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POLARIZATION OF INTERNAL-CONVERSION ELECTRONS EMITTED AFTER BETA DECAY OF ORIENTED NUCLEI

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The correlation of the polarization of conversion electrons and β particles emitted in the decay of oriented nuclei is considered. The calculation is carried out with allowance for the electric field of the nucleus. Formulas are derived for the angular distribution, and longitudinal and transverse polarization of conversion electrons from any shell with an arbitrary multipole mixture. The numerical results for the L_I, L_{II}, and L_{III} shells refer to M1 and E2 multipoles or their mixture, and are presented in the form of tables of the b_{kq}^R coefficients. These coefficients also determine the polarization of conversion electrons emitted after β decay of nonoriented nuclei. The correlation of the β and conversion electrons can be employed to verify the invariance of the β interaction under time inversion.

UNLIKE the β decay of nonoriented nuclei, the daughter nucleus obtained after the β decay of an oriented nucleus would be polarized even if parity in β decay were conserved. The presence of an initial orientation of the nucleus leads to an anisotropy of the angular distribution of the internal-conversion electrons. The polarization of internal-conversion electrons following the β decay of nonpolarized nuclei was considered in a number of works.[1-3] The present article supplements the results of these papers by considering mixed conversion transitions from three L subshells, and pure transitions from the L_{III} subshell.

The investigation of the polarization correlation of β particles and the following conversion electrons in the β decay of oriented nuclei can furnish more complete information on the β-interaction constants.

We consider the

$$I_i \xrightarrow{\beta} I_1 \xrightarrow{\text{c. e.}} I_2$$

cascade (c.e. is a conversion electron). We choose the direction of the spin $\mathbf{I}_i$ of the oriented nucleus as the z axis, we denote the momentum of the emitted β particle with $\mathbf{p}$, and the unit vector along the direction of emission of the conversion electron with $\mathbf{n}$. The correlation function of the two successive nuclear emissions, when the intermediate state is not perturbed, can be written in the form

$$P_{\xi\xi'} = \sum{}' \rho_{M_i} \langle I_1 M_1 \mathbf{p}\sigma | H_\beta | I_i M_i \nu \rangle$$
$$\times \langle I_1 M_1' \mathbf{p}\sigma' | H_\beta | I_i M_i \nu \rangle^* \, \mathfrak{M}^{\xi}_{I_1 M_1 I_2 M_2} \mathfrak{M}^{\xi'}_{I_1 M_1' I_2 M_2}, \qquad (1)$$

where the summation is over all magnetic quantum numbers, and the prime denotes averaging over all unobserved quantities. $\mathfrak{M}_{\dots}$ is the matrix element of the conversion process, ξ and ξ' characterize the polarization of the conversion electron in its c.m.s., and ρ_{M_i} is the density matrix of the initial state. The Hamiltonian of the β interaction is the same as in reference 4, represented in the multipole form analogous to that in reference 5. The electron is described in (1) by its momentum $\mathbf{p}$, and its spin component along $\mathbf{p}$. The wave function of the emitted electron is a solution of the Dirac equation which represents at infinity a superposition of a plane and converging spherical wave:

$$|\mathbf{p}\sigma\rangle = (4\pi)^{1/2} \sum_{\varkappa m} i^{l_2} (2l+1)^{1/2} C(l_2 {}^1/_2 0\sigma;\ j\sigma) D^j_{m\sigma}(z \to p)$$
$$\times \exp[-i\Delta(\varkappa)] \, |\varkappa m\rangle, \qquad (2)$$

$$|\varkappa m\rangle = \begin{pmatrix} -if_\varkappa(r)\,\chi^m_{-\varkappa}(\mathbf{r}) \\ g_\varkappa(r)\,\chi^m_\varkappa(\mathbf{r}) \end{pmatrix}. \qquad (3)$$

The state of the electron is characterized by the magnetic quantum number m, and by the eigenvalue κ of the operator $\beta(\boldsymbol{\sigma}\mathbf{L}+1)$.

The conversion matrix element can be written in the form

$$\mathfrak{M} = \sum_{LM} \int \Psi_2^* Q^*(\pi LM) \Psi_1 d\tau \int \psi_2 B(\pi LM) \psi_1 d\tau \qquad (4)$$

where Ψ_2 and Ψ_1 are the wave functions of the nucleus after and until conversion, ψ_2 and ψ_1 are the corresponding wave functions of the electron, and π indicates the transition parity which is

$(-1)^L$ for electric and $(-1)^{L+1}$ for magnetic multipoles. The electromagnetic interaction is invarient under time inversion; the nuclear matrix elements can, therefore, be considered real without loss of generality.

