EXPERIMENTAL INVESTIGATION OF CYCLOTRON RESONANCE IN METALS

M. Ya. AZBEL' and É. A. KANER

Physico-Technical Institute, Academy of Sciences, Ukr. S.S.R.

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The results of some recent studies of cyclotron resonance in metals are discussed. Some additional opportunities for an experimental investigation of this phenomenon are indicated.

1. Many recent papers have been devoted both to experimental[1-13] and to the theoretical[14-26] investigations of cyclotron resonance in metals. Fawcett,[1] Bezuglyĭ and Galkin,[2] Kip et al.[3] and Khaĭkin[4] observed resonance in tin, while Bezuglyĭ and Galkin carried out additional investigations on lead[5] and indium.[6] Bismuth and its alloys with tin and thallium were studied by Aubrey and Chambers[7] and Galt et al.,[8] copper by Fawcett[1] and Langenberg et al.,[9,10] zinc by Galt et al.[11] and aluminum by Langenberg and Moore[12] and also by Fawcett.[13] Thus, cyclotron resonance has been observed by now in seven metals. It is consequently of interest to evaluate the experimental results, to ascertain the degree of their agreement with theoretical predictions, and to indicate certain supplementary possibilities of experimental investigation of cyclotron resonance in metals. This is the purpose of the present article.

The authors have shown earlier[22-26] that in metals at low temperatures in a constant magnetic field **H** parallel to the surface of the metal there is resonant absorption of energy from the high frequency field. The absorption has a minimum at resonance. The resonance takes place in magnetic fields corresponding to the values of the cyclotron frequency* $\Omega_q \approx \omega/q$, where ω is the frequency of the alternating magnetic field and $q = 1, 2, 3\ldots$. Variation of only the resonant frequencies $\Omega = eH/mc$ or the distances between them (e is the absolute value of the electron charge and c the velocity of light) permits a direct determination of the effective mass m of the electrons ("holes") in the metal and its anisotropy.†

The effective mass will be determined the more accurately, the greater, on the one hand, the average mean free time of the electrons τ_0 (that is, $\omega\tau_0$) and the more accurately, on the other hand, are satisfied in the experiment the theoretical requirements that the magnetic field be parallel to the surface of the metal and that this surface be of highest quality. As $\omega\tau_0$ tends to infinity, the equation $\Omega = \omega/q$, which determines the position of the resonance, becomes exact and not approximate, as in the case of finite value $\omega\tau_0$. If $\omega\tau_0$ does not have the finite value called for by the theory, the absorption minimum shifts towards the smaller magnetic fields. However, independently of the satisfaction of these requirements, the value obtained for the effective mass will be approximately correct.

In contradistinction, the shape of the resonance curve and the sharpness of the resonance are quite sensitive to the parallelness of the magnetic field and to the surface finish; furthermore, the stronger the magnetic field, the more stringent these requirements. Failure to satisfy these requirements may lead to a change in the order of magnitude of the depth and width of the resonance. A quantitative comparison of these characteristics of the resonant curves with theory is possible only when the theoretically predicted law of decrease of surface impedance of the metal, $Z \sim H^{-1/3}$, is satisfied in strong magnetic fields, where $Z = R + iX$ (see references 20, 24, and 25). If the theoretical requirements are satisfied and the dependence $R \sim H^{-1/3}$ is experimentally satisfied in a strong field, a more detailed quantitative comparison of

*We emphasize that the multiplicity of the resonant frequencies is connected with the anomalous character of the skin effect, and not with the deviation of the dispersion from quadratic, and takes place even with a square-law dispersion.

†In the case of a non-quadratic dispersion, when the effective mass depends on the projection p_z of the electron momentum p in the direction of the magnetic field $\mathbf{H} \| z$, we have in mind here the extremal value of $m(p_z)$ with respect to p_z. In this case the sharpness and width of each resonant line differs for the minimum and maximum (with respect to p_z) values of the effective mass.[22-26]

the experimental data with theory is possible. Since the law of dispersion of current carriers in the metals is not known beforehand, and the dependence of Z on H is essentially connected with the form of the dispersion law, comparison with theory must be carried out either near resonance, or in strong fields where all the dependences can be determined in explicit form.

