DECAY OF A PLASMON AT ABSOLUTE ZERO

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We discuss the effect of phonon interactions between electrons on the decay of a plasmon in a solid (in the isotropic model).

1. Although plasmon oscillations in a solid have been considered in a number of papers (for example, references 1-3), processes involving phonons have not been discussed up to the present. In this paper it will be shown that the phonon interaction between electrons changes the frequency and relaxation time of a plasmon. The methods of quantum field theory[3-5] provide tools for examining this question. As is shown in references 3 — 5, the poles of the single particle Green's function give the energy and relaxation time of the quasi-particles. The Green's function of interest is that for the photon, and is given by

$$D_{\mu\nu}(x-x') = i\langle T(A_\mu(x)A_\nu(x'))\rangle \tag{1}$$

where the indices μ and ν run from 1 to 4 and the $A_\mu(x)$ are the electromagnetic field operators in the Heisenberg representation. The average is taken over the ground state of the system. This function describes the interaction between the photon and the surrounding medium, and satisfies the equation

$$(\Box + P)D(x) = -\delta(x) \tag{2}$$

(see reference 5, for example). Here P is the polarization operator, which is related to the compact photon self-energy part Π through $P = i\Pi$.

The homogeneous equation

$$(\Box + P)\langle 0|A_\mu|r\rangle = 0 \tag{3}$$

is satisfied by the wave function corresponding to the given excitation (or, more precisely, by the matrix element $\langle 0 \mid A_\mu \mid r\rangle$ for the transition from the ground state to a single-particle excited state).

Since we are interested only in longitudinal waves, we neglect retardation effects and write these equations as

$$(\Delta + P)D(x) = -\delta(x), \tag{2'}$$

$$(\Delta + P)\langle 0|A_0|r\rangle = 0, \tag{3'}$$

or, in the momentum representation,

$$[\mathbf{k}^2 - P(\mathbf{k},\omega)]D(\mathbf{k},\omega) = 1. \tag{2''}$$

The dispersion relation in which we are interested takes on the form

$$\mathbf{k}^2 - P(\mathbf{k},\omega) = 0. \tag{4}$$

2. Feynman's well known rules can be used to write the polarization operator as

$$P(\mathbf{k},\omega) = \frac{e^2}{i(2\pi)^4}\,\mathrm{Sp}\int G\left(p+\frac{k}{2}\right)\Gamma(p,k)\,G\left(p-\frac{k}{2}\right)d^4p, \tag{5}$$

Here, $\Gamma(p, k)$ is the vertex part, while $G(p)$ is the electron Green's function; in the absence of interactions[4], it is given by

$$G_0(p) = \frac{1}{\varepsilon_p^0 - \varepsilon - i\Delta(p)}, \qquad \Delta(p) \to \begin{cases} +0, & p > p_0 \\ -0, & p < p_0. \end{cases} \tag{6}$$

Carrying out the calculation to first order in e^2, we obtain for $v_0 k \ll \omega_0$:

$$P_0(\mathbf{k},\omega) = \frac{e^2}{i(2\pi)^4}\,\mathrm{Sp}\int G_0\left(p+\frac{k}{2}\right)G_0\left(p-\frac{k}{2}\right)d^4p$$
$$= -\frac{4e^2}{(2\pi)^3}\int\limits_{p<p_0}\frac{(\varepsilon^0_{\mathbf{p}+\mathbf{k}} - \varepsilon^0_{\mathbf{p}})\,d^3p}{(\varepsilon^0_{\mathbf{p}+\mathbf{k}} - \varepsilon^0_{\mathbf{p}})^2 - \omega^2 - i0} \approx \frac{\omega_0^2}{\omega^2}k^2\left[1 + \left(\frac{v_0 k}{\omega_0}\right)^2\right], \tag{7}$$

where ω_0 is the plasma frequency, $v_0^2 = \frac{3}{5}p_0^2$, $m_e = \hbar = 1$.

Substitution of (7) into (4) leads to the well known dispersion relation

$$\omega^2 = \omega_0^2 + v_0^2 k^2. \tag{8}$$

The frequency ω is real, with the consequence that for small k the plasmon does not decay, which is a well known result (see, for example, references 3 and 6). In this approximation, the plasmon decays for $k = k_m$, as determined by the poles of the expression under the integral sign in (7),

$$\omega(k_m) = \tfrac{1}{2}k_m^2 + k_m p_0. \tag{9}$$

If higher order diagrams were to be taken into account, the qualitative picture would not be changed.

3. Phonon interactions between electrons can lead to decay of the plasmon. This interaction would be described by the Hamiltonian

$$H_{int} = \sum_{q<q_m} \alpha_q a^+_{\mathbf{p}+\mathbf{q}} a_{\mathbf{p}} (b_{\mathbf{q}} + b^+_{-\mathbf{q}}), \tag{10}$$

where the $a^+_{\mathbf{p}}(a_{\mathbf{p}})$ and $b^+_{\mathbf{q}}(b_{\mathbf{q}})$ are creation (annihilation) operators for electrons and phonons; $\alpha_q^2 = \lambda_0 \pi^2 \omega_q^0 / p_0$; $\omega_q^0 = c_0 q$; $\lambda_0 \leq 1$; q_m is the maximal phonon momentum and c_0 is the speed of sound.

The Green's function for a free phonon is (see, for example, reference 7),

$$K_0(q) = \alpha_q^2 \left\{ \frac{1}{\omega_q^0 - \omega - i0} + \frac{1}{\omega_q^0 + \omega - i0} \right\}. \tag{11}$$

It is not difficult to find the effect of phonons on the dispersion relation for the plasmon. We calculate the polarization operator only to first order in e^2. Then the function $G(p)$ in (5) should be taken as the propagation function for an electron interacting with phonons; the difference between $\Gamma(p, k)$ and 1 will also be due to phonon interactions.

