SEMI-ANALYTICAL SOLUTION OF EQUATIONS OF THE CHEW-MANDELSTAM TYPE

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Submitted to JETP editor July 23, 1960

J. Exptl. Theoret. Phys. (U.S.S.R.) 39, 1442-1449 (November, 1960)

A simple method for solving equations of the Chew-Mandelstam type is proposed, which rapidly yields an approximate solution. The method is applied to the Chew-Mandelstam equation for the interaction of neutral and charged pions. The results of the first approximation in the case of charged pions are compared with the numerical calculation of Chew, Mandelstam, and Noyes.

1. INTRODUCTION

As is well known, the unitarity conditions and the analyticity properties lead in the two-particle approximation[1-4] to a closed system of equations[5,6] for the spectral functions $A_{ij}(s, s')$ and $a_i(s)$ $(i = 1, 2, 3)$ of the Mandelstam representation. The Chew-Mandelstam equation[4] for the functions $a_i(s)$ results from the above equations if the contributions from the functions $A_{ij}(s, s')$ are neglected. With the help of the solutions of these equations a first approximation can also be obtained for the functions $A_{ij}(s, s')$, and then first-order corrections to the solution $a_i(s)$ of the Chew-Mandelstam equation. Thus the solution of a Chew-Mandelstam type equation may be viewed as the first step of an iteration procedure for the solution of the complete set of equations for the spectral functions $A_{ij}(s, s')$ and $a_i(s)$. We outline below a simple method for solving equations of the Chew-Mandelstam type, which makes it possible to obtain rapidly an approximate solution and investigate its properties.

The method to be proposed is analogous to the well-known method of Dalitz, Dyson, and Castillejo.[7] If the values of the coupling constant lie in a certain region the solution is obtained in the form of analytic functions, the coefficients of these functions being solutions of a system of algebraic equations. They depend on the coupling constant and can be determined by numerical calculations (or even analytically, in the simplest approximation).

The method is applicable in the low-energy region, i.e., precisely where the two-particle approximation, used in the derivation of the equations, makes sense. Even the first approximation gives usually good results in this region.

In the present note we shall demonstrate the basic ideas of this solution on the simplest example: the $\pi\pi$ interaction between neutral and charged mesons. We hope to report later on a number of other cases of physical interest.

2. THE INTERACTION OF NEUTRAL PIONS

The Chew-Mandelstam equation for the amplitude for the interaction of neutral pions

$$A_0(\nu) = [(1+\nu)/\nu]^{1/2} e^{i\delta_0} \sin\delta_0$$

($\nu = q^2/\mu^2$, q is the momentum in the barycentric frame, δ_0 is the phase shift) has the following form:[4,6]

$$A_0(\nu) = -\lambda + \frac{1}{\pi}\int_0^\infty \sqrt{\frac{\nu'}{1+\nu'}}\left(\frac{1}{\nu'-\nu} - \frac{1}{\nu'-\nu_0}\right)|A_0(\nu')|^2 d\nu' + \frac{2}{\pi}\int_{-\infty}^{-1}\left(\frac{1}{\nu'-\nu} - \frac{1}{\nu'-\nu_0}\right)\frac{d\nu'}{\nu'}\int_0^{-\nu'-1}\sqrt{\frac{\nu''}{1+\nu''}} \times |A_0(\nu'')|^2 d\nu'', \quad (1)$$

where $\lambda = -A_0(\nu_0)$, ν_0 being the point at which the subtraction was performed.

Similar to the case of the πN interaction in the static model,[7] it is convenient to deal with the inverse amplitude $h(\nu) = A_0^{-1}(\nu)$. At that we shall consider only that solution of Eq. (1) which has no zeros in the complex ν plane. Then $h(\nu)$ has the same analyticity properties as $A_0(\nu)$; the imaginary part of $h(\nu)$ along the cuts is easily determined from Eq. (1).

