ON THE THEORY OF THE OPTICAL ANISOTROPY OF ATOMIC NUCLEI

A. M. BALDIN and S. F. SEMENKO

P. N. Lebedev Physics Institute, Academy of Sciences, U.S.S.R.

Submitted to JETP editor March 21, 1960

J. Exptl. Theoret. Phys. (U.S.S.R.) 39, 434-437 (August, 1960)

The elastic scattering of gamma rays by oriented nuclei is considered. The possibility of an experimental study of nuclear optical anisotropy parameters, including the vector electric dipole polarizability, of nuclei is discussed.

In earlier papers by one of the present authors[1,2] the concepts of molecular optics (see Placzek's monograph, for example) were extended to photonuclear reactions. It was noted[1] that an induced dipole electric moment in a nucleus may depend on the orientation of the nuclear spins with respect to the electric field. Certain effects were also discussed, whose experimental investigation would enable us to establish the existence of optical anisotropy in nuclei and to measure its basic parameters.[2] The elastic scattering of gamma rays by nuclei was considered in particular, but effects associated with the possible existence of vector polarizability were neglected without justification.

The present note considers the elastic scattering of gamma rays by nuclei, including the previously uninvestigated case of scattering by oriented nuclei. Investigation of the latter effect would, in principle, enable us to measure the vector polarizability.

The elastic scattering of dipole gamma rays is described by the following R matrix:

$$(m\boldsymbol{\lambda}\,|\,R\,|\,m'\boldsymbol{\lambda}') = \Big\{R'\delta_{ij}\delta_{mm'} + \frac{3R^T}{J(2J-1)}\Big(m\,\Big|\,\tfrac{1}{2}\,(\hat{J}_i\hat{J}_j+\hat{J}_j\hat{J}_i) - \tfrac{1}{3}J\,(J+1)\,\delta_{ij}\,\Big|\,m'\Big) + R^V J^{-1} e_{ijl}\,(m\,|\,J_l\,|\,m')\Big\}\lambda_i'^{*}\lambda_j, \quad (1)$$

where m,m′ are the projections of the nuclear spin on the $\mathbf{e}_3$ axis; λ, λ' are the photon polarization vectors before and after scattering, respectively; $\hat{J}_i$ is the nuclear spin projection operator; $R' \equiv R^S - T$; $T = (\omega/c)^3e^2Z^2/2\pi AM\omega^2$ is the amplitude of Thomson scattering by a nuclear charge Z;

$$R^S = (\omega/c)^3c^S/2\pi, \quad R^T = (\omega/c)^3\,c^T/2\pi, \quad R^V = (\omega/c)^3c^V/2\pi, \quad (1a)$$

where c^S, c^T and c^V are the scalar, tensor and vector polarizability, respectively; e_{ijl} is a completely antisymmetric third-rank tensor ($e_{123} = 1$).

The contribution of the vector polarizability to the absorption cross section is

$$\sigma^V = \frac{16\pi^2 m}{(\omega/c)^2 J}\,\mathrm{Re}\,R^V\cdot\mathrm{Im}\,(\boldsymbol{\lambda}_1^*\boldsymbol{\lambda}_2), \quad (2)$$

which differs from zero only in the case of elliptic (or circular) gamma-ray polarization. The results for the absorption cross section given in reference 2 pertain to unpolarized photons and are not affected by the vector polarizability.

The cross section for elastic scattering of gamma rays by unoriented nuclei with vector polarizability taken into account is

$$\frac{d\sigma^{\mathrm{H}}}{d\Omega} = \frac{2\pi^2c^2}{\omega^2}\Big\{|R'|^2[1+(\mathbf{k}'\mathbf{k})^2] + \frac{(J+1)(2J+3)}{5J\,(2J-1)}\left|\frac{R^T}{2}\right|^2[13+(\mathbf{k}'\mathbf{k})^2] + \frac{J+1}{3J}\,|R^V|^2[3-(\mathbf{k}'\mathbf{k})^2]\Big\}, \quad (3)$$

where $\mathbf{k}$ and $\mathbf{k}'$ are the unit wave vector of the photon before and after scattering, respectively.

