ON THE THEORY OF LOW-TEMPERATURE, HIGH-FREQUENCY MAGNETIC SUSCEPTIBILITY OF A FERRODIELECTRIC

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Quantum field theory methods are used to evaluate the transverse components of the magnetic susceptibility tensor of a ferrodielectric, taking both the exchange and the relativistic interaction between spin waves into account. We find the ferromagnetic resonance line width. We show that a knowledge of the magnetic susceptibility enables us to study the relaxation of the magnetic moment of a ferrodielectric, and we evaluate the transverse relaxation time.

1. In ferromagnetic resonance theory the line width is usually introduced phenomenologically by adding to the equation of motion of the magnetic moment a relaxation term either in the Landau-Lifshitz or in the Bloch form. The aim of the present paper is to find the ferromagnetic resonance line shape starting from a microscopic theory of the interaction between spin waves.

It is well known that one can find the average lifetime of a spin wave and the relaxation time of the magnetic moment of a ferrodielectric by using a transport equation which determines the change with time of the spin wave distribution function. The transport equation method does, however, not enable us to determine completely the magnetic susceptibility tensor as a function of frequency and wave vector, since the transverse components of the magnetic moment can not be expressed in terms of the spin-wave occupation number.* To find the magnetic susceptibility we use therefore here field theoretical methods and we connect the magnetic susceptibility with the double-time spin-wave Green's function.

To evaluate the spin-wave Green's function we start from a Hamiltonian which takes both the exchange and the relativistic interaction between spin waves into account; the interactions between the spin-waves and the lattice vibrations will, however, not be taken into account. The method applied is particularly convenient to evaluate the transverse components of the magnetic susceptibility tensor, which are alone of importance for studying the ferromagnetic resonance line shape. Knowledge of the magnetic susceptibility as a function of the frequency and wave vector enables us to determine the behavior of the relaxation of the magnetic moment and, in particular, to track the rotation of the magnetic moment when it approaches its equilibrium value.

*Akhiezer[1] and Kaganov and Tsukernik[2] used the transport equation to evaluate the longitudinal component of the magnetic susceptibility tensor. Kaganov and Tsukernik[3] evaluated the imaginary part of the transverse magnetic susceptibility far from resonance, but their result is incorrect (private communication from the authors), owing to an incorrect application of perturbation theory.

2. We first connect the magnetic susceptibility of a ferrodielectric with the spin-wave Green's function.* The average value of the magnetic moment density is well known to be defined by the formula

$$\mathfrak{M}(\mathbf{r},t) = \operatorname{Sp} \mathbf{M}(\mathbf{r})\,\rho(t), \tag{1}$$

where $\mathbf{M}(\mathbf{r})$ is the magnetic moment density operator in the Schroedinger representation and $\rho(t)$ the system density matrix, which satisfies the equation

$$i\frac{\partial}{\partial t}\rho = [\mathscr{H} + \mathscr{H}^e, \rho], \tag{2}$$

where $\mathscr{H}$ is the total Hamiltonian of the ferrodielectric when there is no external field and $H^e(t)$ the Hamiltonian of the interaction between the ferrodielectric and the variable external magnetic field $\mathbf{h}(\mathbf{r}, t)$:

$$\mathscr{H}^e(t) = -\int \mathbf{M}(\mathbf{r})\,\mathbf{h}(\mathbf{r}, t)\,d\mathbf{r}. \tag{3}$$

To find the magnetic moment in weak magnetic fields we change in Eqs. (1) — (3) to the Heisenberg representation. In the first approximation in the external field, the density matrix in the Heisenberg representation is of the form

*Kubo[4] developed a general theory of irreversible processes using the density matrix.

$$\hat{\rho}(t) = \hat{\rho}_0 - i\int_{-\infty}^{t} [\hat{\mathscr{H}}^e(t'), \hat{\rho}_0]\,dt', \qquad (4)$$

where $\hat{\rho}_0 = \exp(\Omega - \hat{\mathscr{H}})/T$ is the equilibrium density matrix when there is no external field, Ω the thermodynamic potential of the system, and T the temperature. Using (4) one obtains easily the following expression for the magnetic moment density

$$m_i(\mathbf{r}, t) \equiv \mathfrak{M}_i(\mathbf{r}, t) - \mathfrak{M}_i^0 = \int_{-\infty}^{\infty} dt'\,d\mathbf{r}' K_{il}^R(\mathbf{r}-\mathbf{r}', t-t')\,h_l(\mathbf{r}', t'), \qquad (5)$$

where $\mathfrak{M}_i^0$ is the equilibrium value of the magnetic moment at the given temperature and $K_{il}^R(\mathbf{r}, t)$ is the retarded double-time Green's function

$$K_{il}^R(\mathbf{r}-\mathbf{r}', t-t') = i\theta(t-t')\langle[\hat{M}_i(\mathbf{r}, t), \hat{M}_l(\mathbf{r}', t')]\rangle,$$

$$\theta(t) = \begin{cases} 0, & t<0 \\ 1, & t>0 \end{cases}, \qquad \langle f\rangle = \mathrm{Sp}\,\rho_0 f. \qquad (6)$$

Expanding the quantities $m_i(\mathbf{r}, t)$, $h_i(\mathbf{r}, t)$ and $K_{il}^R(\mathbf{r}, t)$ in terms of the Fourier integral,

$$K_{il}^R(\mathbf{r}, t) = \frac{1}{(2\pi)^4}\int_{-\infty}^{\infty} e^{-i\omega t + i\mathbf{k}\mathbf{r}} K_{il}^R(\mathbf{k}, \omega)\,d\mathbf{k}\,d\omega,$$

we can rewrite Eq. (5) as

$$m_i(\mathbf{k}, \omega) = K_{il}^R(\mathbf{k}, \omega)\,h_l(\mathbf{k}, \omega). \qquad (7)$$

The magnetic susceptibility tensor $\chi_{il}(\mathbf{k}, \omega)$ is thus the same as the retarded double-time Green function:

$$\chi_{il}(\mathbf{k}, \omega) = K_{il}^R(\mathbf{k}, \omega) \qquad (8)$$

3. We shall show that a knowledge of the magnetic susceptibility tensor enables us to study the relaxation of the magnetic moment. Let there be initially at time $t = 0$ some distribution of the magnetic moment $\mathbf{m}^0(\mathbf{r})$ in the system which would be an equilibrium distribution if there were present a constant magnetic field $\mathbf{h}^0(\mathbf{r})$ which is connected with the initial magnetic moment distribution through the relation

$$m_i^0(\mathbf{k}) = \chi_{il}(\mathbf{k}, \omega)\,h_l^0(\mathbf{k})|_{\omega=0}.$$

