THE SPATIAL AND CHARGE PARITIES AND TWO-MESON ANNIHILATION OF THE PROTON-ANTIPROTON SYSTEM

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If the proton and the antiproton are described by the Dirac equation, the system $(p, \bar{p})$ has definite charge and spatial parities. It is proposed that experiments on annihilation, $\bar{p} + p \rightarrow \pi^+ + \pi^-$, $\bar{p} + p \rightarrow \pi^0 + \pi^0$, be used to test this consequence of the Dirac equation.

1. INTRODUCTION

As is well known, if the proton and the antiproton are described by the Dirac equation the proton-antiproton system has quite definite spatial and charge parities.

If the proton has spatial parity ± 1 or $\pm i$, then the parity of the antiproton must be ∓ 1 or $\pm i$, respectively. Thus the product of the intrinsic parities of p and $\bar{p}$ is -1 (cf. the book by Akhiezer and Berestetskiĭ,[1] Sec. 8, Art. 3, in particular the note on page 55).

Generally speaking, charge conjugation gives the wave function of the charge-conjugate state apart from an arbitrary phase factor:

$$\hat{C}\psi_p = e^{i\eta}\psi_{\bar{p}}, \quad \hat{C}\psi_{\bar{p}} = e^{i\bar{\eta}}\psi_p. \qquad (1)$$

In order for the wave function $\psi_{\bar{p}}$ of the antiproton (or the field operator) to obey the Dirac equation, we must have $\bar{\eta} = -\eta$, i.e., $\hat{C}^2$ must be equal to $+1$. Then we get for the complete charge parity of a particle-antiparticle system that is in a state with definite orbital and spin angular momenta l and s the value $(-1)^{l+s}$ (cf. the book by Mathews,[2] Chap. 8, Art. 5, and also the paper by Wolfenstein and Ravenhall[3]).

From the fact that the antiproton exists it does not yet follow that the proton and the antiproton obey the Dirac equation (cf. the review by Segre[4]). We recall that the discovery of positive electrons was immediately followed by a series of experiments for the purpose of establishing their identity with the antiparticles whose existence followed from the correctness of the Dirac equation for the electron (cf. the article by de Benedetti and Corben,[5] page 191). In particular, it has been verified that the polarizations of annihilation γ-ray quanta are perpendicular, which is a simple consequency of the pseudoscalar nature of the s state of the positron and the conservation laws.[5,6]

We are suggesting analogous experiments. Let us suppose that there can be values of the spatial and charge parities of the system $(p, \bar{p})$ different from the values that follow from the Dirac equation.* Then from the law of conservation of angular momentum and from invariance with respect to space reflection and charge conjugation one can get different selection rules for the two-meson annihilation $\bar{p} + p \rightarrow \pi + \pi$, and test them experimentally.

2. THE SELECTION RULES FOR TWO-MESON ANNIHILATION FOR VARIOUS TYPES OF PARITIES OF THE SYSTEM $(p, \bar{p})$

As before, we shall regard $\bar{p}$ as the particle charge-conjugate to the proton. With our purpose of testing the correctness of the Dirac equation we cannot, however, define $\hat{C}$ as the operator that converts the Dirac particle into its antiparticle. The example of K^0 and $\bar{K}^0$ shows that the operator $\hat{C}$ can be given a more general definition, analogous to the definition of the isotopic-spin operator (the operator $\hat{C}$ will convert p into $\bar{p}$ like the operator that converts a proton into a neutron).

We assume that for the charge-conjugate particle the mass (inertial), the momentum (or coordinate), the square of the spin, the spin projection, and, in general, all the mechanical characteristics are just the same as for the original particle (we may regard the spatial parity also as a mechanical characteristic, but this is not obligatory). All the

*Suppose, for example, that p and $\bar{p}$ obey two independent two-component equations of the type given by Shirokov,[7] which can describe particles of spin $\frac{1}{2}$ with different (and entirely independent) parities, charges, strangenesses, and so on.

other characteristics, namely the electric charge, the baryon or lepton number, and the strangeness are changed into their opposites by the charge-conjugation operator $\hat{C}$.