Equation (3) shows that investigation of elastic scattering by unoriented nuclei enables us to estimate only the combined effect of the tensor and vector polarizabilities. Information concerning all three parts of the polarizability can, in principle, be obtained by investigating the angular dependence of $d\sigma^{\mathrm{H}}/d\Omega$ or by considering $d\sigma^{\mathrm{H}}/d\Omega$ together with the cross section for absorption by both oriented and unoriented nuclei. These procedures require the solution of a system of equations; the experimental errors could increase as a result of several algebraic operations and thus greatly hinder detection of the effects in question. In connection with the foregoing it is of interest to consider the elastic scattering of gamma rays by oriented nuclei.

The cross section for the scattering of unpolarized photons by a nucleus with spin projection m on the $\mathbf{e}_3$ axis is

$$\frac{d\sigma}{d\Omega}=\frac{d\sigma^{\mathrm{H}}}{d\Omega}+\frac{2\pi^2c^2}{\omega^2}\left\{\left|\frac{3R^T}{2J(2J-1)}\right|^2\left(J^2+J+\frac{5}{2}\right)(m^2-\overline{m^2})\right.$$

$$-\frac{5}{2}(m^4-\overline{m^4})\left[k_3'^2+k_3^2+k_3'^2k_3^2-\frac{11}{15}-\frac{2}{15}(\mathbf{k}'\mathbf{k})^2\right]$$

$$+\left|\frac{3R^T}{2J(2J-1)}\right|^2[(-8J^2-8J+5)(m^2-\overline{m^2})$$

$$+10(m^4-\overline{m^4})]\left[2k_3'^2k_3^2-k_3'k_3(\mathbf{k}'\mathbf{k})-\frac{2}{15}+\frac{1}{15}(\mathbf{k}'\mathbf{k})^2\right]$$

$$-6\operatorname{Re}\left(\frac{3R^{T*}}{2J(2J-1)}R'\right)(m^2-\overline{m^2})\left[k_3'^2+k_3^2-k_3'k_3(\mathbf{k}'\mathbf{k})\right.$$

$$\left.-\frac{2}{3}+\frac{1}{3}(\mathbf{k}'\mathbf{k})^2\right]-6\operatorname{Im}\left(\frac{3R^{T*}}{2J(2J-1)}\frac{R^V}{J}\right)$$

$$\times(m^2-\overline{m^2})(k_3'^2-k_3^2)+\left|\frac{R^V}{J}\right|^2(m^2-\overline{m^2})(\mathbf{k}'\mathbf{k})[3k_3'k_3$$

$$-(\mathbf{k}'\mathbf{k})]+2\operatorname{Re}\left(R'^*\frac{R^V}{J}\right)m(\mathbf{k}'\mathbf{k})[\mathbf{k}'\times\mathbf{k}]_3$$

$$+2\operatorname{Re}\left(\frac{3R^{T*}}{2J(2J-1)}\frac{R^V}{J}\right)$$

$$\times m[5m^2-(3J^2+3J-1)]k_3'k_3[\mathbf{k}'\times\mathbf{k}]_3$$

$$-2\operatorname{Re}\left(\frac{3R^{T*}}{2J(2J-1)}\frac{R^V}{J}\right)$$

$$\left.\times m(m^2-\overline{m^2})(\mathbf{k}'\mathbf{k})[\mathbf{k}'\times\mathbf{k}]_3\right\},\qquad(4)$$

where

$$\overline{m^n}=\frac{1}{(2J+1)}\sum_{m=-J}^{m=J}m^n$$

is the mean value of the n-th power of the spin projection in the case of a nucleus freely oriented in space.