For an electric multipole we have

$$B(ELM) = \left[\frac{L}{L+1}\right]^{1/2} h_L(\omega r) i Y_{LM}(\mathbf{n}) + \left[\frac{2L+1}{L+1}\right]^{1/2} h_{L-1}(\omega r)\, \boldsymbol{\alpha}\mathbf{Y}_{L,\,L-1,\,M}(\mathbf{n}),$$

and for a magnetic multipole

$$B(MLM) = h_L(\omega r)\boldsymbol{\alpha}\mathbf{Y}_{LLM}(\mathbf{n}),$$

where $\omega = E_1 - E_2$ is the energy of the conversion transition, α is the Dirac matrix, and h_L is a spherical Hankel function of the first kind. The initial wave function of the electron is normalized to unity in configuration space, and the final to a δ function of the energy. We can then write

$$\psi_2 = 4\pi \sum_{\varkappa_2 M_2} \sqrt{\frac{E_2}{p_2}} [\chi_{\varkappa_2}^{M_2}(\mathbf{n})]_\xi^* \begin{pmatrix} if_{\varkappa_2}\chi_{-\varkappa_2}^{M_2}(\mathbf{n}) \\ g_{\varkappa_2}\chi_{\varkappa_2}^{M_2}(\mathbf{n}) \end{pmatrix} i^{l_2} \exp[-i\Delta(\varkappa_2)]. \quad (5)$$

The electron matrix elements in (4), after integration over the angular variables, take on the form

$$B_{21}^{\xi}(\pi Lm) = \int \psi_2^* B \psi_1 d\tau$$
$$= \sum_{\varkappa_2 M_2 \mu_2} (-1)^L \left[\frac{E_2}{p_2}(2L+1)(2j_2+1)\right]^{1/2} \times \begin{pmatrix} j_2 & L & j_1 \\ -M_2 & m & M_1 \end{pmatrix} \begin{pmatrix} l_2 & 1/2 & j_2 \\ \mu_2 & \xi & -M_2 \end{pmatrix} \times \begin{pmatrix} j_1 & j_2 & L \\ -1/2 & 1/2 & 0 \end{pmatrix} Y_{l_2\mu_2}(\mathbf{n}) \mathscr{E}_{\varkappa_2}(\pi L),$$

$$\mathscr{E}_{\varkappa_2}(ML) = [L(L+1)]^{-1/2} \exp(i\delta_{\varkappa_2})(\varkappa_1+\varkappa_2)(R_1+R_2),$$

$$\mathscr{E}_{\varkappa_2}(EL) = [L(L+1)]^{-1/2} \exp(i\delta_{\varkappa_2})[(R_3+R_4-R_5+R_6)L - (\varkappa_2-\varkappa_1)(R_5+R_6)],$$

$$\delta_{\varkappa_2} = \Delta(\varkappa_2) - \pi(l_2-1)/2, \quad (6)$$

The radial integrals R_n are defined in the book by Rose,[6] j_1 is the angular momentum of the electron before the conversion, and L is the multipolarity of the transition.

The polarization of the conversion electrons is determined by

$$\langle\boldsymbol{\sigma}\rangle = \mathrm{Sp}\, P\boldsymbol{\sigma}/\mathrm{Sp}\, P, \quad (7)$$

where $\frac{1}{2}\langle\sigma\rangle$ is the mean value of the electron spin in the rest system. The Pauli matrices are conveniently expressed in terms of the coefficients of the vector sum:

$$\sigma_{\xi\xi'}^{m} = \sqrt{6}(-1)^{1/2-\xi} \begin{pmatrix} 1/2 & 1/2 & 1 \\ -\xi' & \xi & -m \end{pmatrix}. \quad (8)$$

