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ANISOTROPIC PROPAGATION OF LONGITUDINAL ELECTROACOUSTICAL WAVES IN A DRIFTING PLASMA

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The experimentally observed anisotropy in the propagation of low-frequency electroacoustical waves in a low-pressure gas-discharge plasma is explained theoretically within the hydrodynamic approximation by including collisions between charged particles and neutral atoms. From the derived dispersion equation it is found that plane waves of a given frequency will propagate under certain conditions from cathode to anode with an increase in amplitude and from anode to cathode with a decrease in amplitude. This anisotropy is due mainly to the combined effect of charged particle — atom collisions and constant drift motion. In the presence of boundaries the anisotropy is enhanced.

LET us consider the propagation of a plane longitudinal low-frequency wave in a plasma consisting of two kinds of charged particles, electrons and ions, drifting in opposite directions in a region filled with a neutral gas. The neutral gas is of such density, moreover, that the collision frequency of electrons and ions with atoms is about the same as their wave frequencies in coordinate systems moving with the drift velocities.

To describe the oscillation process we shall make use of Vlasov's[1] hydrodynamic approximation to the equations of motion, with an additional term to allow for collisions on the following model. We assume that in every collision with a neutral atom a charged particle loses its excess momentum due to the superimposed oscillation. If this excess momentum is directed against the drift velocity, the charged particle will be accelerated in the drift direction by the collision, and vice versa.

Let Δn denote the totality of electrons with velocity $u_0 + u_1$. During a time Δt some of them ($\tilde{q}_1\Delta t$) will collide with neutral atoms ($\tilde{q}_1$ is the fraction of the electron gas that collides with neutral atoms per unit time). We assume that in the collision the electrons lose a fraction, κ_1, of their excess momentum. Thus, the total momentum change is $\Delta p_1 = -\tilde{q}_1\kappa_1 m_1 u_1 \Delta n \Delta t$. If we set $\tilde{q}_1\kappa_1 = q_1$, then the "friction" per unit mass due to collisions with the neutral gas is $-q_1u_1$, and, correspondingly, $-q_2v_1$ for the ion gas. We assume that the wave oscillations are small and take the direction from the cathode to the anode as positive.

The basic linearized equations for one-dimensional flow are as follows:

$$\frac{\partial u_1}{\partial t} + u_0\frac{\partial u_1}{\partial x} = -\frac{v_{T_1}^2}{\rho_0}\frac{\partial \rho_1}{\partial x} + \frac{e_1}{m_1}E_1 - q_1u_1,$$
$$\frac{\partial v_1}{\partial t} - v_0\frac{\partial v_1}{\partial x} = -\frac{v_{T_2}^2}{d_0}\frac{\partial d_1}{\partial x} + \frac{e_2}{m_2}E_1 - a_2v_1; \tag{1}$$

$$\rho_0\frac{\partial u_1}{\partial x} + u_0\frac{\partial \rho_1}{\partial x} = -\frac{\partial \rho_1}{\partial t}, \qquad d_0\frac{\partial v_1}{\partial x} - v_0\frac{\partial d_1}{\partial x} = -\frac{\partial d_1}{\partial t}, \tag{2}$$

$$\frac{\partial E_1}{\partial x} = 4\pi\left(\frac{e_1}{m_1}\rho_1 + \frac{e_2}{m_2}d_1\right). \tag{3}$$

Here (1) is the Bernoulli-Euler equation, (2) the equation of continuity for the electron and ion gas respectively, and (3) Poisson's equation.

In these equations u_0 and v_0 represent the drift velocities, $v_{T_1} = \sqrt{3kT_1/m_1}$ and $v_{T_2} = \sqrt{3kT_2/m_2}$ the sound velocities, u_1 and v_1 the velocity perturbations, ρ_1 and d_1 the density perturbations, T_1 and T_2 the temperature, e_1 and e_2 the charge, m_1 and m_2 the mass of the electron and ion gas respectively, and q_1, q_2 the collision rate between charged and neutral particles multiplied by the average fractional loss of excess momentum.