The magnitude of the magnetic moment at $t > 0$ is determined by Eq. (5), where we must take for the variable magnetic field

$$\mathbf{h}(\mathbf{r}, t) = \theta(-t)\,e^{\varepsilon t}\,\mathbf{h}^0(\mathbf{r}), \qquad \varepsilon \to +0. \qquad (9)$$

Such a choice for $\mathbf{h}(\mathbf{r}, t)$ corresponds to an adiabatic switching on of the field at $t = -\infty$ and an instantaneous switching off at $t = 0$. Using Eqs. (5), (6), and (9), we get

$$m_i(\mathbf{r}, t) = i\int_{-\infty}^{0} dt'\,d\mathbf{r}' e^{\varepsilon t'}\langle[\hat{M}_i(\mathbf{r}, t), \hat{M}_l(\mathbf{r}', t')]\rangle\,h_l^0(\mathbf{r}'). \qquad (10)$$

Noting that according to (6)

$$i\langle[\hat{M}_i(\mathbf{r}, t), \hat{M}_l(\mathbf{r}', t')]\rangle = K_{il}^R(\mathbf{r}-\mathbf{r}', t-t') - K_{li}^{R*}(\mathbf{r}'-\mathbf{r}, t'-t),$$

and using Eq. (8) for the Fourier component of the function K_{il}^R we get the following equation for the relaxation of the magnetic moment

$$m_i(\mathbf{r}, t) = -\frac{i}{(2\pi)^4}\int d\mathbf{k}\,e^{i\mathbf{k}\mathbf{r}}\,h_l^0(\mathbf{k})\int_{-\infty}^{\infty} d\omega\,\frac{e^{-i\omega t}}{\omega}[\chi_{il}(\mathbf{k}, \omega) - \chi_{li}^*(\mathbf{k}, \omega)]. \qquad (11)$$

We see that the relaxation of the magnetic moment is determined by the anti-Hermitian part of the magnetic susceptibility tensor χ_{il}. It is well known[5] that the anti-Hermitian part of the χ_{il} tensor determines also the absorption of energy from a variable magnetic field. One can check this by differentiating the energy of the system $Q(t)$ with respect to the time

$$\dot{Q}(t) = \frac{d}{dt}\mathrm{Sp}\{\hat{\rho}(t)[\hat{\mathscr{H}} + \hat{\mathscr{H}}^e]\} = -\mathrm{Sp}\{\hat{\rho}(t)\int \hat{M}_i(\mathbf{r}, t)\frac{\partial}{\partial t}h_i(\mathbf{r}, t)\,d\mathbf{r}\}.$$

Using Eq. (4) for $\hat{\rho}(t)$ we find

$$\dot{Q}(t) = -\mathrm{Sp}\left\{\rho_0\int \hat{M}_i(\mathbf{r}, t)\frac{\partial}{\partial t}h_i(\mathbf{r}, t)\,d\mathbf{r}\right\} - \int_{-\infty}^{\infty} dt'\,d\mathbf{r}'\,d\mathbf{r}\,h_i(\mathbf{r}', t')\frac{\partial h_l(\mathbf{r}, t)}{\partial t}K_{li}^R(\mathbf{r}-\mathbf{r}', t-t'). \qquad (12)$$

If in the case of a magnetic field which is periodic in time

$$h_i(\mathbf{r}, t) = \frac{1}{(2\pi)^3}\sum_{\omega_n}\int d\mathbf{k}\,h_i(\mathbf{k}, \omega_n)\,e^{i\mathbf{k}\mathbf{r} - i\omega_n t}$$

$$\left(\omega_n = \frac{2\pi n}{t_0},\ n = 0, \pm 1, \ldots\right)$$

we average Eq. (12) over the period t_0, we get the final expression for the average absorption of energy from a variable magnetic field per unit time

$$\bar{\dot{Q}} = -i(2\pi)^{-3}\int d\mathbf{k}\sum_{\omega_n>0}\omega_n h_l(\mathbf{k}, \omega_n)\,[\chi_{li}(\mathbf{k}, \omega_n)] - \chi_{il}^*(\mathbf{k}, \omega_n)]\,h_i^*(\mathbf{k}, \omega_n). \qquad (13)$$

4. We shall now connect the retarded double-time Green's function $K_{il}^R(\mathbf{r}, t)$ with the Matsubara Green's function $\mathscr{K}_{il}(\mathbf{r}, \tau)$, for the evaluation of which we shall apply a diagram technique. The function $\mathscr{K}_{il}(\mathbf{r}, \tau)$ is defined by the equation

$$\mathscr{K}_{il}(\mathbf{r}-\mathbf{r}', \tau-\tau') = \langle T_\tau\{\hat{M}_i(\mathbf{r}, \tau)\,\hat{M}_l(\mathbf{r}', \tau')\}\rangle,$$

$$\hat{M}_i(\mathbf{r}, \tau) = e^{\mathscr{H}\tau}M_i(\mathbf{r})\,e^{-\mathscr{H}\tau}, \qquad (14)$$

where T_τ is the chronological operator for the variable τ.

We expand the function $\mathscr{K}_{il}(\mathbf{r}, \tau)$ in a Fourier series in τ

$$\mathscr{K}_{il}(\mathbf{r}, \tau) = \frac{T}{(2\pi)^3} \sum_{k_4} \int d\mathbf{k} e^{i\mathbf{kr} - ik_4\tau} \mathscr{K}_{il}(\mathbf{k}, k_4)$$

$$(k_4 = 2\pi nT;\ n = 0, \pm 1, \ldots).$$

Using furthermore a method similar to the one applied by Abrikosov, Gor'kov, and Dzyaloshinskii[6] and Fradkin[7] we get, using (8),

$$\chi_{il}(\mathbf{k}, \omega) = K_{il}^R(\mathbf{k}, \omega) = \mathscr{K}_{il}(\mathbf{k}, -i\omega + 0). \tag{15}$$

Writing (14) in the k-representation we find

$$\mathscr{K}_{il}(\mathbf{k}, k_4) = -\frac{1}{2} \int_{-1/T}^{1/T} d\tau e^{ik_4\tau} \sum_{\mathbf{k}'} \langle T_\tau \{\hat{M}_i(\mathbf{k}, \tau) \hat{M}_l(\mathbf{k}', 0)\}\rangle, \tag{16}$$

where $M_i(\mathbf{k}, \tau)$ are the Fourier components of the magnetic moment density operator

$$\hat{M}_i(\mathbf{r}, \tau) = \frac{1}{\sqrt{V}} \sum_{\mathbf{k}} \hat{M}_i(\mathbf{k}, \tau) e^{i\mathbf{kr}}$$

(V is the volume of the system).

