EXCITATION SPECTRUM OF A SYSTEM OF ELECTRONS AND IONS SITUATED IN A HOMOGENEOUS MAGNETIC FIELD

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The effect of a magnetic field on the spectrum of the acoustic branch of excitations of a system of electrons and ions is studied with the aid of the quantum kinetic equation.

At the present time the developed mathematical methods[1-4] of taking into account the collective Coulomb interactions in a system of many particles are being used more and more widely in solid state theory. In this connection the solid (metal) is approximated by an isotropic plasma and in the first approximation the periodic distribution of ions situated at the lattice points of a crystalline lattice is neglected. However, in order to explain those physical properties of such a plasma which are temperature-dependent (for example, electrical conductivity), the motion of the ions should be taken into account.

Silin[5,6] has studied the weakly excited states of electrons and ions by means of a quantum kinetic equation with a self-consistent field. By utilizing the quantum distribution function in the mixed representation, he has established the limits of applicability of the results obtained by means of the classical equation with a self-consistent field.[1] The analysis of the dispersion equation led to the conclusion that in the long-wave domain the excitations are of an essentially collective character.

In the present note we give the results of an investigation by the same method of the effect of a constant homogeneous magnetic field on the spectrum of the longitudinal excitations of the system under discussion. In this discussion our attention is concentrated only on the long-wave part of the spectrum which plays the fundamental role in many physical phenomena.

The dispersion equation can be easily obtained by setting up the equations for the distribution functions for the electrons $f_e(\mathbf{p}, \mathbf{q})$ and for the ions $f_i(\mathbf{p}, \mathbf{Q})$ ($\mathbf{q}$, $\mathbf{Q}$ are the coordinates and $\mathbf{p}$, $\mathbf{P}$ are the momenta of the particles). For weakly excited states we set

$$f_e(\mathbf{q}, \mathbf{p}) = f_{0e}(p) + \varphi(\mathbf{q}, \mathbf{p}, t),$$
$$f_i(\mathbf{Q}, \mathbf{P}) = f_{0i}(P) + \Phi(\mathbf{Q}, \mathbf{P}, t),$$

where we consider that the increments φ and Φ of the equilibrium functions are small.

By going over then to the Fourier components $\varphi_{\mathbf{k}}$, $\Phi_{\mathbf{k}}$, and by solving the system of equations determining them, we obtain the spectrum of the collective oscillations. In doing this we follow Landau's method[7] (we carry out a Laplace-Mellin transformation with respect to time). The dispersion equation will then have the form

$$\begin{vmatrix} 1 - \nu_{ee}(k)\, a_e, & -\nu_{ei}(k)\, a_e \\ -\nu_{ie}(k)\, a_i, & 1 - \nu_{ii}(k)\, a_i \end{vmatrix} = 0, \tag{1}$$

where

$$a_j = -\frac{i}{\Omega_j \hbar} \int \exp\left\{\frac{1}{\Omega_j}\left[\left(s + i\frac{k_z p_z}{m_j}\right)\vartheta + i\frac{k_x p_r}{m_j}\sin\vartheta\right]\right\} p_r\, dp_r\, dp_z\, d\vartheta$$
$$\times \left[\int_0^{\vartheta} \left[f_{0j}\left(\left|\mathbf{p} + \frac{\hbar\mathbf{k}}{2}\right|\right) - f_{0j}\left(\left|\mathbf{p} - \frac{\hbar\mathbf{k}}{2}\right|\right)\right]\right.$$
$$\times \exp\left\{-\frac{1}{\Omega_j}\left[\left(s + i\frac{k_z p_z}{m_j}\right)\beta + i\frac{k_x p_r}{m_j}\sin\beta\right]\right\} d\beta$$
$$+ \frac{\exp\{(s + ik_z p_z/m_j)\, 2\pi/\Omega_j\}}{1 - \exp\{(s + ik_z p_z/m_j)\, 2\pi/\Omega_j\}} \int_0^{2\pi} \left[f_{0j}\left(\left|\mathbf{p} + \frac{\hbar\mathbf{k}}{2}\right|\right)\right.$$
$$\left. - f_{0j}\left(\left|\mathbf{p} - \frac{\hbar\mathbf{k}}{2}\right|\right)\right] \times \exp\left\{-\frac{1}{\Omega_j}\left[\left(s + i\frac{k_z p_z}{m_j}\right)\beta\right.\right.$$
$$\left.\left.\left. + i\frac{k_x p_r}{m_j}\sin\beta\right]\right\} d\beta\right];$$

$$j = e,\ i; \qquad \Omega_j = e_j H / m_j c; \qquad m_e = m, \qquad m_i = M;$$

s is the Laplace-Mellin expansion parameter; the z axis coincides with the direction of the magnetic field $\mathbf{H}$; the angle ϑ is measured from the $\mathbf{H}$, $\mathbf{k}$ plane; $\nu_{ij}(k) = \int U_{ij}(r) \exp(i\mathbf{kr})\, d\tau$ are the Fourier components of the potential energies of interaction $U_{ij}(r)$ between corresponding particles.