Since we are concerned with the propagation of a plane wave with a given frequency ω, we seek solutions of the form

$$\rho_1 = \rho_1^0 e^{i(\omega t + kx)}, \qquad d_1 = d_1^0 e^{i(\omega t + kx)}. \tag{4}$$

Proceeding in the usual way we obtain the dispersion equation:

$$\omega_{01}^2 / [(\omega + u_0k)^2 - v_{T_1}^2k^2 - iq_1\omega - iq_1u_0k]$$
$$+ \omega_{02}^2 / [(\omega - v_0k)^2 - v_{T_2}^2k^2 - iq_2\omega + iq_2v_0k] = 1, \tag{5}$$

where

$$\omega_{01}^2 = 4\pi (e_1/m_1)^2 \rho_0, \qquad \omega_{02}^2 = 4\pi (e_2/m_2)^2 d_0.$$

For frequencies, wave numbers, and drift velocities that satisfy the relations

$$\frac{\omega}{\omega_{01}}, \frac{\omega}{\omega_{02}}, \frac{|k|v_{T_1}}{\omega_{01}}, \frac{|k|v_{T_2}}{\omega_{02}}, \frac{|k|u_0}{\omega_{01}}, \frac{|k|v_0}{\omega_{02}}, \frac{q_1}{\omega_{01}}, \frac{q_2}{\omega_{02}} \ll 1, \tag{6}$$

we can neglect fourth-order terms in comparison with those of the second order.

Equation (5) can be reduced to the form

$$\omega^2 + 2A\omega k + Bk^2 - iDk - iCk = 0, \tag{7}$$

$$A = u_0 m_1/m_2 - v_0, \qquad B = v_0^2 - v_{T_2}^2 + (u_0^2 - v_{T_1}^2) m_1/m_2,$$
$$C = q_1 m_1/m_2 + q_2, \qquad D = q_1 u_0 m_1/m_2 - q_2 v_0,$$

where we have used the fact that $\omega_{01}^2/\omega_{02}^2 = m_2/m_1$, $N_1 = N_2$, and $m_2 \gg m_1$. Substituting $k = \alpha + i\beta$ and equating the coefficients in the real and imaginary parts to zero, we obtain

$$\beta = \tfrac{1}{2}(\alpha D + \omega C)/(\alpha B + \omega A). \tag{8}$$

The dispersion equation for α appears as

$$\omega^2 + 2A\omega\alpha + B\alpha^2 = \frac{B}{4}\left(\frac{\alpha D + \omega C}{\alpha B + \omega A}\right)^2 - \frac{D}{2}\left(\frac{\alpha D + \omega C}{\alpha B + \omega A}\right). \tag{9}$$

Equation (9) has two real roots α_1 and α_2, with a negative sign for a wave propagating from the cathode to the anode and a positive sign for one from the anode to the cathode, while β_1 and β_2 represent the attenuation coefficients of the corresponding waves. If $|D| > |\omega C/\alpha|$ and $|B| > |\omega A/\alpha|$ then β_1 and β_2 have the same sign when α_1 and α_2 are of unlike sign. When $\beta_{1,2} < 0$, the wave from the anode to the cathode decreases exponentially in amplitude, while one from the cathode to the anode increases (the equations are valid as long as the linearization is). This anisotropy is due mainly to the presence of the coefficient D, i.e., to collisions with the atoms at a constant drift velocity. The mechanism responsible for the increase in the wave amplitude can probably be discovered only by kinetic analysis. It should be noted merely that the mechanisms of increase in longitudinal oscillations in the plasma currents, investigated by Filimonov,[2] have nothing to do with collisions.

The oscillations discussed above are of an electroacoustical nature, as can be seen if one obtains the relationship of the wave amplitude on the carrier concentrations.

Equation (9) explains the experimentally observed anisotropy[3] in the propagation of electroacoustical waves in a gas discharge plasma, and yields for the phase velocity of the wave a value that coincides with the experimental value.