5. We turn to an evaluation of the magnetic susceptibility of a ferrodielectric. Following Holstein and Primakoff[8] and introducing spin-wave creation and annihilation operators $c_{\mathbf{k}}^+$ and $c_{\mathbf{k}}$ we write the Hamiltonian of the ferrodielectric in the form*

$$\mathscr{H} = \mathscr{H}_0 + \mathscr{H}_3 + \mathscr{H}_4, \tag{17}$$

where $\mathscr{H}_0$ is the basic Hamiltonian

$$\mathscr{H}_0 = \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} c_{\mathbf{k}}^+ c_{\mathbf{k}} \tag{18}$$

($\epsilon_{\mathbf{k}}$ is the spin wave energy) and $\mathscr{H}_3$ and $\mathscr{H}_4$ are the Hamiltonians of the spin wave — spin wave interaction†

$$\mathscr{H}_3 = \frac{1}{\sqrt{V}} \sum_{1,2,3} \{\Phi(1, 2; 3) c_1^+ c_2^+ c_3 \Delta(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}_3) + \text{compl. conj.} + \Phi_1(1, 2, 3) c_1^+ c_2^+ c_3^+ \Delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) + \text{compl. conj.}\},$$

$$\mathscr{H}_4 = \frac{1}{V} \sum_{1,2,3,4} \{\Psi(1, 2; 3, 4) c_1^+ c_2^+ c_3 c_4 \Delta(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}_3 - \mathbf{k}_4) + \Psi_1(1, 2, 3; 4) c_1^+ c_2^+ c_3^+ c_4 \Delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 - \mathbf{k}_4) + \text{compl. conj.} + \Psi_2(1, 2, 3, 4) c_1^+ c_2^+ c_3^+ c_4^+ \Delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 + \mathbf{k}_4) + \text{compl. conj.}\}. \tag{19}$$

The spin wave energy is $\epsilon_{\mathbf{k}} = \sqrt{A_{\mathbf{k}}^2 - |B_{\mathbf{k}}|^2}$, where

$$A_{\mathbf{k}} = \Theta_c (ak)^2 + \mu(H_0 + \beta M_0) + 2\pi\mu M_0 \sin^2\theta_{\mathbf{k}},$$

$$B_{\mathbf{k}} = 2\pi\mu M_0 \sin^2\Theta_{\mathbf{k}} e^{2i\varphi_{\mathbf{k}}}.$$

Here μ is twice the Bohr magneton, a the lattice constant, Θ_C a quantity of the order of the Curie temperature, M_0 the saturation magnetic moment, β the anisotropy constant, H_0 the constant external magnetic field, and $\theta_{\mathbf{k}}$ and $\varphi_{\mathbf{k}}$ the polar angles of the vector k.

The quantities Φ and Ψ_1 describe the splitting of one spin wave into two or three, respectively, Φ_1 and Ψ_2 describe the creation of three or four spin waves, respectively, and Ψ describes the spin wave — spin wave scattering process. The quantities Φ and Φ_1 are relativistic in origin while the quantities Ψ, Ψ_1, and Ψ_2 are caused by exchange and relativistic interactions.

For actual calculations we restrict ourselves to the range of large wave vectors ($1 \gg ak \gg \sqrt{\mu M_0/\Theta_C}$), when the exchange interaction is the main agency, and to the region of small wave vectors ($ak \ll \sqrt{\mu M_0/\Theta_C}$), where the relativistic interaction plays the main part. In these regions the quantities Φ, Φ_1, Ψ, Ψ_1, and Ψ_2 are determined by the following asymptotic formulae

$$\left.\begin{array}{l}\Psi(1, 2; 3, 4) = -\mu(4M_0)^{-1}\Theta_c a^2(\mathbf{k}_1\mathbf{k}_2 + \mathbf{k}_3\mathbf{k}_4) \\ |\Psi_2| \ll |\Psi_1| \ll |\Psi|\end{array}\right\} \text{when } 1 \gg ak \gg \sqrt{\frac{\mu M_0}{\Theta_c}}, \tag{20}$$

$$\Phi(1, 2; 3) = -\pi\mu\sqrt{2\mu M_0}\{\sin 2\theta_1 (e^{-i\varphi_1} \overset{*}{u}_1 + e^{i\varphi_1} \overset{*}{v}_1)(\overset{*}{u}_2 u_3 + \overset{*}{v}_2 v_3) + \sin 2\theta_2 (e^{-i\varphi_2} \overset{*}{u}_2 + e^{i\varphi_2} \overset{*}{v}_2)(\overset{*}{u}_1 u_3 + \overset{*}{v}_1 v_3) + \sin 2\theta_3 (e^{i\varphi_3} u_3 + e^{-i\varphi_3} v_3)(\overset{*}{v}_1 \overset{*}{u}_2 + \overset{*}{v}_2 \overset{*}{u}_1)\}, \tag{21}$$

$$\left.\begin{array}{l}|\Phi_1| \sim |\Phi|; \quad |\Psi_1| \sim |\Psi_2| \sim |\Psi| \\ \Psi(1, 2; 3, 4) = -\tfrac{1}{2}\mu^2\beta\{\overset{*}{u}_1\overset{*}{u}_2 u_3 u_4 + 4\overset{*}{u}_1\overset{*}{v}_2 v_3 u_4 + \overset{*}{v}_1\overset{*}{v}_2 v_3 v_4\}\end{array}\right\}$$

$$\text{when } ak \ll \sqrt{\frac{\mu M_0}{\Theta_c}}, \tag{22}$$

where*

$$u_{\mathbf{k}} = \sqrt{(A_{\mathbf{k}} + \varepsilon_{\mathbf{k}})/2\varepsilon_{\mathbf{k}}},$$

$$v_{\mathbf{k}} = -e^{2i\varphi_{\mathbf{k}}}\sqrt{(A_{\mathbf{k}} - \varepsilon_{\mathbf{k}})/2\varepsilon_{\mathbf{k}}}.$$

In the present paper we shall be interested in the transverse components of the magnetic susceptibility tensor; to evaluate these it is sufficient to know the transverse components of the magnetic moment. Restricting ourselves to the first terms in the expansion of the moment operators in the spin wave creation and annihilation operators, we have

*The Holstein-Primakoff model presupposes that the Hamiltonian and the moment operators are expanded in the parameter ½s. We note that Dyson[9] has shown that in fact the expansion in the exchange interaction is in a parameter ~(1/10)s (s is the spin of the atom).