This definition still gives no connection between η and $\bar{\eta}$ in Eq. (1). Both the value of $\hat{C}^2$ when applied to the wave function of p or $\bar{p}$ and the charge parity of the system (p, $\bar{p}$) in a state with definite l and s can be complex numbers of unit absolute value (cf. Appendix A). This is an example of a difference between the operator $\hat{C}^2$ and the square of the reflection operator I. In fact, repeated inversion of the coordinates is an identical transformation, and its representation, i.e., the value of I^2, must be unity for single-valued representations and ± 1 for spinor representations. For $\hat{C}$ there is no corresponding fundamental group (such as, for example, a group of coordinate transformations); $\hat{C}$ is defined only through its action on wave functions. Thus after repeated charge conjugation one gets the same wave function only apart from an arbitrary phase factor.

If p and $\bar{p}$ are not described by the Dirac equation, their spatial parities are not related. A particle of half-integral spin can have the spatial parities +1, −1, +i, −i (reference 2, Chap. 7, Art. 3). These same values of the parity can occur for a system of two particles.

It will be clear from the further argument that from the fact of annihilation of (p, $\bar{p}$) into π mesons it follows that the charge parity of (p, $\bar{p}$) can take only the values $\pm(-1)^{l+s}$, and the intrinsic spatial parity of the π meson only the values ± 1. Therefore we shall study four possibilities for the spatial and charge parities of the (p, $\bar{p}$) system: $\{-1, +1\}$ (the ordinary Dirac case), $\{-1, -1\}$, $\{+1, +1\}$, and $\{+1, -1\}$. The first value given is the parity of the π meson, and the second is the coefficient +1 or −1 in the expression $\pm 1(-1)^{l+s}$ for the total charge parity of the system (p, $\bar{p}$).

Assuming the usual conservation laws* for the total angular momentum $\mathbf{l} + \mathbf{s} = \mathbf{l}'$ (l' is the orbital angular momentum of the two mesons), the spatial parity of π: $(-1)^l = (-1)^{l'}$, and the charge parity: $\pm(-1)^{l+s} = (-1)^{l'}$, we get the selection rules

*The expression $(-1)^{l'}$ for the charge parity of a system of two π mesons can be obtained from the following considerations. Granting that π^0 is charge-even (since it decays into two photons) and assuming that π^0, π^+, and π^- form an isotopic triplet (so that $\pi^{\pm} = \mp 2^{-\frac{1}{2}} D_2 \pi^0 + i 2^{-\frac{1}{2}} D_1 \pi^0$, where D_2 and D_1 are counterclockwise rotations through $\pi/2$ around the 2 and 1 axes in the isotopic space), we find that $\hat{C}\pi_+ = -\pi_-$ and $\hat{C}\pi_- = -\pi_+$, and for the charge parity of the system (π_+, π_-) we get the expression $(-1)^{l'}$ (granting that π mesons obey Bose statistics).

TABLE I

Parity signature	Selection rules for two-meson annihilation	Additional selection rule for $\bar{p}+p\to\pi^0+\pi^0$ (l' is even)
{−1, +1}	$s = 1$, $l' = l \pm 1$	l — odd
{−1, −1}	two-meson annihilation is forbidden	
{+1, +1}	$s = 0$, $l = l'$	l — even
{+1, −1}	$s = 1$, $l = l'$	l — even

shown in Table 1 for two-meson annihilation $\bar{p}$ + p → $\pi + \pi$ under the various possibilities for the parities.

In principle we can choose between the possibilities by carrying out a phase-shift analysis of the reaction $\bar{p}$ + p → $\pi + \pi$ (which requires experiments with polarized $\bar{p}$ or p). We shall show that distinctions can be made between the possibilities from the form of the angular distribution and from the energy dependence of the total cross section at small energies of the incident antiprotons.