The following are the specific data for one of the experiments[3]: a discharge with $i = 30$ ma, $E_0 = 2$ v/cm, and $T_1 = 2.05 \times 10^{5\circ}$ K was ignited in a He-filled tube with a radius of 1.5 cm and $p = 0.01$ mm Hg. Induced oscillations with a frequency $\nu = 250$ kc/sec were found to move only from the cathode to the anode. Under these conditions (see Engel's book[4]) $u_0 = 1.18 \times 10^8$ cm/sec $v_0 = 1.20 \times 10^5$ cm/sec, the cross section for electron collisions with helium atoms is $\sigma_1 = 2.53 \times 10^{-16}$ cm^2 while that of ions with helium atoms is $\sigma_2 = 10 \times 10^{-16}$ cm^2. The density of the neutral gas is $N_0 = 3.31 \times 10^{14}$ atoms/cm^3 at 20° C and $T_2 \approx 2000°$K.

On the assumption that the electrons and ions have Maxwellian distributions, the fraction of electrons colliding with neutral atoms per unit time was found to be

$$\tilde{q}_1 = \frac{\sigma_1 N_0 v_{\mathrm{av}}^2}{\sqrt{\pi}\, u_0}\left[\frac{u_0}{v_{\mathrm{av}}}\exp\left\{-\left(\frac{u_0}{v_{\mathrm{av}}}\right)^2\right\} + \left(\frac{1}{2} + \frac{u_0^2}{v_{\mathrm{av}}^2}\right)\sqrt{\pi}\,\Phi\left(\frac{u_0}{v_{\mathrm{av}}}\right)\right], \tag{10}$$

where

$$\Phi(x) = \int_0^x e^{-t^2} dt, \qquad v_{\mathrm{av}} = \sqrt{\frac{2kT_1}{m_1}}.$$

The formula for $\tilde{q}_2$ is similar.

Let us assume that in collisions with the walls, just as in collisions with neutral atoms, the charged particles lose the excess momentum acquired from the wave. For the fraction of particles colliding per unit time at the point (x, t), let us now take the ratio of the number of particles that have collided with the neutral gas and with the wall to the total number of particles in a volume contained between two sections perpendicular to the tube axis.

The result is that the collisions with the wall add a term $v_{\mathrm{av}}/R\sqrt{\pi}$ to Eq. (10) (R is the radius of the tube). An analogous term is also added to the formula for $\tilde{q}_2$.

Under the above conditions, assuming $\kappa_1 = \kappa_2 = 1$, we find that $q_1 = 1.19 \times 10^8$ sec^{-1} and $q_2 = 2.20 \times 10^5$ sec^{-1}. A graphical solution of Eq. (9) yields

$$\alpha_1 = -0.48\,\mathrm{cm}^{-1}, \qquad \beta_1 = -0.05\,\mathrm{cm}^{-1},$$
$$\alpha_2 = 0.45\,\mathrm{cm}^{-1}, \qquad \beta_2 = 0.12\,\mathrm{cm}^{-1}.$$

Hence the phase velocity of the wave is

$$V_{\mathrm{ph}_1} = 32{,}1 \cdot 10^5\ \mathrm{cm/sec}.$$

Since the wave is described by the factor $\exp(i\omega t + i\alpha x - \beta x)$, we see that the first solution describes a wave with slowly increasing amplitude propagating from cathode to anode. No increase in amplitude was observed experimentally, but the phase velocity of the observed wave[3] (31.8×10^5 cm/sec) agrees quite well with the computed value.

The second solution applies to a rapidly attenuated wave from the anode to the cathode, which was not observed experimentally. If the boundaries are ignored, the anisotropy under the same conditions and in the same qualitative circumstances will be less.

In conclusion I wish to thank Prof. A. A. Vlasov for his interest and critical review of this paper and A. A. Zaĭtsev for his aid with the experimental comparison.

[1] A. A. Vlasov, Теория многих частиц (The Many Body Problem), Gostekhizdat, 1950.

[2] G. F. Filimonov, Радиофизика и электроника (Radio Engineering and Electronics) 4, 75 (1959).

[3] Vasil'eva, Zaĭtsev, and Andryukhina, Izv. Akad. Nauk SSSR, Ser. Fiz. 23, 995 (1959), Columbia Tech. Transl. p. 982.

[4] A. von Engel, Ionized Gases, (Clarendon Press, Oxford, 1955).

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