†See, for instance, reference 10.

*To eliminate the indeterminacy in the quantities $u_{\mathbf{k}}$ and $v_{\mathbf{k}}$ as $\mathbf{k} \to 0$, it is necessary to take the shape of the sample into account.[11]

$$\hat{M}^+(\mathbf{k},\tau)=\hat{M}_x(\mathbf{k},\tau)+i\hat{M}_y(\mathbf{k},\tau)$$
$$=\sqrt{2\mu M_0}\{u_\mathbf{k}^*\hat{c}_{-\mathbf{k}}^+(\tau)+v_\mathbf{k}\hat{c}_\mathbf{k}(\tau)\},$$
$$\hat{M}^-(\mathbf{k},\tau)=\hat{M}_x(\mathbf{k},\tau)-i\hat{M}_y(\mathbf{k},\tau)$$
$$=\sqrt{2\mu M_0}\{u_\mathbf{k}\hat{c}_\mathbf{k}(\tau)+v_\mathbf{k}^*\hat{c}_{-\mathbf{k}}^+(\tau)\};\qquad(23)$$

where

$$\hat{c}_\mathbf{k}(\tau)=e^{\mathcal{H}\tau}c_\mathbf{k}e^{-\mathcal{H}\tau},\qquad \hat{c}_\mathbf{k}^+(\tau)=e^{\mathcal{H}\tau}c_\mathbf{k}^+e^{-\mathcal{H}\tau}.$$

We introduce the single-particle spin-wave Green's function:

$$G(\mathbf{k},\tau)\Delta(\mathbf{k}-\mathbf{k}')=\langle T_\tau\{\hat{c}_\mathbf{k}(\tau)\hat{c}_{\mathbf{k}'}^+(0)\}\rangle.\qquad(24)$$

Its Fourier component $G(\mathbf{k},k_4)$ satisfies the Dyson equation

$$G(\mathbf{k},k_4)=G_0(\mathbf{k},k_4)+G_0(\mathbf{k},k_4)\Sigma(\mathbf{k},k_4)G(\mathbf{k},k_4).\qquad(25)$$

Here

$$G_0(\mathbf{k},k_4)=(\varepsilon_\mathbf{k}-ik_4)^{-1}\qquad(26)$$

is the zeroth-approximation spin-wave Green's function and $\Sigma(\mathbf{k},k_4)$ is the mass operator. Substituting (23) into (16) and taking into account that $\langle T_\tau\{\hat{c}_\mathbf{k}^+(\tau)\hat{c}_{\mathbf{k}'}^+(0)\}\rangle$ and $\langle T_\tau\{\hat{c}_\mathbf{k}(\tau)\hat{c}_{\mathbf{k}'}(0)\}\rangle$ are equal to zero in the zeroth approximation in the interaction, we express the function $\mathcal{K}_{il}(\mathbf{k},k_4)$ in terms of the single-particle Green's function $G(\mathbf{k},k_4)$

$$\mathcal{K}_{++}(\mathbf{k},k_4)=2\mu M_0u_\mathbf{k}^*v_\mathbf{k}\{G(\mathbf{k},k_4)+G(-\mathbf{k},-k_4)\},$$
$$\mathcal{K}_{+-}(\mathbf{k},k_4)=2\mu M_0\{|v_\mathbf{k}|^2G(\mathbf{k},k_4)+|u_\mathbf{k}|^2G(-\mathbf{k},-k_4)\},$$
$$\mathcal{K}_{-+}(\mathbf{k},k_4)=2\mu M_0\{|u_\mathbf{k}|^2G(\mathbf{k},k_4)+|v_\mathbf{k}|^2G(-\mathbf{k},-k_4)\},$$
$$\mathcal{K}_{--}(\mathbf{k},k_4)=2\mu M_0u_\mathbf{k}v_\mathbf{k}^*\{G(\mathbf{k},k_4)+G(-\mathbf{k},-k_4)\}.\qquad(27)$$

It follows from (15), (27), and (26) that the transverse components of the magnetic susceptibility tensor tend to infinity in the zeroth approximation of the spin wave — spin wave interaction, if the frequency of the magnetic field and its wave vector are the same as the spin-wave frequency and wave vector:

$$\omega_\mathbf{k}=\pm\varepsilon_\mathbf{k}.\qquad(28)$$

6. We use Eq. (25) to evaluate the complete Green function $G(\mathbf{k},k_4)$:

$$G(\mathbf{k},k_4)=[\varepsilon_\mathbf{k}-ik_4-\Sigma(\mathbf{k},k_4)]^{-1}.\qquad(29)$$

The magnetic susceptibility near resonance is determined by the quantity $G(\mathbf{k},-i\epsilon_\mathbf{k}+0)$; we shall not be interested in the small correction to the position of the resonance line, which is caused by the real part of the function Σ. Introducing therefore a quantity $\gamma(\mathbf{k})$:

$$\gamma(\mathbf{k})=\operatorname{Im}\Sigma(\mathbf{k},-i\varepsilon_\mathbf{k}+0),\qquad(30)$$

we get from (15), (27), and (29) for the magnetic susceptibility near resonance

$$\chi_{xx}(\mathbf{k},\omega)=\tfrac{1}{2}\mu M_0U_1(\mathbf{k})\{[\varepsilon_\mathbf{k}-\omega-i\gamma(\mathbf{k})]^{-1}+[\varepsilon_\mathbf{k}+\omega+i\gamma(\mathbf{k})]^{-1}\},$$
$$\chi_{yy}(\mathbf{k},\omega)=\tfrac{1}{2}\mu M_0U_2(\mathbf{k})\{[\varepsilon_\mathbf{k}-\omega-i\gamma(\mathbf{k})]^{-1}+[\varepsilon_\mathbf{k}+\omega+i\gamma(\mathbf{k})]^{-1}\},$$
$$\chi_{xy}(\mathbf{k},\omega)=-\tfrac{1}{2}i\mu M_0\{U(\mathbf{k})[\varepsilon_\mathbf{k}-\omega-i\gamma(\mathbf{k})]^{-1}-U^*(\mathbf{k})[\varepsilon_\mathbf{k}+\omega+i\gamma(\mathbf{k})]^{-1}\},$$
$$\chi_{yx}(\mathbf{k},\omega)=\tfrac{1}{2}i\mu M_0\{U^*(\mathbf{k})[\varepsilon_\mathbf{k}-\omega-i\gamma(\mathbf{k})]^{-1}-U(\mathbf{k})[\varepsilon_\mathbf{k}+\omega+i\gamma(\mathbf{k})]^{-1}\};\qquad(31)$$