Employing formulas (1) — (8), and averaging over the direction of emission of the neutrino and the polarization of the β particle, we obtain

$$\boldsymbol{\zeta} = \mathrm{Sp}\, P\boldsymbol{\sigma} = \sum_{\substack{\nu k R \pi\pi' \\ L,\, L_1 \leqslant L',\, L_1'}} (-)^{L_1'+R+\nu} f_\nu(I_i) \left[\frac{2\nu+1}{2R+1}\right]^{1/2} X \begin{vmatrix} I_i & I_1 & L_1 \\ I_i & I_1 & L_1' \\ \nu & k & R \end{vmatrix}$$
$$\times b^R(L_1L_1')[F_{R\nu k}(\mathbf{p},\mathbf{n})]^q (2-\delta_{\pi\pi'}\delta_{LL'}) N(\pi L) N(\pi L')$$
$$\times [I_0^{\varkappa_1}(\pi L) I_0^{\varkappa_1}(\pi L')]^{1/2} F_k(LL'I_2I_1) b_{kq}^{\varkappa_1}(\pi L\pi' L') + \text{c.c.} \quad (9)$$

$$\mathrm{Sp}\, P = \sum_{\substack{\nu k R \pi\pi' \\ L,\, L_1 \leqslant L',\, L_1'}} (-)^{L_1'+R+\nu} \left[\frac{1+(-1)^k}{2}\right] f_\nu(I_i) \left[\frac{2\nu+1}{2R+1}\right]^{1/2}$$
$$\times X \begin{vmatrix} I_i & I_1 & L_1 \\ I_i & I_1 & L_1' \\ \nu & k & R \end{vmatrix} b^R(L_1L_1') F_{R\nu k}(\mathbf{p},\mathbf{n})$$
$$\times (2-\delta_{\pi\pi'}\delta_{LL'}) N(\pi L) N(\pi L')$$
$$\times [I_0^{\varkappa_1}(\pi L) I_0^{\varkappa_1}(\pi L')]^{1/2} F_k(LL'I_2I_1) b_{kq}^{\varkappa_1}(\pi L\pi' L') + \text{c.c.} \quad (10)$$

Here F_k are the "geometric factors" that characterize the electromagnetic radiation; they are tabulated in reference 7. We note that

$$F_0(LL'I_2I_1) = \delta_{LL'}.$$

We describe the initial orientation of the nucleus by means of the statistical tensors

$$f_\nu(I_i) = \sum_{M_i} (-1)^{I_i - M_i} C(I_i I_i \nu;\ M_i - M_i) \rho_{M_i}.$$

b_k and b_{kq} are parameters of the conversion electron for the correlation of the directions and for the polarization, respectively:

$$b_k^{\varkappa_1}(\pi L\pi' L') = (-1)^{L+L'} \left[\frac{L(L+1)L'(L'+1)}{(2L+1)(2L'+1)}\right]^{1/2} \frac{M_k^{\varkappa_1}(\pi L\pi' L')}{[D_0^{\varkappa_1}(\pi L) D_0^{\varkappa_1}(\pi' L')]^{1/2}},$$

$$b_{kq}^{\varkappa_1}(\pi L\pi' L') = (-1)^{L+L'+1} \left[\frac{L(L+1)L'(L'+1)}{(2L+1)(2L'+1)}\right]^{1/2} \times \frac{M_{kq}^{\varkappa_1}(\pi L\pi' L')}{[D_0^{\varkappa_1}(\pi L) D_0^{\varkappa_1}(\pi' L')]^{1/2}}, \quad (11)$$

$$M_{kq}^{\varkappa_1}(\pi L\pi' L') = \left[\frac{L(L+1)L'(L'+1)}{(2L+1)(2L'+1)}\right]^{-1/2} \begin{pmatrix} L & L' & k \\ 1 & -1 & 0 \end{pmatrix}^{-1}$$
$$\times \sum_{\varkappa_2\varkappa_3} (-1)^{L+L'+j_1+1/2}(2j_2+1)(2j_3+1)$$
$$\times (2K+1)^{-1/2} \mathscr{E}_{\varkappa_2}(\pi L) \mathscr{E}_{\varkappa_3}^*(\pi' L')$$
$$\times C(j_2 j_1 L;\ 1/2 - 1/2) C(j_3 j_1 L';\ 1/2 - 1/2) C(j_2 j_3 K;\ 1/2 - 1/2)$$
$$\times A_q(\varkappa_2\varkappa_3) W(j_2 L j_3 L';\ j_1 K);$$

$$A_q(\varkappa_2\varkappa_3) = \begin{cases} -1 & q=-1, \\ \varkappa_2+\varkappa_3 & q=1, \\ i(\varkappa_2-\varkappa_3) & q=0. \end{cases} \quad (12)$$

The expression for $M_k^{\varkappa_1}$ is obtained from (12) if it is assumed that $A_q = 1$. Furthermore

$$D_0^{\varkappa_1}(\pi L) = \frac{L(L+1)}{2L+1} M_0^{\varkappa_1}(\pi L)$$

$$= \sum_{\varkappa_2} (2j_2+1) |\mathscr{E}_{\varkappa_2}(\pi L)|^2 C^2 (j_2 j_1 L;\ {}^1\!/_2 - {}^1\!/_2)$$

$$I_0^{\varkappa_1}(\pi L) = \frac{E_2}{p_2} \frac{D_0^{\varkappa_1}(\pi L)}{2L+1};$$

the latter value is proportional to the conversion coefficient up to a nonessential factor.