As a result we obtain for $h(\nu)$ the equation

$$h(\nu) = -\frac{1}{\lambda} - \frac{1}{\pi}\int_0^\infty \sqrt{\frac{\nu'}{1+\nu'}}\left(\frac{1}{\nu'-\nu} - \frac{1}{\nu'-\nu_0}\right)d\nu' - \frac{2}{\pi}\int_{-\infty}^{-1}\left(\frac{1}{\nu'-\nu} - \frac{1}{\nu'-\nu_0}\right)|h(\nu')|^2\frac{d\nu'}{\nu'}\int_0^{-1-\nu'}\sqrt{\frac{\nu''}{1+\nu''}} \times \frac{d\nu''}{|h(\nu'')|^2}. \quad (2)$$

This equation is inconvenient as it stands, because the integrations on the right hand side extend to infinity (and divergent integrals are obtained when the integrands are expanded in powers of ν'). It is useful to introduce in place of ν the variable $x = \nu/(1+\nu)$, which covers the range $0 \le x \le 1$ when $0 \le \nu \le \infty$, and in place of ν' the variable $y = \nu'/(1+\nu')$ in the first integral and $y = (1+\nu')/\nu'$ in the second integral. Equation (2) then reduces to the simple form ($\eta = \nu''/(1+\nu'')$):

$$h(x) = C + \frac{x}{\pi}\int_0^1 \frac{dy}{\sqrt{y}(x-y)} + \frac{2}{\pi}\int_0^1 \frac{y^{3/2} v(y)\,dy}{1/x - y}, \tag{3}$$

$$v(y) = \frac{1-y}{y^{3/2}}\int_0^y \left|\frac{h(1/y)}{h(\eta)}\right|^2 \frac{\sqrt{\eta}\,d\eta}{(1-\eta)^2}, \tag{4}$$

where $C = h(0)$.

To the value $\nu = \nu_0$ corresponds $x = x_0$, with $h(x_0) = -\lambda^{-1}$. Setting in Eq. (3) $x = x_0$ we obtain

$$C = -\lambda^{-1} - \frac{x_0}{\pi}\int_0^1 \frac{dy}{\sqrt{y}(x_0-y)} - \frac{2}{\pi}\int_0^1 \frac{y^{3/2} v(y)\,dy}{1/x_0 - y}. \tag{5}$$

To the value $\nu_0 = -2/3$ corresponds $x_0 = -2$.

We next study the properties of the function $v(y)$. In order to solve the Eqs. (3) and (4) it is sufficient to determine $v(y)$ in the region $0 \le y \le 1$; it follows from Eq. (4) that in that region $v(y) > 0$. Transposing in Eq. (4) the factor multiplying the integral to the left and differentiating with respect to y we obtain

$$\frac{d}{dy}\left[\frac{y^{3/2} v(y)}{(1-y)\,|h(1/y)|^2}\right] = \frac{y^{1/2}}{(1-y)^2\,|h(y)|^2}, \tag{6}$$

$$y(1-y)(\tilde{u}v' - \tilde{u}'v)u + \tfrac{1}{2}(3-y)\tilde{u}uv = \tilde{u}^2, \tag{7}$$

where $u(y) = |h(y)|^2$, $\tilde{u}(y) = |h(1/y)|^2$ and the prime denotes everywhere differentiation with respect to y.

One deduces easily from Eqs. (4) and (3) [or (6) and (3)] that $v(0)$ is a finite quantity. Furthermore, it can be shown that $y = 0$ is altogether not a singular point of the function $v(y)$, i.e., that $v(y)$ can be expanded in a Taylor series about $y = 0$. In addition, it follows from Eqs. (4) and (6) that the function $v(y)$ is finite at $y = 1$ and that as y approaches 1 the function $v(y)$ becomes

$$v(y) \approx 1 + a[\ln(1/y)]^{-1}, \tag{8}$$

where a is some constant.

Thus $v(y)$ is a positive function varying between $v(0) = \alpha_0$ at $y = 0$ and $v = 1$ at $y = 1$. We shall seek an expression for it in the form of a polynomial of degree N:

$$v(y) = \sum_{n=0}^{N} \alpha_n y^n. \tag{9}$$

To find the coefficients α_n we choose $N+1$ points in the interval $0 \le y \le 1$ and require Eq. (7) to be satisfied at these points. We then obtain a system of $N+1$ nonlinear algebraic equations for the $N+1$ coefficients α_n.