The use of a simple formula for the quadratic dispersion law in the reduction of the experimental data, as was done in many papers (see, for example, reference 11), is naturally not permissible in the general case.

2. All the experiments[1-13] have confirmed the theoretically predicted high "sensitivity" of the effect to the parallelness of the constant magnetic field to the specimen surface finish. A large number of harmonics are observed in tin, copper, zinc, and aluminum (up to 15 or 17 in tin,[3,4] up to 12 in copper[10] and up to 8 in zinc[11]), thus splendidly confirming the multiplicity of the resonant frequencies.[10,4]

The anisotropy of the effective mass in one of the crystallographic planes was fully investigated for single crystals of tin,[3,4] bismuth,[7] and copper.[10] In particular, the data of Aubrey and Chambers[7] and of Galt et al.[8] have confirmed the three-ellipsoid Fermi-surface model proposed by Shoenberg[27] for electrons in bismuth, and made possible the calculation of some of their parameters.

The results of the investigation of cyclotron resonance in copper[10] agree with the shape obtained by Pippard[28] for the Fermi surface. The shape of the surface is thus confirmed for copper by four independent methods: the anomalous skin effect (Pippard[28]), the de Haas–van Alphen effect (Shoenberg[29]), galvanomagnetic phenomena (Alekseevskiĭ and Gaĭdukov[30]) and cyclotron resonance (Langenberg and Moore[10]).

The low effective-mass values, $m \sim 0.2m_0$, obtained by Langenberg and Moore[12] and by Fawcett[13] with cyclotron resonance in aluminum, agree with the "cushion-like" Fermi surface for "holes" in aluminum, constructed by Gunnersen[31] from data on the de Haas–van Alphen effect. This Fermi surface, according to Heine,[32] is located in the corners of the first Brillouin zone and corresponds to a "hole" concentration on the order of 4×10^{-3} per atom. The "heavy" carriers with mass on the order of $1.5m_0$, not observed in the de Haas–van Alphen effect and observed by Langenberg and Moore[12] with cyclotron resonance, are connected with the main group of carriers in aluminum, the Fermi surface of which is located in the second Brillouin zone. The smallest electron group ($\sim 10^{-5}$ per atom), responsible for the long-wave oscillations in the de Haas-van Alphen effect and located at the corners of the third Brillouin zone, was naturally not observed in cyclotron-resonance experiments.

3. Many experimenters[3,9,10,12] measured directly not the surface resistance R(H), but its derivative dR/dH with respect to the magnetic field. Plots of dR/dH vs. H have a clearly pronounced resonant character. The resonant cyclotron frequencies and the effective masses are determined from the position of maximum of the derivative dR/dH. At first glance this may appear to be incorrect, since R(H) has a minimum at resonance,[22-25] and it is natural to expect dR/dH to vanish at the resonance point. It must be borne in mind, however, that the function R(H) has a cusp at the resonance points when $\omega\tau_0 = \infty$, and the derivative dR/dH goes to infinity when $H = H_{res} + 0$.

By way of illustration, let us consider a quadratic dispersion law, when

$$Z(H) = 2R(0)\,[1 - \exp(-2\pi i\omega/\Omega)]^{1/3}\exp(i\pi/3),$$

$$\frac{dZ}{dH} = -\frac{2\pi}{3}\,Z(0)\,\frac{1+i\omega\tau_0}{H\Omega\tau_0}\left[1-\exp\left\{-\frac{2\pi(1+i\omega\tau_0)}{\Omega\tau_0}\right\}\right]^{-2/3} \times \exp\{-2\pi i(1+i\omega\tau_0)/\Omega\tau_0\}. \quad (1)$$

The phase $1 - \exp(-2\pi i\omega/\Omega)$ is chosen in the interval $(-\pi/2, \pi/2)$ to make R(H) positive. Near resonance, when $|\kappa_1| \ll 1$ ($\kappa_1 = 1 - q\Omega_{res}/\omega$, and q is an integer),

$$R(H) = 2(2\pi q)^{1/3}R(0)\,|\varkappa_1|^{1/3}\cos(\pi/3 + \tfrac{1}{3}\tan^{-1}(\pi q\varkappa_1)^{-1}). \quad (2)$$