In the case of the Coulomb interaction $\nu_{ee}(k) = 4\pi e_e^2/k^2$; $\nu_{ii}(k) = 4\pi e_i^2/k^2$; $\nu_{ei}\nu_{ie} = \nu_{ee}\nu_{ii}$ and expression (1) reduces to the form

$$1 - \nu_{ee}(k)\, a_e - \nu_{ii}(k)\, a_i = 0.$$

The long waves correspond to small values of $\mathbf{k}$ so that in the integral we can set

$$\frac{1}{\hbar}\left(f_0\left(\left|\mathbf{p}+\frac{\hbar\mathbf{k}}{2}\right|\right)-f_0\left(\left|\mathbf{p}-\frac{\hbar\mathbf{k}}{2}\right|\right)\right)=\mathbf{k}\frac{\partial f_0}{\partial \mathbf{p}} \tag{2}$$

and, moreover,

$$\begin{aligned} &\hbar^2k^2/2m \ll n_{0e}\nu_{ee}(k), \qquad p_e^2/2m \gg \hbar^2k^2/2m; \\ &\hbar^2k^2/2M \ll n_{0i}\nu_{ii}(k), \qquad p_i^2/2M \gg \hbar^2k^2/2M; \end{aligned} \tag{2a}$$

n_{0i}, n_{0e} are the equilibrium concentrations of the ions and of the electrons.

Relation (2) does not exclude the possibility that the distribution in the case of the equilibrium functions is of a quantum nature. In the absence of a magnetic field expression (1) gives for small k two branches of oscillations, acoustic and electronic.[5,6] Thermal motion in a metal excites only the acoustic branch. As regards the electron oscillations, temperatures of the order of 10^4°K are needed to excite them.

We assume that the temperature satisfies $T > \Theta$ where Θ is the Debye temperature. In practice, the electron gas in a metal is degenerate at all temperatures, so that we use the Fermi distribution function for f_{0e}. For the ions we take the Maxwellian function.

In order to take into account the effect of the magnetic field on the acoustic branch we set

$$p_{0e}^2/m^2 \gg \omega^2/k^2 \gg p_i^2/M^2, \tag{3}$$

where p_{0e} is the limiting momentum of the Fermi distribution corresponding to the energy $\epsilon_0 = p_{0e}^2/2m$. We neglect the damping of the waves, and on setting $s = -i\omega$ we obtain

$$a_i = -2\left(\frac{M}{2\pi\varkappa T}\right)^{3/2}\frac{n\pi_{0i}}{\varkappa T}\left\{\frac{(2\pi)^{1/2}(\varkappa T)^{3/2}}{M}\right.$$
$$\left. -\omega\sum\int_{-\infty}^{\infty}\frac{\exp\{-Mv_z^2/2\varkappa T\}\,dv_z}{\omega-k_zv_z-n\Omega_i}\int_0^{\infty}J_n^2\left(\frac{k_xv_r}{\Omega_i}\right)e^{-Mv_r^2/2\varkappa T}v_r\,dv_r\right\}.$$

On taking (3) into account, and on using the expansion for the Bessel function, we obtain approximately

$$a_i = \frac{n_{0i}}{M}\left(\frac{k_z^2}{\omega^2}-\frac{k_x^2}{\omega^2-\Omega_i^2}\right).$$

a_e is evaluated in a somewhat different manner. Then on substituting the resultant values of a_i and a_e into (1) we obtain

$$\omega^2 = \frac{1}{2\beta}\{c^2k^2+\beta\Omega_i^2 \pm [(c^2k^2+\beta\Omega_i^2)^2-4\beta c^2\Omega_i^2k^2\cos^2\alpha]^{1/2}\}, \tag{4}$$

where

$$\beta = 1+(\pi^2/24)(\varkappa T/\varepsilon_0)^2, \qquad c^2 = (p_{0e}^2/3Mm)\,|e_i/e_e|.$$

Expression (4) taken with a minus sign defines the acoustic branch; for small values of k (4) yields

$$\omega = (ck/\sqrt{\beta})\cos\alpha, \tag{5}$$

α is the angle between **H** and **k**.

For $T = 0$ and $\cos\alpha = 1$ (the magnetic field is absent) we recover Silin's result.[6]

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