$N(\pi L)$ are the reduced nuclear matrix elements, and b^R are parameters characterizing the β decay. For allowed transitions $b^R(L_1L_1')$ has the form [for $L_1 \neq L_1'$ the values $b^R(L_1L_1') + b^R(L_1'L_1)$ are given]:

$$b^0(00) = [\,|C_S|^2 + |C_S'|^2 + |C_V|^2$$

$$+ |C_V'|^2 \pm 2\,\mathrm{Re}(C_S^* C_V + C_S'^* C_V')\gamma/\varepsilon]\,|M_F|^2,$$

$$b^0(1,1) = -\sqrt{3}[\,|C_T|^2 + |C_T'|^2 + |C_A|^2 + |C_A'|^2$$

$$\pm 2\,\mathrm{Re}(C_T^* C_A + C_T'^* C_A')\gamma/\varepsilon]\,|M_{GT}|^2,$$

$$b^1(0,1) = [2(C_T^* C_S' + C_T'^* C_S - C_A^* C_V' - C_A'^* C_V) \pm 2(-i)$$

$$\times (C_A^* C_S' + C_A'^* C_S - C_T^* C_V' - C_T'^* C_V)\alpha Z/p]\, M_{GT}^* M_F\, p/\varepsilon;$$

$$b^1(1,1) = \pm\sqrt{2}[2(C_T^* C_T' - C_A^* C_A')$$

$$\mp (-i)\,2(C_T^* C_A' + C_T'^* C_A)\alpha Z/p]\,|M_{GT}|^2 p/\varepsilon.$$

Here the upper sign refers to the electron decay, the lower to positron decay, and ϵ is the energy of the electron. The value $M_{GT}^* M_F$ is assumed real. For first-forbidden transitions one can use the most complete results of Morita.[8]

The function $F_{R\nu k}$ determines the angular dependence

$$F_{R\nu k}(\mathbf{p}, \mathbf{n}) = 4\pi \sum_{\mu} C(\nu k R;\ 0\mu) Y_{R\mu}(\mathbf{p}) Y_{k\mu}^*(\mathbf{n}). \quad (13)$$

The function $[F_{R\nu k}]^q$ is obtained from $F_{R\nu k}$ by substituting $(-1)^{\mu} Y_{k-\mu}$ for $Y_{k\mu}^*$, where

$$\mathbf{Y}_{k-\mu}^q(\mathbf{n}) = \begin{cases} \mathbf{Y}_{k-\mu}^{(-1)}(\mathbf{n}) & q=-1 \\ [k(k+1)]^{-1/2}\mathbf{Y}_{k-\mu}^{(1)}(\mathbf{n}) & q=1 \\ -i\,[k(k+1)]^{-1/2}\mathbf{Y}_{k-\mu}^{(0)}(\mathbf{n}) & q=0 \end{cases}$$

On the right we have the spherical vectors[9] which are conveniently represented in a spherical coordinate system:

$$[\mathbf{Y}_{k-\mu}^{(-1)}]_n = Y_{k-\mu},$$

$$[\mathbf{Y}_{k-\mu}^{(+1)}]_\theta = i\,[\mathbf{Y}_{k-\mu}^{(0)}]_\varphi = [k(k+1)]^{-1/2} \frac{\partial}{\partial\theta} Y_{k-\mu}(\mathbf{n}),$$

$$[\mathbf{Y}_{k-\mu}^{(+1)}]_\varphi = -i\,[\mathbf{Y}_{k-\mu}^{(0)}]_\theta = [k(k+1)]^{-1/2} \frac{1}{\sin\theta}\frac{\partial}{\partial\varphi} Y_{k-\mu}(\mathbf{n}). \quad (14)$$

Hence it is seen that it is sufficient to know the $F_{R\nu k}$ in order to obtain the values of $[F_{R\nu k}]^q$ by simple differentiation.