To the right of resonance ($H > H_{res}$, $\kappa_1 < 0$)

$$R(H) = \sqrt{3}\,(2\pi q)^{1/3}R(0)\,|\varkappa_1|^{1/3}, \quad (3)$$

$$dR/dH = 3^{-1/2}R(0)H^{-1}(2\pi q)^{1/3}\,|\varkappa_1|^{-2/3}, \quad (4)$$

To the left of resonance ($H < H_{res}$, $\kappa_1 > 0$)

$$R(H) = R(0)\,(2\pi q\varkappa_1)^{4/3}, \quad (5)$$

$$dR/dH = -\tfrac{4}{3}R(0)H^{-1}(2\pi q)^{4/3}\,\varkappa_1^{1/3}. \quad (6)$$

Thus, dR/dH tends to zero to the left of resonance and to infinity on the right. Consequently, the position of resonance is best determined from the maximum of dR/dH. The position of the minimum of dR/dH does not coincide with $H = H_{res}$. Figure 1 shows schematically the variation of R(H) and of dR/dH as a function of the magnetic field near one of the resonance points.

Similar curves are obtained in the case of a non-quadratic dispersion, when the resonant frequency corresponds to the minimum effective mass. In the case when the resonance corresponds

to the largest value of effective mass with respect to p_z, $dR/dH = -\infty$ when $H = H_{res} - 0$ and $dR/dH = +\infty$ when $H = H_{res} + 0$ (see Fig. 2). In the latter case the resonant magnetic field can be determined both from the maximum and from the minimum of the derivative dR/dH. The fact that the minimum and the maximum of dR/dH are close together (even for a finite value of $\omega\tau_0$), makes it possible to distinguish between the maximum and minimum effective masses.

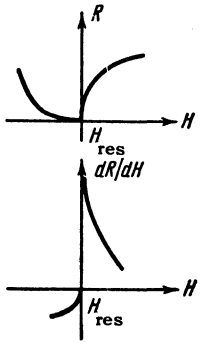

FIG. 1

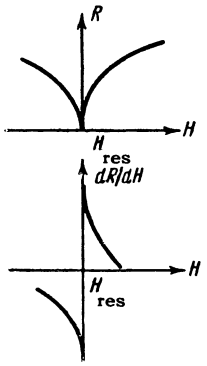

FIG. 2

4. The resonant frequencies can be determined and were determined experimentally both by the minimum of absorption, and by the maximum of its derivative with respect to the magnetic field. Many experimental curves show clearly (see, for example, reference 10) that the resonance is much more clearly pronounced in the dR/dH curves than in the R(H) curves. To understand the reason for this, it is necessary to consider the dependence of R and dR/dH on the magnetic field at large but finite value of $\omega\tau_0 = \xi^{-1}$, $\xi \ll 1$.

Differentiating the expressions for the surface resistance given in Eqs. (3.18), (4.6), (6.3), and (6.8) of reference 24, we readily obtain equations for both the height of the resonance maximum dR/dH and for the shift in the resonant cyclotron frequency relative to ω/q. The results of these calculations and of their comparison with the calculated values of R(H) (see reference 24) are listed in the table, where H_1 is the magnetic field at which $\Omega = \omega$ and τ_0 is the mean free time.*

Thus, only when the dispersion is quadratic does the relative depth of the resonance minimum of R(H) have the order of magnitude of the resonance height of the maximum of dR/dH. For any essentially non-quadratic dispersion law, the relative height of the maximum of dR/dH is greater than the relative depth of the minimum of R(H), by $(\omega\tau_0)^{2/3}$ and $(\omega\tau_0)^{7/18}$ for the maximum and minimum effective masses, respectively. This agrees well with experiment. The comparison of the R(H) and dR/dH curves affords an additional deduction of the dispersion law (naturally, if the theoretical requirements are satisfied).

The relative shift of the resonant cyclotron frequency (magnetic field) for dR/dH, for any dispersion law, is positive and on the order of $\xi = (\omega\tau_0)^{-1} \ll 1$, i.e., it is of opposite sign and substantially smaller than the corresponding shift for R(H) when the dispersion is quadratic ($\kappa \sim \xi^{1/2}$), or non-quadratic with $m = m_{min}$ ($\kappa_1 \sim \xi^{2/3}$).[24] This conclusion can be used for an experimental determination of the effective mass.

