RADIATION EMITTED BY A CHARGED PARTICLE PASSING THROUGH A PLATE

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Results of calculation of the angular distribution of the radiation emitted by a charged particle passing through an isotropic ferrodielectric plate and through a crystalline plate are presented. In the case of thick plates the main contribution is due to the Cerenkov radiation. An investigation of the solution shows that in the frequency range in which the projections of the wave vector and the Poynting vector on the particle velocity have opposite signs, the Cerenkov radiation is emitted through the back wall of the plate.

The radiation emitted by a charged particle moving through thin dielectric plates is also considered.

1. The problem of energy transfer through boundaries is of a considerable importance in the theory of the Vavilov-Cerenkov effect. In particular, it is interesting to study the angular distribution of the radiation emitted by a charged particle penetrating through a plate. Omitting the calculations, which are analogous to those carried out in reference 1, we give here the final results. The angular distribution of the radiation energy behind the plate is:

a) for the case of a ferrodielectric plate

$$\frac{dW_\omega}{d\Omega} = \frac{e^2v^2}{\pi^2c^3}\frac{\sin^2\vartheta\cos^2\vartheta}{(1-\beta^2\cos^2\vartheta)^2}\,|f(\omega,\vartheta)|^2,$$

$$f(\omega,\vartheta) = \{[(\varepsilon-1)(1-\beta x)-\beta^2(\varepsilon\mu-1)](1+\beta x)$$
$$\times(x+y)e^{i\omega xd/c}+[(\varepsilon-1)(1+\beta x)-\beta^2(\varepsilon\mu-1)](1-\beta x)$$
$$\times(x-y)e^{-i\omega xd/c}-2x[(\varepsilon-1)(1-\beta^2x^2)$$
$$-\beta^2(\varepsilon\mu-1)(1+\beta y)]e^{i\omega d/v}\}\times[(x+y)^2e^{i\omega xd/c}$$
$$-(x-y)^2e^{-i\omega xd/c}]^{-1}(1-\beta^2x^2)^{-1},$$

$$x=\sqrt{\varepsilon\mu-\sin^2\vartheta}\quad(\operatorname{Im}x<0),$$

$$y=\varepsilon\cos\vartheta,\quad d\Omega=\sin\vartheta\,d\vartheta\,d\varphi; \tag{1}$$

b) for the case of a plate cut from a uniaxial crystal perpendicular to its optical axis*

$$\frac{dW_\omega}{d\Omega} = \frac{e^2v^2}{\pi^2c^3}\frac{\sin^2\vartheta\cos^2\vartheta}{(1-\beta^2\cos^2\vartheta)^2}\left|\frac{\varepsilon_0}{\varepsilon_e}F(\omega,\vartheta)\right|^2,$$

$$F(\omega,\vartheta)=[(\gamma_1+\chi\gamma_2)(\chi+\eta)e^{i\omega\chi d/c}+(\gamma_1-\chi\gamma_2)(\chi-\eta)e^{-i\omega\chi d/c}$$
$$-2\chi(\gamma_1+\eta\gamma_2)e^{i\omega d/v}][(\chi+\eta)^2e^{i\omega\chi d/c}$$
$$-(\chi-\eta)^2e^{-i\omega\chi d/c}]^{-1}(1-\beta^2\chi^2)^{-1},$$

$$\gamma_1=\beta^2\sin^2\vartheta+1-\beta^2-\varepsilon_0\beta^2\sin^2\vartheta-\varepsilon_e+\varepsilon_0\varepsilon_e\beta^2,$$

$$\gamma_2=\beta(1-\beta^2-\varepsilon_e/\varepsilon_0+\varepsilon_e\beta^2),\qquad\chi=[\varepsilon_0-(\varepsilon_0/\varepsilon_e)\sin^2\vartheta]^{1/2}$$

$$(\operatorname{Im}\chi<0),\quad\eta=\varepsilon_0\cos\vartheta. \tag{2}$$

*Formula (2) has been derived earlier.[4] The results given in the present article, in the part concerning the crystal plate, were obtained in that investigation.