The main idea of our method has to do with the circumstance that the Eq. (1) was derived in the two-particle approximation and therefore can be valid only for small values of x (not close to unity). The whole scheme will be self consistent if on the right hand side of Eq. (3) values of $v(y)$ for y near unity are unimportant. It is therefore natural to require Eq. (7) to be satisfied at the point $y = 0$ for the case $N = 0$, at the points $y = 0$ and $y = 1/2$ for the case $N = 1$, etc.

In the first approximation we set $v(y) = \alpha_0$. Then Eq. (7), evaluated at the point $y = 0$, yields

$$\tfrac{3}{2}\alpha_0 u(0) = \tilde{u}(0). \tag{10}$$

On the other hand, substituting Eq. (9) into Eq. (3) we obtain

$$h(x) = C + \frac{2}{\pi}\Big[xJ_0(1/x) + 2\sum_{n=0}^{N} \alpha_n J_{2n+4}(x)\Big] - i\sqrt{x}\,\theta(1-x) + 2ix^{-3/2}\sum_{n=0}^{N} \alpha_n x^{-n}\theta(x-1),$$

$$J_n(x) = \tfrac{1}{2}\,\mathrm{P}\int_0^1 \frac{y^{n-1)/2}}{1/x - y}dy = -\frac{1}{n-1} + \frac{1}{x}J_{n-2}(x). \tag{11}$$

Here P denotes the principal-value integral.

In order to calculate $u(0) = |h(0)|^2$ and $\tilde{u}(0) = |h(\infty)|^2$ we make use of Eq. (11) in which we keep only the first term in the summation over n. As a result we obtain from Eq. (10)

$$\alpha_0 = \frac{2}{3}\,\frac{(1-\eta)^2}{1 - 8\eta(1-\eta)/9 + \sqrt{1 - 16\eta(1-\eta)/9}},$$

$$\eta = -\frac{2}{\pi C}. \tag{12}$$

In this approximation the constant η is related to the four-boson interaction constant λ by

$$\frac{2}{\pi\eta} = \frac{1}{\lambda} + \frac{2}{\pi}x_0 J(1/x_0) + \frac{4}{\pi}\alpha_0 J_4(x_0). \tag{13}$$

In the next higher approximation we seek a $v(y)$ in the form $v(y) = \alpha_0 + \alpha_1 y$ and determine α_0 and α_1 from Eq. (7) evaluated at the points $y = 0$ and $y = 1/2$. The equations for α_0 and α_1 are of the form

$$\tfrac{3}{2}\alpha_0 = \left[1 - \eta\left(1 - \tfrac{2}{3}\alpha_0 - \tfrac{2}{5}\alpha_1\right)\right]^2,$$

$$\alpha_1 = 2\,\frac{4\tilde{u}^2(1/2) - 5\tilde{u}(1/2)u(1/2)\alpha_0 + \tilde{u}'(1/2)u(1/2)\alpha_0}{u(1/2)[7\tilde{u}(1/2) - \tilde{u}'(1/2)]}. \tag{14}$$

Here $\tilde{u}(1/2)$, $u(1/2)$, and $\tilde{u}'(1/2)$ are expressed in terms of α_0 and α_1 according to Eq. (11). These equations are solved very simply by the iteration

TABLE I

η	−0.4	−0.3	−0.2	−0.1	0	0.1	0.15
α_0	0.82	0.84	0.8	0.74	0.67	0.57	0.49
α_1	−0.34	−0.05	−0.02	0.06	0.19	0.48	0.88

method, where we ignore in the first approximation those terms in the right hand sides which contain α_1. Then the first equation in (14) yields for α_0 the same value as is given by Eq. (12), and the second equation in (14) gives α_1 in terms of C and α_0, whose dependence on λ is already determined with the help of (14).

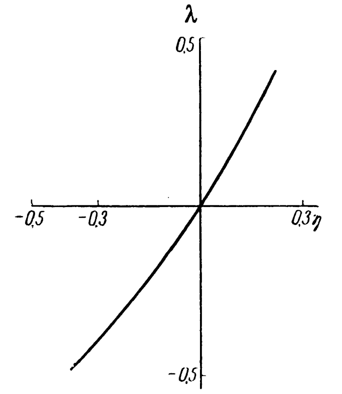

FIG. 1

We have solved the system (14) numerically. In Table I we give values of α_0 and α_1 for various values of η. If we substitute into Eq. (5) $v(y) = \alpha_0 + \alpha_1 y$ and use for α_0 and α_1 values from Table I, we can obtain η as a function of λ (Fig. 1). From here one obtains easily α_0 and α_1 as functions of λ (Fig. 2). A solution for the system (14) exists only in the region $-0.4 \le \eta \le 0.15$ or $-0.5 \le \lambda \le 0.25$. At the limits of this region α_1 becomes infinite from which it follows that the amplitude $A_0(\nu)$ vanishes identically.