The minimum of dR/dH does not have a resonant character when the dispersion is quadratic and the mass is a minimum (with respect to p_z), since it is far removed from $H = H_{res}$. For the maximum mass (with respect to p_z)

$$(dR/dH)_{min} \sim -(R(0)/H_1)q^{4/3}(\omega\tau_0)^{5/6}, \tag{7}$$

$$\frac{\Omega_{res} - \omega/q}{\omega/q} = -\left(\omega\tau_0 \tan\frac{\pi}{11}\right)^{-1}; \tag{8}$$

the minimum of dR/dH is a resonant one, and $(dR/dH)_{min}$ tends to $-\infty$ as $\tau_0 \to \infty$, as already mentioned (see Fig. 2).

5. A characteristic feature of the experimental curves is their considerable asymmetry in the vicinity of resonance: the derivative of dR/dH to the left of the maximum (like the value of R(H) to the right of the minimum) changes much more rapidly than on the right (see, for example, references 3 and 10). This feature is manifest also in the theoretical curves — see Figs. 1 and 2. A similar variation of dR/dH and R(H) with the magnetic field can be seen also in Fig. 5 of reference 24; it must be borne in mind, however, that R is plotted in this figure as a function of 1/H and not of H.

6. Certain experimental curves exhibit an increase in the height of the maximum with increasing magnetic field (this is seen particularly clearly on the figure of reference 10), beginning with harmonics of a definite order. This can be attributed to the fact that when $2\pi/\Omega\tau_0 > 1$, the derivative dR/dH diminishes exponentially [$\sim \exp(-2\pi/\Omega\tau_0)$] with diminishing magnetic field and the resonance maxima disappear. This affords a convenient method for estimating the value of τ_0 from the number q_0 of the distinctly noticed resonant harmonics $\tau_0 \sim 2\pi q_0/\omega$. Thus, for example, $q_0 \sim 10$ in the experiments of Kip, Langenberg, et al.[3] on tin and $q_0 = 9$ in the experiments of Langenberg and Moore[10] on copper. When $\omega = 2\pi \cdot 24 \times 10^9$ cps this yields

*The mean free time τ_0 for the anomalous skin effect was cogently introduced in reference 26.

Resonant values of the surface resistance R_{res}, the derivative $(dR/dH)_{res}$, and the relative shift of the minimum of R and of the maximum of dR/dH; $-\kappa_1 = q\Omega_{res}/\omega - 1$

	$R_{res}/R(0)$	$(dR/dH)_{res}/R(0)H_1$	$-\varkappa_1$ for $R(H)$	$-\varkappa_1$ for dR/dH
Quadratic dispersion law, $m(p_z)=\text{const}$	$(2\pi q/\omega\tau_0)^{2/3}$	$\sim (q^2\omega\tau_0)^{2/3}$	$-(2\pi q\,\omega\tau_0)^{-1/2}$	$\left(\omega\tau_0 \cot\frac{\pi}{10}\right)^{-1}$
Non-quadratic dispersion law, $m(p_z)=m_{min}$	$\sim(q^2/\omega\tau_0)^{4/5}$	$\sim q^{4/5}(\omega\tau_0)^{5/6}$	$\sim -(q\,\omega\tau_0)^{-2/3}$	$\left(\omega\tau_0 \cot\frac{\pi}{22}\right)^{-1}$
Non-quadratic dispersion law, $(p_z)=m_{max}$	$\sim(q^2/\omega\tau_0)^{1/6}$	$\sim q^{4/3}(\omega\tau_0)^{5/6}$	$-\left(\omega\tau_0\tan\frac{\pi}{5}\right)^{-1}$	$\left(\omega\tau_0 \cot\frac{3\pi}{22}\right)^{-1}$

$\tau_0 \sim 3\times10^{-10}$ sec, in good agreement with the values $\tau_0 \sim 3\times10^{-10}$ sec given in these papers.

