φ is the angle between the spin projections on (111) and the twofold symmetry axis, m is the ferromagnetic moment, and d, a, g, B are expansion coefficients [see Eq. (3a) in reference 7]. Figure 2 gives the experimentally observed arrangement of magnetic moments in $CoCO_3$, corresponding to state II. It should be noted that the angle β is very close to the direction of the shortest distance from the central ion to its nearest neighbor. This picture differs somewhat from the theoretical prediction for state II, since the coefficient ratio d/a is of the order of unity, whereas the theory predicts the very small ratio $\sim v^2/c^2$.

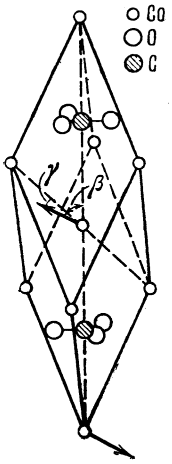

FIG. 2

In connection with this additional observed type of antiferromagnetic structure for isomorphous carbonates of the iron group, it should be remembered that in $FeCO_3$ magnetic moments are directed along the [111] axis, while in $MnCO_3$ they lie in the (111) plane.

The weak ferromagnetism of $CoCO_3$, which is indicated by the small magnetic contribution to the (211) reflections, shows that the magnetic moments of the ions form a small angle γ with the plane of symmetry, thus producing a total ferromagnetic moment along the twofold axis (Fig. 2). The ratio between ferromagnetic and antiferromagnetic contributions to the (211) and (100) reflections indicates $15 \pm 5°$ as the magnitude of γ. The existence of the ferromagnetic moment (8.6%) in the (111) plane was recently observed by Borovik-Romanov and Ozhogin,[8] who investigated the weak ferromagnetism of $CoCO_3$ in crystals obtained from the same source as ours.[†] They calculated $\gamma = 7°$, from their absolute data for the ferromagnetic moment and the calculated saturation moment of the ion. Aside from experimental errors, uncertainty regarding the saturation moment of Co^{++} is the most likely source of a discrepancy regarding γ.

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DIPOLE MOMENT OF UNSTABLE ELEMENTARY PARTICLES

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PARITY nonconservation makes it possible for an elementary particle with spin to have a dipole moment.[1] Landau's theory of combined inversion[2] leads to time-reversal invariance, from which Landau concludes that elementary particles do not have an electric dipole moment.

However, this conclusion cannot be extended to apply to decaying unstable particles. To prove Landau's assertion, we consider a particle with spin and a dipole moment. Time reversal changes the spin direction, but leaves the direction of the dipole moment unchanged; it follows that there can be no time-reversal-invariant relation between spin and dipole moment.

An unstable particle is represented by an exponentially-decaying state amplitude surrounded by outgoing waves for the decay products. Under time reversal, an unstable particle is transformed not into the same particle with reversed spin but into something entirely different — into a state with exponentially-increasing amplitude surrounded by incoming waves for the decay products. Therefore, the proof that a dipole moment is not present does not apply to unstable particles.

We shall use an example to show the connection between a particle's instability and its dipole moment. We consider a neutral particle A^0 with spin $\frac{1}{2}$. Suppose the Hamiltonian contains terms corresponding to the two different reactions

$$A^0 \rightleftarrows B^+ + C^-, \quad A^0 \rightleftarrows D^0 + E^0.$$

Further suppose that $m_B + m_C > m_A > m_D + m_E$, so that A^0 actually decays into D^0 and E^0, while the transformation of A^0 into B^+ and C^- proceeds virtually, i.e., the B^+ and C^- particles form a cloud around the A^0, but do not escape to infinity.

The dipole moment depends only on the distribution of the charged B^+ and C^- particles. We assume that the interaction $A^0 \rightleftharpoons B^+ + C^-$ is parity nonconserving, so that the B^+ and C^- cloud is a superposition of $S_{1/2}$ and $P_{1/2}$ waves. Let B^+ have spin $\frac{1}{2}$ and C^- have spin 0; the wave function of the system has two components corresponding to the two possible spin orientations of B^+.

Suppose A^0 is in the state with $s_z = +\frac{1}{2}$. Then the wave function for B^+ and C^- has the form

$$\psi = \begin{vmatrix} af_0(r) - ib\sqrt{1/3}\, f_1(r)\cos\theta \\ ib\sqrt{2/3}\, f_1(r)\sin\theta\, e^{i\varphi} \end{vmatrix},$$

where r, θ, φ describe the relative position of B^+ and C^-.

$$f_0(r) = r^{-1}e^{-\varkappa r}, \quad f_1(r) = df_0/dr = -(r^{-2} + \varkappa r^{-1})\, e^{-\varkappa r},$$

$$\varkappa = \sqrt{2\mu\Delta}, \quad \mu = m_B m_C/(m_B + m_C), \quad \Delta = E_B + E_C - E_A.$$

The charge density is $|\psi|^2 = \psi^*\psi$. As has been shown by the author[3] (cf. the derivation of the corresponding formulae), a and b are real in a time-reversal-invariant theory, so that if κ is real the interference term (proportional to $\cos\theta$) in the charge density vanishes and the dipole moment is identically equal to zero.

For an unstable particle, time-reversal invariance requires that the constants coupling the A particle to the S- and P-wave states of B and C (and consequently, also the amplitudes, a and b, of these waves) remain real. The difference lies in the fact that, owing to the decay of the A^0 into D^0 and E^0, the energy of the A^0 becomes complex: $E_A = m_A c^2 - iw/2$, where w is the decay probability of the A (units with $\hbar \equiv 1$). Therefore, κ also becomes complex; for small w, we have $\kappa = \kappa_0(1 + iw/4\Delta)$. With this, the dipole moment is easily found to be*

$$d_z = e\int r\cos\theta\, \psi^*\psi\, dv = -\frac{\pi}{3\sqrt{3}}\frac{e}{\varkappa_0}\frac{w}{\Delta}ab.$$

The integral which gives d_z converges, although $\int \psi^*\psi\, dv$ diverges at the origin.

Thus, with a time-reversal-invariant Hamiltonian, the combination of parity nonconservation and particle instability leads to the appearance of a dipole moment, i.e., to an apparent violation of time-reversal invariance. In this sense, the result agrees with that of Behrends,[4] who discussed the radiative decay $\Lambda \rightarrow n + \gamma$ along with the main decay channel $\Lambda \rightarrow p + \pi^-$. In this case the result contains terms obviously contradicting time-reversal invariance, notwithstanding the time-reversal invariance of the Hamiltonian.

From a general point of view, this problem is of interest as an example of the special properties of unstable particles; statements which are true for stable particles cannot be automatically extended to unstable particles.

***Note that the non-zero result comes from the term κr^{-1} in $f_1(r)$ and not from $\exp(-\kappa r)$.**

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