The mixing coefficient of the multipoles L and L′ for γ rays is determined in the following manner:

$$\delta(\pi L \pi' L') = N(\pi' L')/N(\pi L) = \pm[I_\gamma(\pi' L')/I_\gamma(\pi L)]^{1/2},$$

where $I_\gamma(\pi L)$ is the intensity of the pure πL-pole γ radiation. For conversion electrons one can introduce the corresponding mixing parameters

$$\bar{\delta}(\pi L \pi' L') = \left[\frac{c(\pi' L')}{c(\pi L)}\right]^{1/2} \delta(\pi L \pi' L') = \left[\frac{I_0(\pi' L')}{I_0(\pi L)}\right]^{1/2} \delta(\pi L \pi' L'),$$

where $c(\pi L)$ is the conversion coefficient for a πL multipole. Selecting an arbitrary multipole $\pi_0 L_0$ as the standard, we can write the polarization of the conversion electrons for an arbitrary multipole mixture in the form

$$\langle\sigma\rangle = \sum_{\substack{\nu k R \pi\pi'; \\ LL_1 \leqslant L' L_1'}} (-)^{L_1' + R + \nu} f_\nu(I_i) \left[\frac{2\nu+1}{2R+1}\right]^{1/2} X \begin{vmatrix} I_i I_1 L_1 \\ I_i I_1 L_1' \\ \nu\ k\ R \end{vmatrix}$$

$$\times b^R(L_1 L_1')[F_{R\nu k}(\mathbf{p}, \mathbf{n})]^q (2 - \delta_{\pi\pi'}\delta_{LL'})$$

$$\times \delta(\pi_0 L_0;\ \pi L)\bar{\delta}(\pi_0 L_0 \pi L)$$

$$\times F_k(LL' I_2 I_1) b_{kq}^{\varkappa_1}(\pi L \pi' L') + \mathrm{c.c.}\Big\} W^{-1}; \quad (15)$$

$$W = \sum_{\substack{\nu k R \pi\pi'; \\ L, L_1 \leqslant L', L_1'}} (-)^{L_1' + R + \nu} \left[\frac{1 + (-1)^k}{2}\right]$$

$$\times f_\nu(I_i) \left[\frac{2\nu+1}{2R+1}\right]^{1/2} X \begin{vmatrix} I_i I_1 L_1 \\ I_i I_1 L_1' \\ \nu\ k\ R \end{vmatrix}$$

$$\times b^R(L_1 L_1') F_{R\nu k}(\mathbf{p}, \mathbf{n})(2 - \delta_{\pi\pi'}\delta_{LL'})\bar{\delta}(\pi_0 L_0 \pi L)$$

$$\times \bar{\delta}(\pi_0 L_0 \pi' L')\ F_k(LL' I_2 I_1) b_k^{\varkappa_1}(\pi L \pi' L') + \mathrm{c.c.} \quad (16)$$

The quantity W determines the angular distribution of the conversion electrons following the β decay of oriented nuclei.

The angular correlation of the conversion electron from the β decay of an oriented nucleus can be used to verify the invariance of the β interaction under time inversion. One of the methods of verification consists in measuring the upward-downward asymmetry of the β intensities for the correlation of the β particle and the conversion electron (with regard to both direction and polarization) in the decay of oriented nuclei with respect to the plane containing $\mathbf{I}_i$ and $\mathbf{n}$.

If it is assumed that there occurs an AV and TS interaction (the assumption that there is no interference between the AV and STP interactions is in agreement with present-day data on β decay), it follows from (15) and (16) that the asymmetry arises from terms with odd $\nu + R + k$ corresponding to the interference between β-decay matrix elements of various rank (this follows from the properties of the Fano coefficients for odd $\nu + R + k$). All these terms contain the factor

$$\operatorname{Im}(C_T^* C_S' + C_T'^* C_S - C_A^* C_V' - C_A'^* C_V).$$

To observe the contribution of such terms to the polarization of the conversion electrons, it is sufficient to know the dipole polarization of the initial nucleus (the term which contains $F_{111} \sim \mathbf{I}[\mathbf{p} \times \mathbf{n}]$). The absence of such terms in the experiment could serve as a proof of the invariance of the β interaction under time inversion. However, if another combination of the interaction variants occurs (interference of the AV and STP interactions is present), then terms proportional to $\alpha Z/p$ will enter into the correlation of β particles and conversion electrons. These terms may cause an upward-downward asymmetry of the β intensities with respect to the plane containing $\mathbf{I}_i$ and $\mathbf{n}$, even when time parity is conserved; an example is the term

$$\operatorname{Re}(C_A^* C_S' + C_A'^* C_S - C_T^* C_V' - C_T'^* C_V)\, \alpha Z/p.$$

Hence it is clear that in this case both terms which do not conserve time parity, and Coulomb terms which do conserve time parity will contribute to the same phenomenon. It must be noted that in this case (interference between the AV and STP interactions) it is possible to check time parity by investigating the correlation between the polarization of the β particle and the conversion electron without employing oriented nuclei.