$$U_1(\mathbf{k})=|u_\mathbf{k}|^2+|v_\mathbf{k}|^2+u_\mathbf{k}^*v_\mathbf{k}+u_\mathbf{k}v_\mathbf{k}^*,$$
$$U_2(\mathbf{k})=|u_\mathbf{k}|^2+|v_\mathbf{k}|^2-u_\mathbf{k}^*v_\mathbf{k}-u_\mathbf{k}v_\mathbf{k}^*,$$
$$U(\mathbf{k})=1+u_\mathbf{k}^*v_\mathbf{k}-u_\mathbf{k}v_\mathbf{k}^*.\qquad(32)$$

We consider now the relaxation of the transverse part of the magnetic moment. Substituting (31) into (11) and integrating over ω, we get

$$m_x(\mathbf{r},t)=\frac{\mu M_0}{(2\pi)^3}\int\frac{d\mathbf{k}}{\varepsilon_\mathbf{k}}e^{i\mathbf{kr}-\gamma(\mathbf{k})t}\{h_x^0(\mathbf{k})U_1(\mathbf{k})\cos\varepsilon_\mathbf{k}t+h_y^0(\mathbf{k})\operatorname{Im}[U(\mathbf{k})e^{-i\varepsilon_\mathbf{k}t}]\},$$
$$m_y(\mathbf{r},t)=\frac{\mu M_0}{(2\pi)^3}\int\frac{d\mathbf{k}}{\varepsilon_\mathbf{k}}e^{i\mathbf{kr}-\gamma(\mathbf{k})t}\{h_x^0(\mathbf{k})\operatorname{Im}[U(\mathbf{k})e^{i\varepsilon_\mathbf{k}t}]+h_y^0(\mathbf{k})U_2(\mathbf{k})\cos\varepsilon_\mathbf{k}t\}.\qquad(33)$$

We see that the quantity $\gamma(\mathbf{k})$ determines the relaxation time of the transverse components of the magnetic moment. That the quantity $\gamma(\mathbf{k})$ is positive follows from the Lehmann representation of the Green's function

$$G(\mathbf{k},-i\omega+0)=\int_{-\infty}^{\infty}d\omega'\rho(\mathbf{k},\omega')/(\omega'+\omega+i0),\qquad(34)$$

where

$$\rho(\mathbf{k},\omega)=\sum_{n,n'}[\exp\{(\Omega-E_n)/T\}-\exp\{(\Omega-E_{n'})/T\}]\langle n|c_\mathbf{k}|n'\rangle\langle n'|c_\mathbf{k}^+|n\rangle\delta(\omega+E_n-E_{n'})$$

(E_n are the energy levels of the system). One sees easily that the quantity $\rho(\mathbf{k},\omega)$ is real, an odd function of ω, and positive for $\omega>0$. From (34) it follows that the imaginary part of the Green's function is of the form

$$\operatorname{Im}G(\mathbf{k},-i\omega+0)=-\pi\rho(\mathbf{k},-\omega).$$

From Eq. (29) it follows that the sign of the imaginary part of $\Sigma(\mathbf{k},-i\omega+0)$ is the same as the sign of the imaginary part of $G(\mathbf{k},-i\omega+0)$; the quantity $\gamma(\mathbf{k})$, which is the same as $\operatorname{Im}\Sigma(\mathbf{k},-i\omega+0)$, is thus positive for $\omega=\epsilon_\mathbf{k}>0$.

7. We turn now to finding the resonance line width $\gamma(\mathbf{k})$, which is connected with the mass operator $\Sigma(\mathbf{k},k_4)$ through Eq. (30). We use perturbation theory and diagram methods to evaluate the quantity $\Sigma(\mathbf{k},k_4)$. We show in Figs. 1 — 5 the diagrams of the first and second approximation for the quantity Σ. The solid lines in the

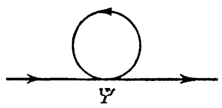

FIG. 1

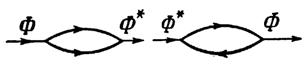

FIG. 2

diagrams correspond to the function G_0, and the vertices to the functions Φ, Φ_1, Ψ, Ψ_1, Ψ_2, and a δ-function expressing the conservation of **k** and k_4. One integrates over the momentum **k** of internal lines and sums over k_4. The general coefficient in front of the analytical expression corresponding to an arbitrary diagram is equal to $(-1)^{n+1}qT^p(2\pi)^{-3p}$, where q is the number of equivalent diagrams, n the number of vertices, and p the number of independent momenta of virtual spin waves.

In the first perturbation-theory approximation (see Fig. 1) the mass operator is equal to

$$\Sigma_1(\mathbf{k}) = -4(2\pi)^{-3}\int d\mathbf{k}'\,\Psi(\mathbf{k},\mathbf{k}';\mathbf{k},\mathbf{k}')\,n_{\mathbf{k}'}, \qquad (35)$$

where $n_{\mathbf{k}} = [\exp(\epsilon_{\mathbf{k}}/T) - 1]^{-1}$ is the Bose distribution function of the spin waves. The quantity $-\Sigma_1(\mathbf{k})$ is real, independent of k_4, and is a small correction to the spin wave energy.