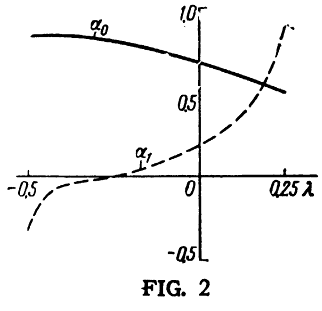

FIG. 2

Equation (1) was also solved numerically. The limiting values for λ as obtained by this method were almost the same: $-0.6 \le \lambda \le 0.25$. The curves in Figs. 3 and 4 represent the dependence of $-\frac{1}{2}\pi\eta\sqrt{x}\cot\delta_0 = -\frac{1}{2}\pi\eta \operatorname{Re} h(x)$ on ν

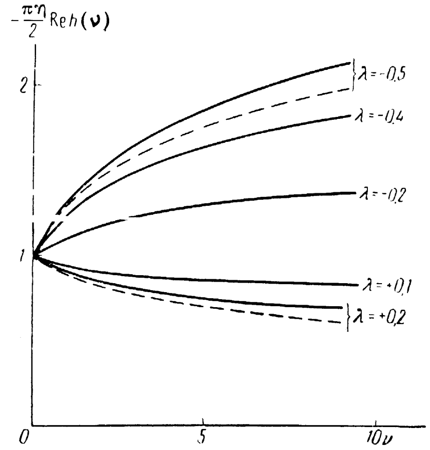

FIG. 3

$= 1/(1-x) = q^2/\mu^2$. The solid curves represent the first approximation constructed according to Eqs. (11) and (14). For all λ, with the exception of values in the immediate vicinity of the limiting values, the first and second approximations prac-

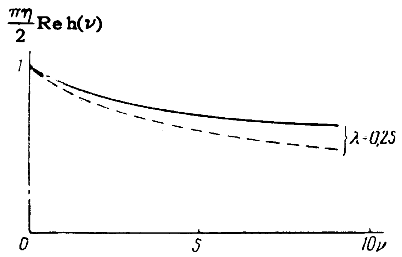

FIG. 4

tically coincide (for such λ a single curve is shown in Fig. 3). For values close to the limiting values the second approximation gives a significant correction. The second approximation curves are shown in Figs. 3 and 4 by dotted lines.

3. INTERACTION OF CHARGED MESONS

The equations appropriate to this case were given by Chew and Mandelstam.[4] As in the neutral mesons case we introduce the quantities $h_0(x) = [A_0^0(x)]^{-1}$, $h_2(x) = [A_0^2(x)]^{-1}$, $h_1(x) = [A_1^1(x)]^{-1}$; the upper index denotes the isotopic spin, the lower index denotes the orbital angular momentum. We then obtain the following system of equations for the functions $h_i(x)$ $(i = 0, 2)$ and $h_1(x)$:

$$h_i(x) = C_i + \frac{x}{\pi}\int_0^1 \frac{dy}{\sqrt{y}\,(x-y)} + \frac{1}{\pi}\int_0^1 \frac{y^{3/2} v_i(y)\,dy}{1/x - y}, \quad (15)$$

$$h_1(x) = C_1 + \frac{x}{\pi}\int_0^1 \frac{\sqrt{y}\,dy}{x-y} + \frac{1}{\pi}\int_0^1 \frac{y^{5/2} v_1(y)\,dy}{1/x - y}; \quad (16)$$

$$C_i = -\begin{pmatrix}(5\lambda)^{-1}\\(2\lambda)^{-1}\end{pmatrix} - \frac{x_0}{\pi}\int_0^1 \frac{dy}{\sqrt{y}\,(x_0-y)} - \frac{1}{\pi}\int_0^1 \frac{y^{3/2} v_i(y)\,dy}{1/x_0 - y}, \quad (17)$$

$$C_1 = \left\{ \frac{1}{\pi} \int_0^1 \left[\frac{\sqrt{y}}{|h_1(y)|^2} - \frac{y^{3/2} v_1(y)}{|h_1(1/y)|^2} \right] dy \right\}^{-1}. \quad (18)$$