We denote the pseudovector of the β-particle polarization in the rest system by $\zeta_1(\chi, \omega)$. The angles χ and ω are taken in a coordinate system whose z axis is along the direction of $\mathbf{p}$. The following expression for the longitudinal polarization of the conversion electrons, when the β electron and its polarization are observed and the initial nucleus is not oriented, can then be obtained:

$$\begin{aligned}
\langle\sigma\rangle_{\mathbf{n}} = \sum_{\pi\pi' L \leqslant L'} &\{[2p \operatorname{Re} Q_m + 2\alpha Z \operatorname{Im} Q_n - \lambda_{I_1 I_i}(p \operatorname{Re} Q_1 \\
&+ \alpha Z \operatorname{Im} Q_1)] E^{-1} \cos\theta_n - \tfrac{1}{3}(2 \operatorname{Re} D + \lambda I_1 I_2 G) \\
&\times (\cos\theta_n \cos\chi + \sin\chi \sin\theta_n \cos\omega) \\
&+ [-2p \operatorname{Im} Q_n + 2\alpha Z \operatorname{Re} Q_m + \lambda_{I_1 I_i}(p \operatorname{Im} Q_1 \\
&- \alpha Z \operatorname{Re} Q_1)] E^{-1} \sin\theta_n \sin\chi \sin\omega \\
&+ \tfrac{1}{3}(1 - \gamma/E)[-2 \operatorname{Re}(D_0 + D_1) \\
&+ \lambda_{I_1 I_i}(M_1 + N_1)](2\cos\theta_n \cos\chi - \sin\theta_n \sin\chi \cos\omega)\} \\
&\times (2 - \delta_{\pi\pi'}\delta_{LL'})\, \bar{\delta}(\pi_0 L_0 \pi L)\, \bar{\delta}(\pi_0 L_0 \pi' L') \\
&\times F_1(LL' I_2 I_1)\, b^{\varkappa_1}_{1(-1)}(\pi L \pi' L') \\
&\times \Big\{\sum_{\pi L} \bar{\delta}^2(\pi_0 L_0 \pi L) \sqrt{3}\Big[b^0(0,0) + M_1 - \frac{\gamma}{E} N_1 \\
&+ E^{-1}[p \operatorname{Re}(Q_0 + Q_1) + \alpha Z \operatorname{Im}(Q_0 + Q_1)] \cos\chi\Big]\Big\}^{-1}.
\end{aligned} \tag{17}$$

The transverse polarization is obtained from the same formula by substituting $b_{11}^{\kappa_1}$ for $b_{1(-1)}^{\kappa_1}$, and by using relations (14). Formula (17) is written in a coordinate system whose z axis is directed along $\mathbf{p}$, and in which $\mathbf{n}$ lies in the zx plane. In (17) we have put

$$\lambda_{I_1 I_i} = [I_1(I_1 + 1) - I_i(I_i + 1) + 2]/2[I_1(I_1 + 1)]^{1/2}.$$

The remaining quantities in (17) are defined as follows (cf. reference 10):

$$\begin{aligned}
&Q_m = -(\beta_{ST} - \beta_{VA}) M_F M_{GT}, \quad Q_n = (\beta_{VT} - \beta_{SA}) M_F M_{GT}^*, \\
&D_0 = -(\alpha_{ST} + \alpha_{VA}) M_F M_{GT}^*, \quad D_1 = (\alpha_{VT} + \alpha_{SA}) M_F M_{GT}^*, \\
&\operatorname{Re} Q_1 = (\beta_{TT} - \beta_{AA}) |M_{GT}|^2, \quad \operatorname{Im} Q_1 = -2 \operatorname{Im} \beta_{AT} |M_{GT}|^2, \\
&\operatorname{Re} Q_0 = (\beta_{SS} - \beta_{VV}) |M_F|^2, \quad \operatorname{Im} Q_0 = -2 \operatorname{Im} \beta_{VS} |M_F|^2, \\
&M_1 = (\alpha_{TT} + \alpha_{AA}) |M_{GT}|^2, \quad N_1 = -2 \operatorname{Re} \alpha_{TA} |M_{GT}|^2, \\
&D = (D_0 - \gamma D_1/E) - 2(D_1 - \gamma D_0/E), \\
&G = (M_1 - \gamma N_1/E) - 2(N_1 - \gamma M_1/E),
\end{aligned}$$

where

$$\alpha_{xy} = C_x C_y^* + C_x' C_y'^*, \qquad \beta_{xy} = C_x C_y'^* + C_x' C_y^*.$$