We emphasize at once that we must exclude from consideration cases with preliminary formation of "protonium" [we use this name for the bound system (p, $\bar{p}$) analogous to positronium]. The angular distribution of the π mesons from the annihilation of "protonium" is always isotropic, and if there indeed is a dependence of the cross section σ on the energy it is determined just by the Coulomb interaction and is the same for all the parity possibilities. In Appendix B it is shown that if annihilation has occurred at a $\bar{p}$ energy larger than 0.25 Mev in the laboratory system the effect of the formation of "protonium" can be neglected.

Furthermore, in many of the proposed experiments annihilation "in flight" must occur in s or p states ($l = 0, 1$) of the proton-antiproton system. This requires that the quantity kr_0, be small, where $\hbar k$ is the relative momentum of $\bar{p}$ and p in the center-of-mass system and r_0 is the range of the strong (mesonic) interaction of $\bar{p}$ and p [we assume that $r_0 = \hbar/\mu c = 1.4 \times 10^{-13}$ cm, the Compton wavelength of the π meson (see the paper of Rarita and Schwed[8] and the bibliography given there)]. Then in case there is no Coulomb interaction (for example, for the annihilation of an antineutron and a neutron) the matrix element for the transition from the nucleon-antinucleon state with orbital angular momentum l, total spin s, and momentum $\hbar k$ to the $\pi + \pi$ state with orbital angular momentum l' has for $(kr_0)^2 \ll |4l - 2|$ the following basic sort of dependence on k and l (cf. Table 1 in reference 9):

$$R_{sl}^{l'} \sim (kr_0)^{l+1/2}/(l+1)(2l-1)!! \qquad (2)$$

The wave function of the initial state $\bar{p} + p$ (charged particles) differs only slightly from a plane wave if the antiproton energy E is larger than 0.25 Mev.[10] Therefore we may suppose that the dependence (2) holds for $\bar{p} + p \rightarrow \pi + \pi$ over the range from 0.25 Mev to some tens of Mev. More exactly, it can be shown that for the ratios of transition matrix elements with different values of l, in which we shall be interested in what follows, the Coulomb corrections are smaller than 10 percent at $E = 0.25$ Mev.

From Eq. (2) we get for the energy dependence of the total cross section of the exothermal reaction in the energy range specified the result $\sigma \sim (E^{1/2})^{2l_0-1}$, where l_0 is the lowest orbital angular momentum for the given hypothesis about the parities, which predominates over all larger values in virtue of Eq. (2) if $(kr_0)^2 \ll |4l-2|$ (cf. the previously mentioned paper by Wigner,[9] Sec. IIIA, "neutral case." We note that for $E < 0.025$ Mev we have the "charged case," and then $\sigma \sim 1/E$ independent of the parities).

In estimating the contribution of higher angular momenta one must also take into account the coefficients with which the quantities $R_{sl}^{l'}$ occur in the expressions for the differential and total cross sections (cf. Appendix B).

3. PROPOSED EXPERIMENTS

The first problem is to establish the existence or nonexistence of the two-meson annihilation $\bar{p} + p \rightarrow \pi + \pi$ at any energy of the $\bar{p}$ (including the case with preliminary formation of "protonium"). If it does not exist, the parities are those for $\{-1, -1\}$. On the other hand if we suppose that the existence of two-meson annihilation* of proton and antiproton gets established experimentally, then three of our possibilities for the parities remain. Under suitable limitations on the energy of $\bar{p}$ the annihilation cross section will be mainly determined by the elements $R_{sl}^{l'}$ of the transition matrix with the smallest values of l. Assuming further that the matrix elements $R_{sl}^{l'}$ behave essentially as shown in Eq. (2), we get the forms shown in Table II for the energy dependence of the total cross section for two-meson annihilation with the various parity possibilities, correct to deviations of not more than 10 percent within the energy ranges indicated in the table (meaning in all cases the kinetic energy of the antiproton in the laboratory system).

*More than 3000 $\bar{p}p$ annihilations are now known; among them there has not been registered a single case of two-meson annihilation.[11] Further investigations are necessary to establish whether this is due to the Dirac parity type $\{-1,-1\}$ or to other causes.