The expressions for $v_i(y)$ and $v_1(y)$ are of the form

$$v_i(y) = (1-y) \left| h_i\left(\frac{1}{y}\right)\right|^2 y^{-3/2} \int_0^y \frac{\eta d\eta}{(1-\eta)^2} \left\{ \frac{\alpha_{i0}}{|h_0(\eta)|^2} + \frac{\alpha_{i2}}{|h_2(\eta)|^2} + \alpha_{i1} 3\left(1 - 2\frac{y(1-\eta)}{\eta(1-y)}\right) \frac{\eta^2}{|h_1(\eta)|^2} \right\}, \quad (19)$$

$$v_1(y) = (1-y) \left| h_1\left(\frac{1}{y}\right)\right|^2 y^{-3/2} \int_0^y \frac{\sqrt{\eta} d\eta}{(1-\eta)^2} \left(1 - \frac{1-y}{1-\eta}\right) \times \left\{ \frac{\alpha_{10}}{|h_0(\eta)|^2} + \frac{\alpha_{12}}{|h_2(\eta)|^2} + 3\alpha_{11}\left(1 - 2\frac{y(1-\eta)}{\eta(1-y)}\right) \frac{\eta^2}{|h_1(\eta)|^2} \right\}, \quad (20)$$

where α_{ik} is a known matrix:[4]

$$\alpha_{ik} = \begin{pmatrix} 2/3 & 10/3 & 2 \\ 2/3 & 1/3 & -1 \\ 2/3 & -5/3 & 1 \end{pmatrix}. \quad (21)$$

Transfering the factor multiplying the integrals on the right hand sides of the Eqs. (19) and (20), and differentiating with respect to y, we obtain equations for $v_i(y)$ and $v_1(y)$ analogous to Eq. (7) in the case of neutral mesons. These equations are now much more involved:

$$y(1-y)(v_i'\tilde{u}_i - v_i\tilde{u}_i') + \tfrac{1}{2}(3-y)\tilde{u}_i v_i = \tilde{u}_i^2 \left\{ \frac{\alpha_{i0}}{u_0(y)} + \frac{\alpha_{i2}}{u_2(y)} - \frac{3\alpha_{i1}y^2}{u_1(y)} - 6\alpha_{i1} y^{-1/2} \int_0^y \frac{\eta^{3/2} d\eta}{(1-\eta)|h_1(\eta)|^2} \right\}, \quad i = 0.2; \quad (22)$$

$$y(1-y)(v_1'\tilde{u}_1 - v_1\tilde{u}_1') + \tfrac{1}{2}(3-y)\tilde{u}_1 v_1 = \tilde{u}_1^2 y^{-1/2} \left\{ (1-y)^2 \int_0^y \frac{\sqrt{\eta} d\eta}{(1-\eta)^3} \left[\frac{2}{3u_0(\eta)} - \frac{5}{3u_2(\eta)} + 3\left(1 - \frac{2y(1-\eta)}{\eta(1-y)}\right) \frac{\eta^2}{u_1(\eta)} \right] - 6\int_0^y \frac{\eta^{3/2} d\eta}{(1-\eta)} \left(1 - \frac{1-y}{1-\eta}\right) [u_1(\eta)]^{-1} \right\}. \quad (23)$$

It is easy to show from Eqs. (22) and (23) that $v_0(0)$ and $v_2(0)$ are not equal to zero, whereas $v_1(0)$ equals zero. As in the preceding section it can be shown that y = 0 is not a singular point of the functions $v_i(y)$. Following our method we now look for $v_i(y)$ in the form of a polynomial in y. The problem is then reduced to the solution of nonlinear algebraic equations, whose number equals the number of unknown coefficients of the various powers of y in the polynomials (24):

$$v_0(y) = \sum_{k=0}^{N} \alpha_k y^k; \quad v_2(y) = \sum_{k=0}^{N} \beta_k y^k; \quad v_1(y) = \sum_{k=1}^{N} \gamma_k y^k. \quad (24)$$

