $k = \sqrt{2}$.

The curves $\alpha = \alpha$ (k, $\cos \varphi_{13}$) have an extremum at

$$\alpha = (5k^4 + k^2 - 4) / (k^4 - k^2 - 2), \tag{14}$$

except for the monotonic curve

$$\alpha_{\pi/2} = 5 - 6 / (k^2 + 2),$$

which corresponds to $\varphi_{13} = \pi/2$.

It is seen from Fig. 1 that for $\alpha = \sigma_+/\sigma_0 = 3.4$ the possible values of k^2 lie in the interval

$$1/20 < k^2 < 64. \tag{15}$$

If $\sigma_+/\sigma_0 = \alpha = 5$ then the possible values of k^2 lie in the interval $1/8 < k^2 < \infty$. Therefore the equality $\sigma_+/\sigma_0 = 5$ can under no circumstances be taken as proof of validity of the resonant theory.

It should also be noted that if k is constant but the relative phase of A_{13} and A_{11} changes for some reason, then the quantity α will change. In the opinion of the authors such a process may take place in pion production on bound nucleons in a nucleus, for example in a deuteron.

In Fig. 1 we also show the curve corresponding to $\Delta\sigma_{10,11} = 0$ [$\alpha = 3k^2/(2-k^2)$], i.e., to the case of emission of the proton and the neutron symmetrically relative to the π^+ meson.

It is interesting to note that for α very large (~ 10 and larger), as is the case for energies near threshold, one has $|A_{13}| \sim 1.5\,|A_{11}|$ and the phase difference (up to π) is very small. Precisely such a picture would follow from the assumption that for pion energies close to zero the phases of A_{13} and A_{11} should be small, and consequently their difference must be small.

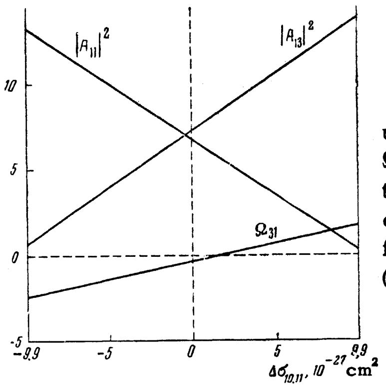

FIG. 2. Possible values of $|A_{13}|^2$, $|A_{11}|^2$ and $\Omega_{3,1}$ (in 10^{-27} cm^2) as functions of the difference of cross sections $\Delta\sigma_{10,11}$, for 660 Mev protons ($\alpha = 3.4$).

In Fig. 2 are shown the possible values of $|A_{13}|^2$, $|A_{11}|^2$ and $\Omega_{3,1}$ as a function of the difference of cross sections $\Delta\sigma_{10,11}$. It is seen from Fig. 2 that $|A_{13}|^2 \approx |A_{11}|^2$, and the amplitudes are nearly orthogonal, if $\Delta\sigma_{10,11} = 0$. For 660 Mev protons the maximum possible value of $\Delta\sigma_{10,11}$ is 9.9×10^{-27} cm^2.

CONCLUSION

Thus, in order to settle the question of the validity of the resonant theory of pion production by nucleons in the 600 Mev region it is necessary and sufficient to measure the difference of total cross sections $\Delta\sigma_{10,11}$, which is related to the asymmetry in the emission of protons and neutrons relative to the direction of flight of the π^+ meson.

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