In Figs. 2 and 3 we give the second-order diagrams for the mass operator which are caused by the term $\mathcal{H}_3$ in the interaction Hamiltonian. The analytical expression for the diagrams of Fig. 2 is of the form

$$\Sigma_\Phi(\mathbf{k}, k_4) = 2T(2\pi)^{-3}\sum_{k'_4}\int d\mathbf{k}'\,|\Phi(\mathbf{k}', \mathbf{k} - \mathbf{k}'; \mathbf{k})|^2 G_0(\mathbf{k}', k'_4)G_0(\mathbf{k}-\mathbf{k}', k_4 - k'_4)$$
$$+ 4T(2\pi)^{-3}\sum_{k'_4}\int d\mathbf{k}'\,|\Phi(\mathbf{k}, \mathbf{k}'; \mathbf{k} + \mathbf{k}')|^2 G_0(\mathbf{k}', k'_4)\,G_0(\mathbf{k}+\mathbf{k}', k_4 + k'_4). \qquad (36)$$

Summing over k'_4 and performing the substitution $k_4 = -i\omega + 0$ we get

$$\Sigma_\Phi(\mathbf{k}, -i\omega + 0) = 2(2\pi)^{-3}\int d\mathbf{k}'\,|\Phi(\mathbf{k}', \mathbf{k} - \mathbf{k}'; \mathbf{k})|^2\,(n_{\mathbf{k}'} + n_{\mathbf{k}-\mathbf{k}'} + 1)(\varepsilon_{\mathbf{k}'} + \varepsilon_{\mathbf{k}-\mathbf{k}'} - \omega - i0)^{-1}$$
$$+ 4(2\pi)^{-3}\int d\mathbf{k}'\,|\Phi(\mathbf{k}, \mathbf{k}'; \mathbf{k} + \mathbf{k}')|^2(n_{\mathbf{k}'} - n_{\mathbf{k}+\mathbf{k}'}) \times (\varepsilon_{\mathbf{k}+\mathbf{k}'} - \varepsilon_{\mathbf{k}'} - \omega - i0)^{-1}.$$

The imaginary part of this expression which we need to evaluate the ferromagnetic resonance line width is equal to

FIG. 4

$$\operatorname{Im}\Sigma_\Phi(\mathbf{k}, -i\omega + 0) = (2\pi)^{-2}\int d\mathbf{k}'\,|\Phi(\mathbf{k}', \mathbf{k} - \mathbf{k}'; \mathbf{k})|^2[(n_{\mathbf{k}'} + 1)(n_{\mathbf{k}-\mathbf{k}'} + 1) - n_{\mathbf{k}'}n_{\mathbf{k}-\mathbf{k}'}]\,\delta(\omega - \varepsilon_{\mathbf{k}'} - \varepsilon_{\mathbf{k}-\mathbf{k}'})$$
$$+ 2(2\pi)^{-2}\int d\mathbf{k}'\,|\Phi(\mathbf{k}, \mathbf{k}'; \mathbf{k}+\mathbf{k}')|^2[n_{\mathbf{k}'}(n_{\mathbf{k}+\mathbf{k}'} + 1) - (n_{\mathbf{k}'} + 1)\,n_{\mathbf{k}+\mathbf{k}'}]\,\delta(\omega + \varepsilon_{\mathbf{k}'} - \varepsilon_{\mathbf{k}+\mathbf{k}'}). \qquad (37)$$

The terms of the mass operator which correspond to the diagrams of Fig. 3 are determined by the expressions

$$\Sigma_a(\mathbf{k}) = \frac{4}{(2\pi)^3}\left(\exp\frac{\varepsilon_0}{T} - 1\right)^{-1}\Phi(\mathbf{k}, 0; \mathbf{k})\int d\mathbf{k}'\,\Phi^*(\mathbf{k}', 0; \mathbf{k}')\,n_{\mathbf{k}'} + \text{compl. conj.},$$
$$\Sigma_b(\mathbf{k}, k_4) = \frac{18T}{(2\pi)^3}\sum_{k'_4}\int d\mathbf{k}'\,|\Phi_1(\mathbf{k}, \mathbf{k}', -\mathbf{k} - \mathbf{k}')|^2 \times G_0(\mathbf{k}', k'_4)G_0(-\mathbf{k}-\mathbf{k}', -k_4 - k'_4). \qquad (38)$$

By integrating the first of Eqs. (38) over the angles of the vector **k′** we verify easily that the quantity $\Sigma_a(\mathbf{k})$ vanishes. Summing the second of Eqs. (38) over k'_4 and separating off the imaginary part of the quantity $\Sigma_b(\mathbf{k}, -i\omega + 0)$ we get

$$\operatorname{Im}\Sigma_b(\mathbf{k}, -i\omega + 0) = -\frac{9}{(2\pi)^2}\int d\mathbf{k}'\,|\Phi_1(\mathbf{k}, \mathbf{k}', -\mathbf{k} - \mathbf{k}')|^2[(n_{\mathbf{k}'} + 1)(n_{\mathbf{k}+\mathbf{k}'} + 1) - n_{\mathbf{k}'}n_{\mathbf{k}+\mathbf{k}'}]\,\delta(\omega + \varepsilon_{\mathbf{k}'} + \varepsilon_{\mathbf{k}+\mathbf{k}'}).$$

One sees easily that near resonance ($\omega = \epsilon_{\mathbf{k}}$) this quantity tends to zero and thus does not contribute to the line width $\gamma(\mathbf{k})$.

We show in Figs. 4 and 5 the second-order diagrams for the mass operator which are caused by the term H_4 in the interaction Hamiltonian. We are led to analytical expressions only for those diagrams the imaginary part of which differs from zero for $k_4 = -i\omega + 0$ near the resonance point $\omega = \epsilon_{\mathbf{k}}$ (Fig. 4)