In the first approximation we may substitute $v_0(y) = \alpha_0$, $v_2(y) = \beta_0$ and $v_1(y) = \gamma_0 y$ into the Eqs. (22) and (23) and then set in them y = 0. We then obtain for α_0, β_0, and γ_0 the following system of equations:

$$\begin{aligned} \tfrac{3}{2}\alpha_0 &= \tilde{u}_0(0)(2/3\,u_0(0) + 10/3\,u_2(0)), \\ \tfrac{3}{2}\beta_0 &= \tilde{u}_2(0)(2/3\,u_0(0) + 1/3\,u_2(0)), \\ \tfrac{5}{2}\gamma_0 &= \tfrac{2}{3}\tilde{u}_1(0)(2/3\,u_0(0) - 5/3\,u_2(0)), \end{aligned} \quad (25)$$

where

$$\tilde{u}_i(x) = |h_i(1/x)|^2, \qquad u_i(x) = |h_i(x)|^2.$$

As in the neutral meson case, $h_i(x)$, $u_i(x)$ and $\tilde{u}_i(x)$ may be expressed in terms of the coefficients α_k, β_k and γ_k. For example

$$h_0(x) = C_0 + \frac{2}{\pi}\left[xJ_0\left(\frac{1}{x}\right) + \sum_{n=0}^{N} \alpha_n J_{2n+4}(x) \right] + i\sqrt{x}\theta(1-x) + i\theta(x-1)x^{-3/2} \sum_{n=0}^{N} \alpha_n x^{-n}. \quad (26)$$

Keeping in Eq. (26) only the coefficients α_0, β_0 and γ_0 we obtain after substitution into (25)

$$\begin{aligned} u_i(0) &= C_i^2, \quad \tilde{u}_0(0) = \left[C_0 + \frac{2}{\pi}\left(1 - \tfrac{1}{3}\alpha_0\right)\right]^2, \\ \tilde{u}_2(0) &= \left[C_2 + \frac{2}{\pi}\left(1 - \tfrac{1}{3}\beta_0\right)\right]^2, \\ \tilde{u}_1(0) &= \left[C_1 + \frac{2}{\pi}\left(\tfrac{1}{3} - \tfrac{1}{7}\gamma_0\right)\right]^2. \end{aligned} \quad (27)$$

The first two equations in (25) are quadratic in α_0 and β_0 and do not contain γ_0. Solving these equations yields the functions $\alpha_0(C_0, C_2)$ and $\beta_0(C_0, C_2)$. On the other hand by making use of Eq. (17) we can express C_0 and C_2 in terms of α_0 and α_2:

$$C_0 = -\frac{1}{5\lambda} - \frac{2}{\pi}\left[x_0 J_0\left(\frac{1}{x_0}\right) + \alpha_0 J_4(x_0) \right], \quad (28)$$

$$C_2 = -\frac{1}{2\lambda} - \frac{2}{\pi}\left[x_0 J_0\left(\frac{1}{x_0}\right) + \beta_0 J_4(x_0) \right]. \quad (29)$$

From here α_0 and β_0 are obtained by iteration of which the first step is obtained by substituting for C_0 and C_2 in $\alpha_0(C_0, C_2)$ and $\beta_0(C_0, C_2)$ the values given by Eqs. (28) and (29) with $\alpha_0 = \beta_0 = 0$.

The third equation in (25) is quadratic in γ_0; solving it using the already determined $\alpha_0(\lambda)$ and $\beta_0(\lambda)$ yields $\gamma_0(C_1, \lambda)$. According to Eq. (18)

$$C_1^{-1} = \frac{1}{\pi}\int_0^1 \left[\frac{\sqrt{y}}{u_1(y)} - \frac{\gamma_0 y^{5/2}}{\tilde{u}_1(y)} \right] dy. \quad (30)$$

Finally $\gamma_0(\lambda)$ is obtained by iteration of which the first step is obtained by substituting in $\gamma_0(C_1, \lambda)$ the value $C_1^{(0)}$ obtained from Eq. (30) with $\gamma_0 = 0$. Table II lists the coefficients $\alpha_0(\lambda)$ and $\beta_0(\lambda)$ obtained in this manner.