TABLE II

Parity signature	$\bar{p}+p \rightarrow \pi^+ + \pi^-$: Total cross section $\sigma_+(E)$	$\bar{p}+p \rightarrow \pi^+ + \pi^-$: Energy, Mev	$\bar{p}+p \rightarrow \pi^0 + \pi^0$: Total cross section $\sigma_0(E)$	$\bar{p}+p \rightarrow \pi^0 + \pi^0$: Energy, Mev
$\{-1, +1\}$	$\sigma \backsim E^{-1/2}$	$0.5 \div 10$	$\sigma \backsim E^{1/2}$	0.5 – 50
$\{+1, +1\}$	$\sigma \backsim E^{-1/2}$	$0.5 \div 10$	$\sigma \backsim E^{-1/2}$	0.5 – 50
$\{+1, -1\}$	$\sigma \backsim E^{1/2}$	$0.5 \div 40$	$\sigma \backsim E^{3/2}$	0.5 – 100

Other possibilities for distinguishing between the parity possibilities are provided by the angular distribution of two-meson annihilations "in flight." It is shown in Appendix C that for $\{+1, -1\}$ the angular distribution $\sigma_+(\vartheta)$ of the reaction $\bar{p} + p \rightarrow \pi^+ + \pi^-$ at any energy must be of the form $\sin^2\vartheta$ in certain ranges of angles near 0° and 180°. The larger the energy of the antiproton, the smaller the range of angles. In particular, for $0.25 < E \leq 10$ Mev the "combined" angular distribution $\sigma_+(\vartheta) + \sigma_+(\pi - \vartheta)$ is of the form $\sin^2\vartheta$ at all angles to within maximum deviations of the order of 10 percent (at angles ϑ close to 90°). For $0.25 < E \leq 40$ Mev it is of the form $\sin^2\vartheta$ in the range 0° – 30° (with the same deviation at angles close to 30°). The lower limit of the energy can of course also be larger than 0.25 Mev.

For this same parity type the angular distribution $\sigma_0(\vartheta)$ of the reaction $\bar{p} + p \rightarrow \pi^0 + \pi^0$ must be of the form $\sin^2\vartheta\cos^2\vartheta$ for angles close to 0° and 90°. In particular, $\sigma_0(\vartheta) \sim \sin^2\vartheta\cos^2\vartheta$ (to accuracy 10 percent) at all angles for $0.25 < E \leq 20$ Mev, and in the ranges 0° – 30°, 60° – 90° for $0.25 < E \leq 50$ Mev (to the same accuracy).

These features of the energy dependence and angular dependence of the cross section enable us to indicate some possible series of experiments for distinguishing between the three parity possibilities.

1. One measures $\sigma_+(\vartheta)$ for $E < 40$ Mev. If it does not go to zero at $\vartheta = 0°$, the possibility $\{+1, -1\}$ can be rejected. To distinguish between $\{-1, +1\}$ and $\{+1, +1\}$ one has only to determine whether $\sigma_0(E)$ decreases or increases as the energy is decreased.* For example, one can measure and compare the mean cross sections for the cases with antiproton energies $10 \leq E < 30$ Mev and those with $30 < E < 50$ Mev.

*In the case $\{+1,+1\}$ the cross section $\sigma_0(\vartheta)$ must be isotropic as long as we can neglect R_{02}^{0}. But this fact cannot be used to distinguish between the two cases, since even for $E \lesssim 5$ Mev the deviation from isotropy can reach 15 percent; furthermore in the Dirac case $\sigma_0(\vartheta)$ can be accidentally isotropic (cf. Appendix C).