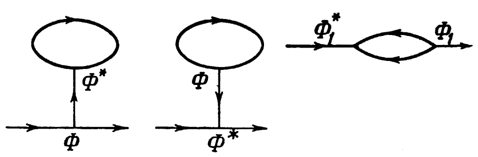

FIG. 3

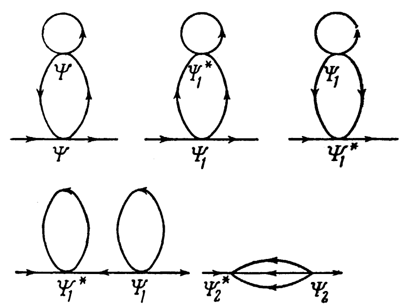

FIG. 5

$$\Sigma_\Psi(\mathbf{k}, k_4) = 8T^2(2\pi)^{-6} \sum_{k_4', k_4''} \int d\mathbf{k}' d\mathbf{k}'' |\Psi(\mathbf{k}, \mathbf{k}'; \mathbf{k}'', \mathbf{k} + \mathbf{k}' - \mathbf{k}'')|^2 G_0(\mathbf{k}', k_4') G_0(\mathbf{k}'', k_4'') G_0(\mathbf{k} + \mathbf{k}' - \mathbf{k}'', k_4 + k_4' - k_4'') + 18T^2(2\pi)^{-6} \sum_{k_4', k_4''} \int d\mathbf{k}' d\mathbf{k}'' |\Psi_1(\mathbf{k}, \mathbf{k}', \mathbf{k}''; \mathbf{k} + \mathbf{k}' + \mathbf{k}'')|^2 G_0(\mathbf{k}', k_4') G_0(\mathbf{k}'', k_4'') G_0(\mathbf{k} + \mathbf{k}' + \mathbf{k}'', k_4 + k_4' + k_4'') + 6T^2(2\pi)^{-6} \sum_{k_4', k_4''} \int d\mathbf{k}' d\mathbf{k}'' |\Psi_1(\mathbf{k}', \mathbf{k}'', \mathbf{k} - \mathbf{k}' - \mathbf{k}''; \mathbf{k})|^2 G_0(\mathbf{k}', k_4') G_0(\mathbf{k}'', k_4'') G_0(\mathbf{k} - \mathbf{k}' - \mathbf{k}'', k_4 - k_4' - k_4''). \quad (39)$$

For the imaginary part of the quantity $\Sigma_\Psi(\mathbf{k}, -i\omega + 0)$ we get

$$\operatorname{Im} \Sigma_\Psi(\mathbf{k}, -i\omega + 0) = \frac{4}{(2\pi)^5} \int d\mathbf{k}' d\mathbf{k}'' |\Psi(\mathbf{k}, \mathbf{k}'; \mathbf{k}'', \mathbf{k} + \mathbf{k}' - \mathbf{k}'')|^2 [n_{\mathbf{k}'}(n_{\mathbf{k}''} + 1)(n_{\mathbf{k}+\mathbf{k}'-\mathbf{k}''} + 1) - (n_{\mathbf{k}'} + 1) n_{\mathbf{k}''} n_{\mathbf{k}+\mathbf{k}'-\mathbf{k}''}] \delta(\omega + \varepsilon_{\mathbf{k}'} - \varepsilon_{\mathbf{k}''} - \varepsilon_{\mathbf{k}+\mathbf{k}'-\mathbf{k}''}) + \frac{9}{(2\pi)^5} \int d\mathbf{k}' d\mathbf{k}'' |\Psi_1(\mathbf{k}, \mathbf{k}', \mathbf{k}''; \mathbf{k} + \mathbf{k}' + \mathbf{k}'')^2| [n_{\mathbf{k}'} n_{\mathbf{k}''} \times (n_{\mathbf{k}+\mathbf{k}'+\mathbf{k}''} + 1) - (n_{\mathbf{k}'} + 1)(n_{\mathbf{k}''} + 1) n_{\mathbf{k}+\mathbf{k}'+\mathbf{k}''}] \delta(\omega + \varepsilon_{\mathbf{k}'} + \varepsilon_{\mathbf{k}''} - \varepsilon_{\mathbf{k}+\mathbf{k}'+\mathbf{k}''}) + \frac{3}{(2\pi)^5} \int d\mathbf{k}' d\mathbf{k}'' |\Psi_1(\mathbf{k}', \mathbf{k}'', \mathbf{k} - \mathbf{k}' - \mathbf{k}''; \mathbf{k})|^2 [(n_{\mathbf{k}'} + 1)(n_{\mathbf{k}''} + 1)(n_{\mathbf{k}-\mathbf{k}'-\mathbf{k}''} + 1) - n_{\mathbf{k}'} n_{\mathbf{k}''} n_{\mathbf{k}-\mathbf{k}'-\mathbf{k}''}] \delta(\omega - \varepsilon_{\mathbf{k}'} - \varepsilon_{\mathbf{k}''} - \varepsilon_{\mathbf{k}-\mathbf{k}'-\mathbf{k}''}). \quad (40)$$

One verifies easily that the imaginary part of the terms of the mass operator which are given by Fig. 5 tend to zero near resonance.

It is clear from Eqs. (30), (37), and (40) that the quantity $\gamma(\mathbf{k})$ is proportional to the probability that a spin wave makes a transition from an initial state with momentum $\mathbf{k}$ to any of all the possible final states.

8. To determine the shape of the homogeneous resonance line we must know the quantity $\gamma(0) = \operatorname{Im} \Sigma(0, -i\epsilon_0 + 0)$ where ϵ_0 is the homogeneous resonance frequency which is determined by the shape of the body.[12] If the ferrodielectric has the shape of a disc and if the axis of easiest magnetization lies in the plane of the disc, we have*

$$\varepsilon_0 = \mu (H_0 + \beta M_0)^{1/2} (H_0 + \beta M_0 + 4\pi M_0)^{1/2}. \quad (41)$$

The exchange interaction does not contribute to the homogeneous resonance width. The quaternary relativistic interaction is small compared with the ternary relativistic interaction so that the main contribution to the line width $\gamma(0)$ comes from the quantity $\operatorname{Im} \Sigma_\Phi(0, -i\epsilon_0 + 0)$ which is proportional to the probability that a spin wave with $\mathbf{k} = 0$ splits up into two spin waves

*The x axis is along the normal to the disc.

$$\gamma(0) = \operatorname{Im} \Sigma_\Phi(0, -i\varepsilon_0 + 0) = \frac{1}{(2\pi)^2} \operatorname{cth} \frac{\varepsilon_0}{4T} \int d\mathbf{k} |\Phi(\mathbf{k}, -\mathbf{k}; 0)|^2 \delta(2\varepsilon_{\mathbf{k}} - \varepsilon_0). \quad (42)^*$$

This quantity differs from zero in the region where the anisotropy of the ferrodielectric and the external magnetic field are sufficiently small:

$$H_0 / M_0 + \beta < 4\pi / 3.$$

We give the expression for the quantity $\gamma(0)$ in two limiting cases

$$\gamma(0) = \begin{cases} \frac{2}{5\pi}\left(\frac{3}{8}\right)^{7/2} \operatorname{cth} \frac{\varepsilon_0}{4T} \left(\frac{\mu M_0}{\Theta_c}\right)^{3/2} \left(\frac{4\pi}{3} - \frac{H_0}{M_0} - \beta\right)^{5/2} \mu M_0, & \frac{4\pi}{3} - \frac{H_0}{M_0} - \beta \ll 1 \\ A \operatorname{cth} \frac{\varepsilon_0}{4T} \left(\frac{\mu M_0}{\Theta_c}\right)^{3/2} \left(\frac{H_0}{M_0} + \beta\right)^{1/2} \mu M_0, & \frac{H_0}{M_0} + \beta \ll 1 \end{cases} \quad (43)^*$$

$$A = \frac{\pi}{4} \int_0^{\pi/6} \cos^2 \vartheta \sqrt{1 - 4\sin^2 \vartheta}\, d\vartheta.$$