TABLE II

λ	0	0.1	0.3	1
α_0	0.8	0,58	0.305	0,04
β_0	3	2.1	1,05	0,18

The dashed lines in Fig. 5 depict the dependence of $-5\lambda\sqrt{x}\cot\delta_0 = -5\lambda \operatorname{Re} h_0(x)$ on $\nu = 1/(1-x) = q^2/\mu^2$ in the first approximation constructed according to Eqs. (15) and (24). The dashed lines in Fig. 6 depict the dependence of the function $-2\lambda\sqrt{x}\cot\delta_2$ on ν.

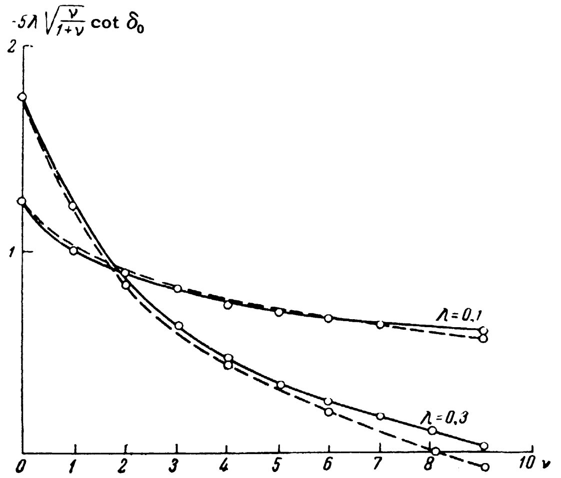

FIG. 5

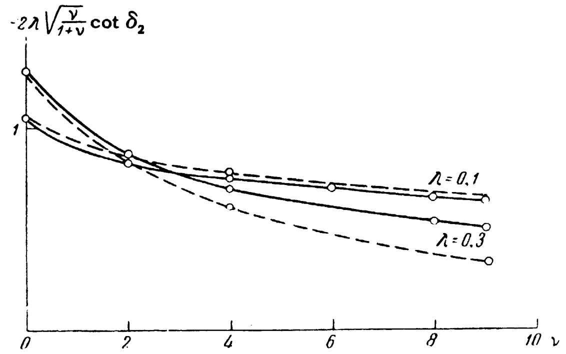

FIG. 6

4. CONCLUSIONS

A comparison of the results of the first and second approximations for the function $-(\pi\eta/2) \times \sqrt{x}\cot\delta = -(\pi\eta/2)\operatorname{Re} h(x)$ (Figs. 3 and 4) shows, that the second order corrections may be neglected, except for values of λ close to the limiting values when $\alpha_1(\lambda)$ becomes large. However, as is seen from Fig. 2, this exceptional region is not large. It must also be remembered that the range of validity of Eqs. (1) and (2) is restricted to the limits $0 \le \nu \lesssim 3$. This is just the region in which the corrections attain their smallest values and for $\nu = 0$ the first and second approximations are identical. Furthermore our first approximation is in good agreement with the numerical solution of Eq. (1) (E. P. Vedeneev and A. L. Krylov, private communication).

We may also check the appropriateness of our first approximation in the charged meson theory. The comparison of our first approximation (dashed lines) and of the numerical solutions of the Chew-Mandelstam equations for the scattering of charged mesons[8] (solid lines) is given in Figs. 5 and 6 for the functions $-5\lambda\sqrt{x}\cot\delta_0$ and $-2\lambda\sqrt{x}\cot\delta_2$. It is seen that the agreement, particularly in the region $0 \le \nu \lesssim 3$, is very good. It is not surprising that the solution found by us in the first approximation is, in essence, the S-wave dominant solution of Chew, Mandelstam, and Noyes. The reason for this is that in the first approximation the S-wave scattering amplitudes are obtained independently of the P-wave scattering amplitude [the first two equations in (25) are not coupled to the third one].

Thus, in all cases considered, already the first approximation yields reasonably good results. When it is noted that the first approximation can be obtained quite easily it becomes clear that the method here outlined could be very useful for the investigation of solutions of equations of the Chew-Mandelstam type.

Our method was applied here to the Chew-Mandelstam equations for the scattering of neutral and charged pions. It may also be applied to other equations of analogous structure, say πN or πK scattering.

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