The tensor and vector polarizabilities are revealed most strongly in the azimuthal scattering asymmetry. The evaluation of

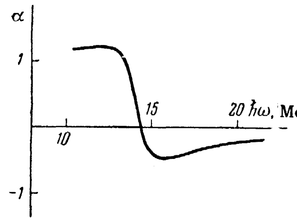
$$\alpha=\frac{\left[\frac{d\sigma}{d\Omega}(k_3'=0,\ k_3=0,\ \mathbf{k}'\mathbf{k}=0)-\frac{d\sigma}{d\Omega}(k_3'=1,\ k_3=0,\ \mathbf{k}'\mathbf{k}=0)\right]}{(d\sigma^{\mathrm{H}}/d\Omega)(\mathbf{k}'\mathbf{k}=0)}$$

where m = J and the expressions for the tensor and scalar polarizabilities given in reference 2 are used as well as the giant resonance parameters for $_{73}\mathrm{Ta}^{181}$ given in reference 4, yields

$$\alpha(\hbar\omega_1=12.5\ \mathrm{MeV})=1.2;\qquad\alpha(\hbar\omega_2=15.5\ \mathrm{MeV})=-0.4.$$

The energy dependence of α for Ta^{181} is shown in the figure, assuming $R^V=0$. A comparison of our results with estimates of the influence of tensor polarizability on photoabsorption by oriented nuclei indicates that the tensor polarizability is considerably more important in the case of scattering by oriented nuclei.

The existence of vector polarizability can be established by measuring the difference $d\sigma(\mathbf{k},\mathbf{k}')/d\Omega-d\sigma(\mathbf{k}',\mathbf{k})/d\Omega$. In the region of dipole resonance frequencies the vector polarizability of an axisymmetric nucleus can be put into



the form

$$c^V=\frac{K}{J+1}\sum_r\frac{\hbar\omega+i\gamma_r/2}{(E_r-E_0)^2-\hbar^2\omega^2-i\hbar\omega\gamma_r}$$

$$\times[(0|d_\eta|r)(r|d_\xi|0)-(0|d_\xi|r)(r|d_\eta|0)],$$

where K is the projection of the nuclear spin on its ζ symmetry axis; ξ, η, ζ is a right-handed system of orthogonal axes attached to the nucleus; **d** is the dipole moment operator. It is easily seen that if the nuclear energy levels E_r merge, so that all of them can be effectively replaced by one or two levels, we have $c^V=0$. Thus vector polarizability is a manifestation of the fine structure of giant resonance.

An indication of the possible presence of vector polarizability follows from the condition

$$\sum_r i(E_r-E_0)[(0|d_\xi|r)(r|d\ \ |0)-(0|d_\eta|r)(r|d_\xi|0)]$$

$$=\frac{e^2\hbar^2}{M}\left(\frac{N^2}{A^2}\sum_p m_p+\frac{Z^2}{A^2}\sum_n m_n\right),\qquad(5)$$

where m_p, m_n are the projections of the proton and neutron orbital moment, respectively, on the nuclear symmetry axis.* The ratio of the right-hand part of (5) to the corresponding sum for scalar polarizability is of the order 1/A. On the basis of the foregoing the vector polarizability would be small in heavy nuclei, where giant resonance is unlikely to have a fine structure.

The authors wish to thank Dr. E. Fuller for discussions of a number of questions concerning the optical anisotropy of nuclei. Dr. Fuller also privately communicated to us the advisability of investigating effects associated with vector polarizability.

[1] A. M. Baldin, Nuclear Phys. **9**, 237 (1958).

[2] A. M. Baldin, JETP **37**, 202 (1959), Soviet Phys. JETP **10**, 142 (1960).

[3] G. Placzek, Rayleigh-Streuung und Raman-Effekt, Handbuch der Radiologie Bd. 6, II, 209, 1934, Russ. Transl. ONTI, Moscow, 1935.

[4] E. G. Fuller and M. S. Weiss, Phys. Rev. **112**, 560 (1958).

*Effects associated with exchange forces are not being taken into account here.

Translated by I. Emin
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