Formula (17) describes electron decay. To obtain positron decay, the following substitutions have to be made:

$$Z \to -Z, \qquad C_i \to -C_i'^*, \qquad C_i' \to C_i'^* \qquad (i = A, S)$$

and

$$C_j \to C_j'^*, \qquad C_j^* \to -C_j'^* \qquad (j = V, T).$$

Finally, we bring the expression for the polarization of the conversion electrons for the case when an M1—E2 multipole mixture is considered, the initial nucleus is not polarized, and the β transition is allowed (no polarization of β particles is observed). From (15) or (17) we obtain

$$\begin{aligned}
\langle\sigma\rangle = \frac{\alpha}{(1 + \bar{\delta}^2)\sqrt{3}} &\Big\{\mathbf{n}\Big(\frac{\mathbf{p}}{p}\mathbf{n}\Big)[F_1(11 I_2 I_1)\, b^{\varkappa_1}_{1(-1)}(M1) \\
&+ 2\bar{\delta} F_1(12 I_2 I_1)\, b^{\varkappa_1}_{1(-1)}(M1E2) + \bar{\delta}^2 F_1(22 I_2 I_1)\, b^{\varkappa_1}_{1(-1)}(E2)] \\
&+ \frac{1}{2p}[\mathbf{p} - (\mathbf{p}\mathbf{n})\mathbf{n}][F_1(11 I_2 I_1)\, b^{\varkappa_1}_{11}(M1) + 2\bar{\delta} F_1(12 I_2 I_1) \\
&\times b^{\varkappa_1}_{11}(M1E2) + \bar{\delta}^2 F_1(22 I_2 I_1)\, b^{\varkappa_1}_{11}(E2)]\Big\}.
\end{aligned}$$

Here $\bar{\delta} = \bar{\delta}(M1E2)$ and α differ by a factor $-[(I_1 + 1)/I_1]^{1/2}$ from the expression employed by Geshkenbeĭn.[2] For reference we list the values of F_1:

$$F_1(LLI_2I_1) = \frac{\sqrt{3}\,[L(L+1) + I_1(I_1+1) - I_2(I_2+1)]}{2L(L+1)\,[I_1(I_1+1)]^{1/2}},$$

$$F_1(12I_2I_1) = \frac{[3(I_2+I_1-1)(I_2+I_1+3)(I_1-I_2+2)(I_2-I_1+2)]^{1/2}}{4\,[5I_1(I_1+1)]^{1/2}}$$

Our consideration of the β-e correlation is applicable when the nucleus is free in the intermediate state. If the nucleus is acted upon in the intermediate state by a torque, arising as a result of the interaction of the magnetic dipole moment μ with the external magnetic field **B**, or from the interaction of the electrical quadrupole moment Q with the gradients of the electric fields $\partial^2V/\partial z^2$, then the β-e correlation, generally speaking, decreases. The magnitude of this perturbation depends mainly on the average lifetime τ_{I_1} of the nucleus in the intermediate state. If the perturbation is described by the precession frequency ω, then for magnetic interactions ω is equal to the Larmor frequency, and for the quadrupole interaction $\omega \sim Q$ and $\partial^2V/\partial z^2$. A rough criterion of the applicability of our considerations can be obtained from the condition $\omega\tau_{I_1} < 0.1$. Hence $\tau_{I_1} < 10^{-10}$ sec.

In conclusion, I express my deep gratitude to B. V. Geshkenbeĭn and I. S. Shapiro for their interest in the work and its discussion.

APPENDIX

Below we list the values of the polarization parameters calculated with the aid of tables of radial integrals compiled by L. A. Sliv (private

TABLE I. Polarization parameter $b_{1(-1)}^{L_I}$ (M1E2) for the longitudinal polarization for an M1–E2 mixture and conversion from the L_I shell.

Z	k = 0.10	0.15	0.2	0.3	0.5	0.7
57	0.045	0.18	0.25		0.45	0.54
65	−0.047	0.047	0.14	0.26	0.36	0.47
73	0.11	−0.046	0.02	0.14	0.29	0.39
81	0.41	0.075	−0.47	0.031	0.19	0.31

TABLE II. Polarization parameter $b_{1(1)}^{L_I}$ (M1E2) for the transverse polarization for an M1–E2 mixture and conversion from the L_I shell.