The case in which $\sigma_+(\vartheta)$ is small near 0° requires special examination. This is also possible for the parity types $\{-1, +1\}$ and $\{+1, +1\}$, but only for a certain quite definite relation between the elements $R^1_{0,1}$, $R^0_{1,1}$, $R^2_{1,1}$, . . . and $R^0_{0,0}$, $R^0_{1,0}$, . . . , which can occur accidentally. If, however, E < 10 Mev, this relation is in contradiction with the relations (2). Thus to be sure about the legitimacy of the type $\{+1, -1\}$ one has, strictly speaking, still to see whether $\sigma_+(0°)$ remains small for 0.25 < E < 10 Mev. As can be seen from Table II, this experiment can be replaced by a measurement of $\sigma_+(E)$ in the range 0.5 — 10 Mev. If it decreases with decreasing E, then we have the case $\{+1, -1\}$; if it increases, there remain the other two cases [which can be distinguished by means of the behavior of $\sigma_0(E)$, see above]. But this experiment is evidently more difficult than the measurement of $\sigma_+(\vartheta)$ for 0.25 < E ≤ 10 Mev.

2. One measures $\sigma_0(\vartheta)$ for E ≤ 50 Mev. If it does not go to zero at 0° and 90°, the possibility $\{+1, -1\}$ can be rejected, and the other two possibilities are distinguished by means of $\sigma_0(E)$, as indicated above.

If $\sigma_0(0°)$ and $\sigma_0(90°)$ are small and this situation holds also for E < 30 Mev, the possibility $\{+1, -1\}$ is the correct one. For the other two possibilities the vanishing of $\sigma_0(\vartheta)$ at both 0° and 90° would require the existence of an even more special relation between the $R^{l'}_{sl}$, which is contradicted by Eq. (2) for E < 30 Mev (elements would have to be equal that differ by an order of magnitude according to Eq. (2).

3. One measures $\sigma_0(E)$ with enough accuracy to distinguish between the $E^{1/2}$ and $E^{3/2}$ laws over the range 0.5 — 50 Mev in the case of decrease of $\sigma_0(E)$ with decrease of the energy. As can be seen from Table II, this suffices to distinguish between all three possibilities.

We repeat that all of these arguments are based on the assumption that a number of conservation laws hold (cf. Sec. 2) and that the relations (2) are valid. If the cross section does not behave as required by the Dirac case, then one could assume, for example, that parity is not conserved in the annihilation. This, however, would be a deduction of no less fundamental importance than the inapplicability of the Dirac equation to p and $\bar{p}$.

APPENDIX A

The Charge Parity of $(p, \bar{p})$. In the representation of the spin components m_1, m_2, the signs of charge ϵ_1 and ϵ_2, and the angles of the relative momentum the correct antisymmetric wave function of the system $(\bar{p}, p)$ must be of the form

$$\psi = 2^{-1/2}\{\langle \mathbf{n} m_1 m_2 \varepsilon_1 \varepsilon_2 \,|\, l\mu s m\rangle - \langle -\mathbf{n} m_2 m_1 \varepsilon_2 \varepsilon_1 \,|\, l\mu s m\rangle\},$$

where μ and m are the projections of the orbital angular momentum and the total spin. The interchange of the indices on ϵ that characterize state and antistate, which is required for the antisymmetrization, can be accomplished by means of the operator $\hat{C}$:

$$\hat{C}\langle \varepsilon_1\varepsilon_2 \,|\,\rangle = \langle \varepsilon_2\varepsilon_1 \,|\,\rangle e^{i\alpha}.$$

Here $\alpha = \eta + \bar{\eta}$ [cf. Eq. (1)]; $\alpha = 0$ if p and $\bar{p}$ are described by the Dirac equation. Taking this fact into account we have

$$\langle -\mathbf{n} m_2 m_1 \varepsilon_2 \varepsilon_1 \,|\, l\mu s m\rangle \equiv N Y_{l\mu}(-\mathbf{n})\, C^{sm}_{1/2 m_2 1/2 m_1} \langle \varepsilon_2\varepsilon_1 \,|\,\rangle$$
$$= N(-1)^{l+s-1} Y_{l\mu}(\mathbf{n})\, C^{sm}_{1/2 m_1 1/2 m_2}\, \hat{C}\langle \varepsilon_1\varepsilon_2 \,|\,\rangle e^{-i\alpha}$$
$$= (-1)^{l+s-1}\hat{C}\langle \mathbf{n} m_1 m_2 \varepsilon_1\varepsilon_2 \,|\, l\mu s m\rangle e^{-i\alpha},$$
$$\psi = 2^{-1/2}[1 + (-1)^{l+s}\hat{C}e^{-i\alpha}]\langle \mathbf{n} m_1 m_2 \varepsilon_1\varepsilon_2 \,|\, l\mu s m\rangle,$$