If the anisotropy of the ferrodielectric and the external magnetic field are such that

$$H_0 / M_0 + \beta \geqslant 4\pi/3,$$

then one spin wave with $\mathbf{k} = 0$ cannot split into two and $\gamma(0)$ is determined by the quantity $\operatorname{Im} \Sigma_\Psi(0, -i\epsilon_0 + 0)$ and only the first of the three terms in (40) is different from zero and is proportional to the probability that a spin wave is scattered by a background spin wave:

$$\gamma(0) = \operatorname{Im} \Sigma_\Psi(0, -i\varepsilon_0 + 0) = 4(2\pi)^{-5} (e^{\varepsilon_0/T} - 1) \times \int d\mathbf{k}' d\mathbf{k}'' |\Psi(0, \mathbf{k}', \mathbf{k}'', \mathbf{k}' - \mathbf{k}'')|^2 (n_{\mathbf{k}'} + 1) n_{\mathbf{k}''} n_{\mathbf{k}-\mathbf{k}''} \delta \times (\varepsilon_0 + \varepsilon_{\mathbf{k}'} - \varepsilon_{\mathbf{k}''} - \varepsilon_{\mathbf{k}'-\mathbf{k}''}). \quad (44)$$

In the temperature range $\epsilon_0 \ll T \ll \Theta_c$ the homogeneous resonance line width is of the form

$$\gamma(0) \approx \frac{\mu M_0}{\Theta_c} \left(\frac{T}{\Theta_c}\right)^2 \mu M_0, \qquad \frac{H_0}{M_0} + \beta \gtrsim \frac{4\pi}{3};$$

$$\gamma(0) = \frac{\beta^2}{192\pi} \frac{\mu M_0}{\Theta_c} \left(\frac{T}{\Theta_c}\right)^2 \mu M_0, \qquad \beta \gg \frac{4\pi}{3}. \quad (45)$$

We turn now to the evaluation of the quantity $\gamma(\mathbf{k})$ in the region of large values of the wave vector $\mathbf{k}$ ($ak \gg \sqrt{\mu M_0/\Theta_c}$). The main part is then played by the quantity $\Sigma_\Phi(\mathbf{k}, -i\epsilon_{\mathbf{k}} + 0)$ which is caused by the exchange interaction between the spin waves. Using (40) and (20) we get

$$\gamma(\mathbf{k}) = \operatorname{Im} \Sigma_\Psi(\mathbf{k}, -i\varepsilon_{\mathbf{k}} + 0) = \frac{1}{4(2\pi^5)} \left(\frac{\mu\Theta_c}{M_0}\right)^2 \left(\exp \frac{\varepsilon_{\mathbf{k}}}{T} - 1\right) \times a^4 \int d\mathbf{k}' d\mathbf{k}'' (\mathbf{k}\mathbf{k}' + \mathbf{k}\mathbf{k}'' + \mathbf{k}'\mathbf{k}'' - \mathbf{k}''^2)^2 (n_{\mathbf{k}'} + 1) + n_{\mathbf{k}''} n_{\mathbf{k}+\mathbf{k}'-\mathbf{k}''} \delta(\varepsilon_{\mathbf{k}} + \varepsilon_{\mathbf{k}'} - \varepsilon_{\mathbf{k}''} - \varepsilon_{\mathbf{k}+\mathbf{k}'-\mathbf{k}''}). \quad (46)$$

When $\epsilon_{\mathbf{k}} \sim T$ the quantity $\gamma(\mathbf{k})$ determines the time it takes to establish thermal equilibrium in the spin system; in that case

*Cth = coth.

$$\gamma(\mathbf{k}) \approx (T/\Theta_c)^4 \Theta_c. \tag{47}$$

Comparing Eqs. (47) and (45) we see easily that $\gamma(\mathbf{k})$ for $\epsilon_\mathbf{k} \sim T$ is much larger than $\gamma(0)$ if the temperature is such that $T/\Theta_c \gg \sqrt{\mu M_0/\Theta_c}$. This means, according to (33) that first, after a time on the order of $\gamma(\mathbf{k})^{-1}$, a uniform distribution of the magnetic moment is established in the ferrodielectric. Then, after a time on the order of $\gamma(0)^{-1}$, the transverse part of the magnetic moment $\mathbf{m}_\perp$ slowly approaches its equilibrium value rotating with the homogeneous resonance frequency ϵ_0. The end point of $\mathbf{m}_\perp$ describes then the ellipse

$$m_x^2(t)/U_1(0) + m_y^2(t)/U_2(0) = \text{const}. \tag{48}$$

9. In the solutions of the Maxwell equations there occurs usually a tensor $\tilde{\chi}_{il}$ which connects the magnetic moment density $\mathbf{M}$ with the total magnetic field in the system $\mathbf{h} + \mathbf{h}^i$:

$$M_i = \tilde{\chi}_{il}(h_l + h_l^i). \tag{49}$$

We shall connect the tensor χ_{il} occurring in Sec. 2 with the tensor $\tilde{\chi}_{il}$. Solving the Maxwell equations one finds easily the connection between the field $\mathbf{h}^i$ and the magnetic moment density $\mathbf{M}$; in the magnetostatics approximation we have

$$\mathbf{h}^i = -4\pi\mathbf{k}/k^{-2}(\mathbf{kM}). \tag{50}$$

Eliminating the quantity $\mathbf{h}^i$ from Eq. (49) and using (50) we get

$$(\delta_{il} + 4\pi\tilde{\chi}_{im}k_m k_l/k^{-2})M_l = \tilde{\chi}_{il}h_l.$$

Comparing this formula with Eqs. (7) and (8) we get the following equation which connects the quantities χ_{il} and $\tilde{\chi}_{il}$

$$(\delta_{im} + 4\pi\tilde{\chi}_{in}k_n k_m/k^{-2})\chi_{ml} = \tilde{\chi}_{il}. \tag{51}$$

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