Z	k = 0.10	0.2	0.3	0.5	0.7
57	−0.098	−0.35	−0.49	−0.63	−0.68
65	0.008	−0.23	−0.37	−0.50	−0.61
73	−0.13	−0.081	−0.23	−0.40	−0.51
81	−0.42	−0.001	−0.094	−0.28	−0.41

TABLE III. Polarization parameter $b_{1(-1)}^{L_{II}}$ (M1E2) for the longitudinal polarization for an M1–E2 mixture and conversion from the L_{II} shell.

Z	k = 0.1	0.3	0.7
57	−1.0	−0.99	−0.97
65	−1.0	−0.99	−0.97
73	−1.0	−0.98	−0.95
81	−0.97	−0.95	−0.92

TABLE IV. Polarization parameter $b_{1(-1)}^{L_{II}}$ (M1E2) for the transverse polarization for an M1–E2 mixture and conversion from the L_{II} shell.

Z	k = 0.1	0.15	0.3	0.7
57	−0.94	−0.83	−0.81	−0.76
65	−0.92	−0.87	−0.84	−0.79
73	−0.93	−0.88	−0.85	−0.81
81	−0.91	−0.88	−0.86	−0.80

TABLE V. Polarization parameter $b_{1(-1)}^{L_{III}}$ (M1) for longitudinal polarization in the case of a pure conversion transition from the L_{III} shell.

Z	k = 0.1	0.2	0.3	0.5	0.7
57	−0.06	−0.21	−0.32	−0.43	−0.49
65	0.006	−0.14	−0.27	−0.41	−0.48
73	0.08	−0.12	−0.24	−0.38	−0.46
81	0.13	−0.08	−0.19	−0.34	−0.44

TABLE VI. Polarization parameter $b_{1(1)}^{L_{III}}$ (M1) for transverse polarization in the case of a pure conversion transition from the L_{III} shell.

Z	k = 0.1	0.2	0.3	0.5	0.7
57	0.19	0.29	0.38	0.41	0.40
65	0.05	0.23	0.31	0.37	0.38
73	−0.03	0.16	0.23	0.32	0.34
81	−0.09	0.08	0.16	0.24	0.27

TABLE VII. Polarization parameter $b_{1(-1)}^{L_{III}}$ (M1E2) for longitudinal polarization for an M1–E2 mixture and conversion from the L_{III} shell.

Z	k = 0.1	0.15	0.2	0.3	0.5	0.7
57	0.55	0.58	0.56	0.48	0.28	−0.011
65	0.51	0.56	0.52	0.45	0.26	0.011
73	0.44	0.49	0.48	0.43	0.24	0.046
81	0.38	0.45	0.43	0.38	0.24	0.12

TABLE VIII. Polarization parameter $b_{1(1)}^{L_{III}}$ (M1E2) for transverse polarization for an M1–E2 mixture and conversion from the L_{III} shell.

Z	k = 0.1	0.15	0.2	0.3	0.5	0.7
57	0.68	0.62	0.61	0.56	0.65	0.69
65	0.82	0.74	0.68	0.66	0.69	0.75
73	0.89	0.82	0.72	0.71	0.74	0.81
81	0.89	0.83	0.77	0.72	0.79	0.91

TABLE IX. Polarization parameter $b_{1(-1)}^{L_{III}}$ (E2) for longitudinal polarization in the case of a pure conversion transition from the L_{III} shell.

Z	k = 0.1	0.2	0.3	0.5	0.7
57	0.35	0.35	0.34	0.28	0.082
65	0.32	0.33	0.33	0.23	0.1
73	0.27	0.29	0.29	0.21	0.12
81	0.21	0.24	0.23	0.19	0.115

TABLE X. Polarization parameter $b_{1(1)}^{L_{III}}$ (E2) for the transverse polarization in the case of a pure conversion transition from the L_{III} shell.

Z	k				
	0.1	0.2	0.3	0.5	0.7
57	0.83	0.59	0.37	0.006	—0.42
65	0.97	0.71	0.47	0.14	—0.27
73	1.15	0.87	0.64	0.27	—0.062
81	1.29	0.98	0.79	0.43	0.15

communication). The calculations are carried out for nuclei with Z = 57, 65, 73, and 81, and transition energies from 0.1 to 0.7 in units of m_ec^2 for conversion from the L_I, L_{II}, and L_{III} shells.

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