ELECTRON SCATTERING IN THE THOMAS-FERMI MODEL

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We calculate the differential cross section for electron scattering by atoms in the Thomas-Fermi approximation with the Latter potential.[1] Results of numerical integration are presented for Ne, Si, and S, and these are compared with results obtained by the Hartree method.[3]

SEVERAL authors have used numerous different kinds of statistical potentials to calculate atomic energy levels and the corresponding eigenfunctions. Latter[1] and Gambos[2] have given complete reviews of the literature concerned with these problems.

According to Latter the best results for the eigenfunctions and eigenvalues are obtained when one uses a statistical potential of the form

$$V(r)=\begin{cases}-(Ze^2/r)\,\Phi(x), & Z\Phi(x)>1\\ -e^2/r, & Z\Phi(x)<1,\end{cases} \qquad (1)$$

where $\Phi(x)$ is the well known Thomas-Fermi function for a neutral atom. He has shown that this potential gives more accurate values for the atomic energy levels than can be obtained in a calculation using the Hartree method.[3] It is thus of interest to calculate the differential cross section for elastic electron scattering using this potential, and to compare the result so obtained with the result of a numerical calculation by the Hartree method in the first Born approximation.

In the first Born approximation the differential cross section for the potential of (1) is given by the expression

$$I(\vartheta)=\frac{3^{8/3}h^4Z^{2/3}}{2^{24/3}\pi^{4/3}e^4m^2P^2}\left[\int_0^{x_0}\Phi(x)\sin Px\,dx+\frac{1}{Z}\int_{x_0}^{\infty}\sin Px\,dx\right]^2. \qquad (2)$$

The quantity P appearing in (2) is defined by the formula

$$P=\frac{3^{8/3}h\nu\sin(\vartheta/2)}{2^{7/3}\pi^{1/3}e^2Z^{1/3}}=\frac{3^{8/3}h^2}{2^{7/3}\pi^{1/3}e^2mZ^{1/3}}\,\frac{\sin(\vartheta/2)}{\lambda}, \qquad (3)$$

where ϑ is the scattering angle, v is the electron velocity, and λ is the corresponding wave length. The parameter x_0 in Eq. (2) is obtained from the relation $Z\Phi(x_0)=1$.

Noting that

$$\int_{x_0}^{\infty}\sin Px\,dx=\lim_{\alpha\to0}\int_{x_0}^{\infty}e^{-\alpha x}\sin Px\,dx, \qquad (4)$$

we find that $I(\vartheta)$ is given by

$$I(\vartheta)=\frac{3^{8/3}h^4Z^{2/3}}{2^{24/3}\pi^{4/3}e^4m^2P^2}\left[\int_0^{x_0}\Phi(x)\sin Px\,dx+\frac{\cos Px_0}{ZP}\right]^2. \qquad (5)$$

Numerical values of x_0 and $\Phi(x)$ are found in the tables of Kobayashi and Taima.[4]

The Table gives the dependence of $I(\vartheta)/5.66\times10^{-20}$ on $\lambda^{-1}\sin(\vartheta/2)\times10^{-8}$ for three values of Z, as obtained from numerical calculations based on Eq. (5). For comparison we give also the results obtained using the Hartree method.[3] Detailed comparison shows that the two methods give equal results for large Z, whereas they differ considerably for low Z.

$\frac{10^{-8}}{\lambda}\sin\frac{\vartheta}{2}$, $\frac{1}{cm}$	$\frac{10^{20}}{5,66}I(\vartheta)$, cm²					
	Ne		Si		S	
	Hartree method	Equation (5)	Hartree method	Equation (5)	Hartree method	Equation (5)
0.1	4900	26708	70000	38346	57600	44555
0.2	3900	6315	13200	11053	17400	13707
0.3	2180	2936	4150	5189	6130	6428
0.4	1220	1465	1830	2468	2600	3021
0.5	700	716	1000	1206	1340	1487
0.6	420	375	610	660	770	832
0.7	256	221	400	412	480	527
0.8	156	146	275	275	324	350
0.9	106	101	188	185	222	233
1.0	72	70	130	125	160	141
1.1	49	49	94	89	116	97

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[1] R. Latter, Phys. Rev. 99, 510 (1955).

[2] P. Gambos, Die statistische Theorie des Atoms und ihre Anwendungen, Springer-Wien, 1949.

[3] N. Mott and H. Massey, Theory of Atomic Collisions, Oxford 1949.

[4] S. Kobayashi and T. Taima, Table of the Exact Values of the T. F. Function, Memoires of the Faculty of Liberal Arts, Education Kagawa, Japan, 1957.

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