from which it follows that the charge parity of the system $(p, \bar{p})$ with definite l and s is $(-1)^{l+s}e^{i\alpha}$:

$$C\psi = 2^{-1/2}[\hat{C} + (-1)^{l+s}e^{2i\alpha}e^{-i\alpha}]\,|\, l\mu s m\rangle = (-1)^{l+s}e^{i\alpha}\psi.$$

APPENDIX B

The process of the formation of "protonium" in the collision of a fast antiproton with a proton is the inverse of the photoelectric effect $\gamma + (p, \bar{p}) \rightarrow p + \bar{p}$. Using the well known expression for the cross section for the photoelectric effect[12] and the principle of detailed balancing, we find that for energies of $\bar{p}$ much larger than 25 kev (in the l. s.) electromagnetic capture leads to the formation of "protonium" mainly in the 1s state with a total cross section that falls off rapidly with increase of the energy of the $\bar{p}$:

$$\sigma = \frac{2^7\pi}{3\cdot 137^3}\,\frac{a_0^2}{(ka_0)^5}\,\frac{\xi}{2},$$

where $a_0 = \hbar^2/Me^2 = 0.58 \times 10^{-11}$ cm; $k = \hbar^{-1} \times (ME/2)^{1/2}$, where E is the kinetic energy in the l.s.; $\xi = 1/2$ if the "protonium" is formed in the singlet state, and $\xi = 3/2$ if it is in the triplet state.

For E = 0.25 Mev we get for σ a value of the order of 0.01 mb. On the other hand it can be expected that the cross section for two-meson annihiliation owing to the strong (mesonic) interaction is of the order of several millibarns at low energies (if it is not forbidden).[13]

APPENDIX C

1. The general formula for the cross section for two-meson annihilation is (cf. e. g., the review article by Blatt and Biedenharn[14]):

$$\sigma(\vartheta) \propto \sum_{m_a, m_b} \Big| \sum_{l'\mu' lsm} Y_{l'\mu'}(\vartheta, \varphi) R_{sl}^{l'} C_{sml0}^{l'\mu'} C_{1/2 m_a 1/2 m_b}^{sm} \sqrt{2l+1} \Big|^2$$

$$= \sum_{s,m} \Big[\sum_{l_1', l_1} Y_{l_1' m}(\vartheta, 0) C_{sml_10}^{l_1' m} \sqrt{2l_1+1} R_{sl_1}^{l_1'} \Big]$$

$$\times \Big[\sum_{l_2' l_2} Y_{l_2' m}(\vartheta, 0) C_{sml_20}^{l_2' m} \sqrt{2l_2+1} R_{sl_2}^{l_2'} \Big]^*. \qquad (C.1)$$

The z axis is chosen parallel to the momentum $\mathbf{p}_0$ of the antiproton; m_a and m_b are the spin projections of $\bar{p}$ and p; and $C^{::}_{::}$ are Clebsch-Gordan coefficients.

2. If we confine ourselves only to the matrix elements R_{11}^0 and R_{11}^2 we get from Eq. (C.1) for the reaction $\bar{p} + p \to \pi^0 + \pi^0$ in the "usual" parity possibility (s = 1, l odd) the result

$$\sigma_0(\vartheta) \propto |R_{11}^0|^2 + \frac{5}{2}|R_{11}^2|^2 + \sqrt{10}\,\mathrm{Re}\,(R_{11}^0 R_{11}^{2*})$$

$$+ \cos^2\vartheta \Big[\frac{15}{2}|R_{11}^2|^2 - 3\sqrt{10}\,\mathrm{Re}\,(R_{11}^0 R_{11}^{2*})\Big] \equiv A + B\cos^2\vartheta.$$

Regarding R_{11}^0 and R_{11}^2 we know only that $|R_{11}^0|^2 + |R_{11}^2|^2 \le 1$ (the unitarity relation). For the quantity A we have $A = |R_{11}^0 + (5/2)^{1/2} R_{11}^2|^2 \ge 0$; B can also be negative (but $A + B \ge 0$), and in particular can be equal to zero (isotropy).