In deriving the formulas it was assumed that the particle moves perpendicularly to the plate with constant velocity. The following notation has been used in Eqs. (1) and (2): e and v are the charge and velocity of the particle, respectively, β is the ratio of the particle velocity to the velocity of light in vacuum, d is the plate thickness, ϑ is the angle between the normal to the plate and the direction of observation, φ is the azimuth angle, and ϵ_0 and ϵ_e are the transverse and longitudinal components (with respect to the crystal axis) of the dielectric permittivity tensor. In the derivation, all the quantities were expanded in Fourier integrals and, therefore, in the presence of damping it was assumed that the imaginary parts of ϵ, μ, ϵ_0, and ϵ_e are negative.

The formulas for the angular distribution of the radiation energy in front of the plate can be obtained from Eqs. (1) and (2) by replacing β by $-\beta$.* It should be noted that (1) and (2) are valid at sufficiently large distances, where the total field forms a spherical wave.

It is of interest to consider the results obtained from the point of view of the Vavilov-Cerenkov effect. For simplicity, let us assume in the following that the plates are ideally transparent. For the direction of the refracted Cerenkov angles, satisfying the equations

$$\text{a) } \beta^2(\varepsilon\mu-\sin^2\vartheta)=1,\quad\text{b) } \beta^2(\varepsilon_0-\varepsilon_0\varepsilon_e^{-1}\sin^2\vartheta)=1. \tag{4}$$

an infinity of the type $\lim_{\alpha\to 0}\sin^2(\alpha d)/\alpha^2$, where d

*For $\mu = 1$ and $\varepsilon_0 = \varepsilon_e = \varepsilon$, Eqs. (1) and (2) coincide with the results of the author given in references 1 and 2. [In reference 2, typographical errors occurred in the signs of the second and third exponents in the numerator of Eq. (2)].

is the plate thickness, occurs in (1) and (2). In the case of thick plates (compared with the wavelength), (1) and (2) exhibit sharp intensity maxima, proportional to the square of the plate thickness. The width of these maxima is inversely proportional to d. As the result of reflection of the Cerenkov radiation from the front surface, similar maxima appear in front of the plate also. A study of the formulas reveals that the ratio of the Cerenkov radiation energy emitted backward (W_b) to the energy emitted forward (W_f) is, in the two cases considered, given by the expressions*

$$\text{a)}\ \frac{W_b}{W_f} = \frac{(1 - \varepsilon\beta\cos\vartheta_r)^2}{(1 + \varepsilon\beta\cos\vartheta_r)^2}, \qquad \text{b)}\ \frac{W_b}{W_f} = \frac{(1 - \varepsilon_0\beta\cos\vartheta_r)^2}{(1 + \varepsilon_0\beta\cos\vartheta_r)^2}, \tag{4}$$

where ϑ_r are the refracted Cerenkov angles. It can be seen from Eq. (4) that, in the frequency range where the Cerenkov light ray makes an obtuse angle with the direction of motion of the particle,[4] more energy is emitted backward than forward. The forward radiation is a result of reflection from the back wall. It is interesting that when the radiation is emitted at Brewster's angle, for which in the given case the denominators in Eq. (4) vanish, the forward Cerenkov radiation disappears completely.

2. Let us consider the radiation in thin dielectric plates. We shall use the formula for radiation energy[2] correct for plates of arbitrary thickness. Assuming that $|\sqrt{\epsilon}|\,\omega d/c \ll 1$ and that $\omega d/v \ll 1$, and expanding all the exponents in Eq. (2)[2] in a series, we obtain for the case of a relativistic particle the angular distribution of radiation intensity in front of the plate in the form

$$\frac{dW_\omega(1)}{d\Omega} = \frac{e^2\omega^2 d^2}{4\pi^2 c^3}\,|\varepsilon - 1|^2 \frac{\sin^2\vartheta\cos^2\vartheta}{(1 - \beta^2\cos^2\vartheta)^2}. \tag{5}$$