Migdal has shown[7] that when phonon interactions are included, $G(p)$ differs from $G_0(p)$ only for $|\epsilon - \epsilon_0| \sim c_0 q_m$ and $|p - p_0| \ll p_0$. On the other hand, it is easy to see that $G(p)$ contributes to the polarization operator of the plasmon only for $|p - p_0| \sim k \ll p_0$ and $|\epsilon - \epsilon_0| \sim \omega_0 \gg c_0 q_m$. Hence in calculating $P(\mathbf{k}, \omega)$ we can use perturbation theory, i.e., we can expand in powers of the photon interaction constant and keep only the first order terms, which are those corresponding to the diagrams shown in Fig. 1.

FIG. 1

The first two terms give small contributions to the imaginary part and affect only the frequency of the plasmon; the principal contribution comes from the third diagram. The imaginary part can be written

$$\operatorname{Im} P_1(\mathbf{k}, \omega) = \frac{e^2}{i(2\pi)^4} \operatorname{Re} \operatorname{Sp} \int G_0\left(p + \frac{k}{2}\right) \Gamma_1(p, k)\, G_0\left(p - \frac{k}{2}\right) d^4 p, \tag{12}$$

where the vertex part $\Gamma_1(p, k)$ is given by Fig. 2.

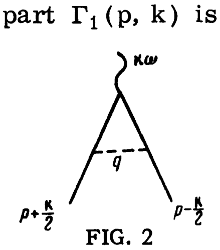

FIG. 2

Assuming that $\Gamma_1(p, k)$ has no poles, we find that

$$\operatorname{Im} P_1(\mathbf{k}, \omega) = -\frac{2e^2}{(2\pi)^3} \left[\int\limits_{\substack{p<p_0 \\ |\mathbf{p}+\mathbf{k}|>p_0}} \frac{\operatorname{Im} \Gamma_1\left(\mathbf{p} + \frac{\mathbf{k}}{2}, \varepsilon_p^0 + \frac{\omega}{2}; \mathbf{k}, \omega\right)}{\varepsilon^0_{\mathbf{p}+\mathbf{k}} - \varepsilon^0_{\mathbf{p}} - \omega} d^3 p \right.$$

$$\left. + \int\limits_{\substack{p<p_0 \\ |\mathbf{p}+\mathbf{k}|>p_0}} \frac{\operatorname{Im} \Gamma_1\left(-\mathbf{p} - \frac{\mathbf{k}}{2}, \varepsilon_p^0 - \frac{\omega}{2}; \mathbf{k}, \omega\right)}{\varepsilon^0_{\mathbf{p}} - \varepsilon^0_{\mathbf{p}+\mathbf{k}} + \omega} d^3 p \right]$$

$$= \frac{2e^2}{(2\pi)^6} \int\limits_{\substack{p<p_0 \\ |\mathbf{p}+\mathbf{k}|>p_0}} d^3 p \int\limits_{\substack{p_1<p_0 \\ |\mathbf{p}_1-\mathbf{p}|<q_m}} d^3 p_1 \alpha^2_{\mathbf{p}-\mathbf{p}_1} \{ [(\varepsilon^0_{\mathbf{p}+\mathbf{k}} - \varepsilon^0_{\mathbf{p}} - \omega)$$

$$\times (\varepsilon^0_{\mathbf{p}_1} - \varepsilon^0_{\mathbf{p}} - \omega^0_{\mathbf{p}-\mathbf{p}_1} + i0)\, (\varepsilon^0_{\mathbf{p}_1+\mathbf{k}} - \varepsilon^0_{\mathbf{p}} - \omega^0_{\mathbf{p}-\mathbf{p}_1} - \omega)]^{-1}$$

$$+ [(\varepsilon^0_{\mathbf{p}} - \varepsilon^0_{\mathbf{p}+\mathbf{k}} + \omega)(\varepsilon^0_{\mathbf{p}_1} - \varepsilon^0_{\mathbf{p}} - \omega^0_{\mathbf{p}-\mathbf{p}_1} + i0)$$

$$\times (\varepsilon^0_{\mathbf{p}_1+\mathbf{k}} - \varepsilon^0_{\mathbf{p}} - \omega^0_{\mathbf{p}-\mathbf{p}_1} + \omega)]^{-1} \}$$

$$\approx -i \begin{cases} \dfrac{\lambda_0 \pi}{80 c_0^2} k^4, & k < 2c_0 \\ \lambda_0 \pi c_0 \left(\dfrac{\varkappa}{2p_0}\right)^3 k, & 2c_0 \ll k \ll p_0, \end{cases}$$

where $\kappa = \min\{q_m, 2p_0\}$.

Upon substituting the polarization operator $P = P_0 + P_1$ in (4), we find that the decay constant is

$$\gamma \approx \begin{cases} \omega_0 \dfrac{\pi \lambda_0}{40} \left(\dfrac{k}{2c_0}\right)^2, & k < 2c_0 \\ \omega_0 \dfrac{\pi \lambda_0}{4} \left(\dfrac{\varkappa}{2p_0}\right)^3 \dfrac{2c_0}{k}, & 2c_0 \ll k \ll p_0, \end{cases} \tag{13}$$

which is fairly large. From the third diagram in Fig. 1, it is clear that this decay constant may be considered to be due to the decay of a plasmon into an electron and a hole, with the emission of a phonon. This differs from the decay described by Landau and which may be considered as the inverse of the Vavilov-Cerenkov effect.

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