With inclusion of l = 1, 3 the total cross section is given by

$$\sigma_0 = \frac{1}{16} k^{-2} \{|R_{11}^0|^2 + 5|R_{11}^2|^2 + 5|R_{13}^2|^2 + 9|R_{13}^4|^2\}.$$

At E = 50 Mev we have $(kr_0)^2 = 0.9$. Then the condition of applicability of Eq. (2), $(kr_0)^2 \ll |4l - 2|$, is well satisfied for R_{13}^2, R_{13}^4, and poorly for R_{11}^0, R_{11}^2. Using nevertheless the estimate (2), we find

$$(5+9)|R_{13}|^2/(1+5)|R_{11}|^2 \approx 0.2\%,$$

where the error of this estimate itself is probably 100 percent. Thus we can suppose that $\sigma \sim k^{-2} \times |R_{11}|^2 \sim E^{1/2}$ in the range 0.5 — 50 Mev, with accuracy not worse than 0.5 percent.

3. For the case {+1, −1} we have s = 1 and $l = l'$. In view of the fact that $C_{10l0}^{l0} = 0$, the sum over m reduces to the sum of equal terms with m = ±1:

$$\sigma(\vartheta) \propto \Big|\sum_l Y_{l,1}(\vartheta, 0) C_{11l0}^{l1} \sqrt{2l+1} R_{1l}^l\Big|^2. \qquad (C.2)$$

The spherical function $Y_{l,1}(\vartheta, 0)$ is proportional to the associated Legendre polynomial of the first kind. Therefore

$$Y_{l,1}(\vartheta, 0) \propto (1-\cos^2\vartheta)^{1/2} dP_l(\cos\vartheta)/d\cos\vartheta,$$

$$\sigma_+(\vartheta) \propto \sin^2\vartheta\,\{|R_{11}^1|^2 + \ldots\} \qquad (C.3)$$

In particular, if we can neglect all the elements R_{1l}^l with $l > 2$, then

$$\sigma_+(\vartheta) \propto \sin^2\vartheta \Big\{|R_{11}^1|^2 + (10/\sqrt{3})\,\mathrm{Re}\,(R_{11}^1 R_{12}^{2*})\cos\vartheta + \frac{25}{3}|R_{12}^2|^2\cos^2\vartheta\Big\},$$

$$\sigma_+(\vartheta) + \sigma_+(\pi - \vartheta) \propto \sin^2\vartheta\Big\{|R_{11}^1|^2 + \frac{25}{3}|R_{12}^2|^2\cos^2\vartheta\Big\}.$$

For E = 10 Mev we get $kr_0 = 0.245$ and $(25/3)|\times|R_{12}^2|^2/|R_{11}^1|^2 \approx 7$ percent.

For the reaction $\bar{p} + p \to \pi^0 + \pi^0$ in this parity signature l must be even. At any energy the cross section is a linear combination of products of pairs of functions $Y_{21}(\vartheta, 0)$, $Y_{41}(\vartheta, 0)$, $Y_{61}(\vartheta, 0)$, etc. They are all proportional to $\sin\vartheta\, dP_l(\cos\vartheta)/d\cos\vartheta$, and since for even l the Legendre polynomial $P_l(\cos\vartheta)$ depends only on even powers of $\cos\vartheta$, for practical purposes $Y_{l,1}(\vartheta, 0) \sim \sin\vartheta\cos\vartheta$, so that

$$\sigma_0(\vartheta) \propto \sin^2\vartheta\cos^2\vartheta\,\{|R_{12}^2|^2 + \ldots\}.$$

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