Since the result is independent of the sign of the particle velocity, the angular distribution behind the plate is identical. The corresponding spectral density of the radiation energy is given by the formula

$$W_\omega(1) = \frac{e^2\omega^2 d^2}{4\pi c^3}\,|\varepsilon - 1|^2\left[\ln\frac{4}{1-\beta^2} - 3\right]. \tag{6}$$

It is obvious that the perturbations due to separate patches of low inhomogeneity, encountered along the particle path, are coherent. This explains why the radiation energy varies as the square of the plate thickness.*

From the results given above one can conclude that the radiation energy in a thin stack of thin plates will be proportional to the square of the number of plates. For a large separation between the plates, the coherence condition is violated, and the energy is proportional to the number of plates. In order to determine the coherence condition let us consider the angular distribution of radiation energy for the passage of a charged particle through a stick consisting of m thin plates. Omitting the calculations, the final result for a relativistic particle is

$$\begin{aligned} dW_\omega(m)/d\Omega = |\,1 &+ \exp[i\omega c^{-1}(l - l\beta\cos\vartheta + d)] \\ &+ \exp[2i\omega c^{-1}(l - l\beta\cos\vartheta + d)] \\ &+ \ldots + \exp[(m-1)\, i\omega c^{-1}(l - l\beta\cos\vartheta \\ &+ d)]\,|^2\, dW_\omega(1)/d\Omega, \\ (|\sqrt{\varepsilon}|\,\omega dm/c \ll 1), \end{aligned} \tag{7}$$

where l is the distance between the plates and $dW_\omega(1)/d\Omega$ is determined from Eq. (5). The radiation energy density per unit solid angle in Eq. (5) has a maximum in the direction $\vartheta \sim \sqrt{1 - \beta^2}$. All particles will radiate coherently if, in the direction $\vartheta \sim \sqrt{1 - \beta^2}$, The largest exponent in Eq. (7) is much smaller than unity,

$$\omega c^{-1}\,[l\,(Mc^2/E)^2 + d]\,m \ll 1, \tag{8}$$

where M is the particle mass and E is its total energy. The spectral radiation energy density is proportional to m^2:$W_\omega(m) = m^2 W_\omega(1)$. Under the condition

$$\omega c^{-1}[l\,(Mc^2/E)^2 + d] \gg 1 \tag{9}$$

the coherence condition for adjoining plates is violated and, therefore, $W_\omega(m) = mW_\omega(1)$.†

A study of Eq. (5) reveals that the logarithmic increase in the spectral energy density of the transition radiation (6) from each plate is due to the emission at all angles large compared with

*At small distances from the plate, the refracted Cerenkov waves are cylindrical. In the case of an isotropic dielectric plate these waves were studied by Garibyan and Chalikyan.[3] It should be taken into account that the last bracket of Eq. (7)[3] should read $(\varepsilon\sqrt{1+v^2c^{-2}(1-\varepsilon)}-1)$ instead of $(\sqrt{1+v^2c^{-2}(1-\varepsilon)}-1$. The ratio of the Cerenkov radiation energy emitted backward [Eq. (7) of reference 3] to the energy emitted forward [Eq. (8)[3]] is then identical with the expression obtained in reference 1 for an isotropic plate. It follows from Eq. (4) as a particular case for $\mu = 1$ and $\varepsilon_0 = \varepsilon_e = \varepsilon$.

*The calculations of the energy loss for a particle traversing a thin plate were carried out also by Garibyan.[5] The author calculated the work done by the force acting upon a particle, limiting himself to the term linear in the plate thickness. Since the Poynting-vector flux through a remote sphere is proportional to the square of the thickness, the estimates obtained in reference 5 should be attributed to other, non-radiative losses, i.e., ionization of the atoms in the plate.

†Garibyan[6] found the radiation field due to the passage of a charged particle through a stack of plates. His conclusions regarding the radiation energy do not follow, however, from the solution obtained.

$\vartheta \sim \sqrt{1-\beta^2}$. Since, at large angles, the coherence condition is more stringent, the above fact will lead to a lesser variation of the coherence condition with the energy of an ultra-relativistic particle.

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26