There is, however, a circumstance that casts doubts on this conclusion. The point is that, according to theory, the instant that a clearly pronounced resonance appears, the height of the resonance maxima should decrease with increasing magnetic field (i.e., with decreasing number of the harmonic q):

$$(dR/dH)_{max} \sim q^{4/3} \sim H_q^{-4/3}.$$

The opposite takes place in the experiments of Langenberg and Moore: the resonant maxima increase with increasing H. This can be explained only by assuming that even in relatively strong magnetic fields on the order of 2000 oe, we get $\Omega\tau_0 \gtrsim 2\pi$, owing to failure to maintain the magnetic field strictly parallel to the surface of the metal and to the insufficiently smooth surface; an alternative is to consider the presence of anomalous zones. Such factors can cause the value of τ_0, which determines the surface impedance, to be itself dependent on the magnetic field and to be considerably less than in the bulk metal.[24] In this case, when the effective magnetic field is weak, the formulas of references 22 — 25 are naturally inapplicable.

When $\Omega\tau/2\pi \ll 1$, strictly speaking, there is no resonance, but the impedance Z has a small periodic increment ΔZ:

a) in the case of quadratic dispersion

$$\Delta Z/Z \approx \frac{1}{3}\exp\left\{-\frac{2\pi}{\Omega\tau} - 2\pi i\,\frac{\omega}{\Omega}\right\},$$

b) in the general case

$$\Delta Z/Z = \frac{1}{6p_0}\left\{\frac{1}{2}\left[\frac{d^2}{dp_z^2}(\Omega\tau)\right]_{p_z=p_0}\right\}^{1/2} \times \exp\left\{-\frac{2\pi}{(\Omega\tau)_{max}} - 2\pi i\frac{\omega}{\Omega(p_0)}\right\}$$

[p_0 is the point of maximum of $\Omega(p_z)\tau(p_z)$]. There exists, naturally, in addition to this increment, the increment obtained in reference 24, which is monotonic in H and of order $(l^2/\delta_{eff})^2$ (l is the mean free path, r the radius of the orbit, and ω_0 the plasma frequency). Obviously

$$d\ln X/dH \sim q^n \exp(-2\pi q/\omega\tau)$$

(n = 2 for quadratic dispersion and n = 3/2 for a nonquadratic law).

Unfortunately, the knowledge of $\Omega(p_0)$ determines the effective mass at an unknown point and thus affords merely an estimate of the order of magnitude of m*. The substantial temperature dependence of d ln X/dH in the experiments of Khaĭkin[4] shows that the measurements have been performed, possibly for the most part, in the region where $\Omega\tau/2\pi \gtrsim 1$. Nevertheless, the effect is quite noticeable because of the high accuracy of the experiment.

When $\Omega\tau \gg 1$, the derivative d ln X/dH depends little on the temperature and $(d\ln X/dH)_{res} \approx \pm\omega\tau_0/24H\pi q$ when $(\Omega_{res} - \omega/q)/(\omega/q) = \pm 2\pi q/\omega\tau_0$. The high-frequency oscillations (relative to the magnetic field) observed in Khaĭkin's experiments may be due to the quantum oscillations predicted in reference 33 for the high-frequency region, oscillations which are quite useful for the construction of the Fermi surface.

For a detailed comparison of the theory with experiment, an experimental verification of the theoretical requirements becomes particularly important. It is convenient to study for this purpose the measurements of dR/dH and d ln X/dH in a strong magnetic field. In a strong field we have, independently of the dispersion law

$$dR/dH \sim H^{-1/3} \quad \text{for} \quad 2\pi\omega \ll \Omega \ll \pi\omega^2\tau_0,$$

$$dR/dH \sim H^{-4/3},\ d\ln X/dH \approx -\tfrac{1}{3} \quad \text{for} \quad \Omega \gg \pi\omega^2\tau_0.$$

We emphasize that the law of decrease of dR/dH in a strong magnetic field strictly parallel to the surface of the metal should be independent of the rotation of the vector **H** in the plane of the sample. Another criterion for the satisfaction of the theo-

retical requirements is the "regular" dependence of the height of the maximum $(dR/dH)_{max}$ or $(d \ln X/dH)_{res}$ on the number of the harmonic q ($dR/dH \sim q^{4/3}$, $d \ln X/dH \sim q^{-1}$).

7. In spite of the large number of experiments on cyclotron resonance in metals, the dependence of the absorption on the direction of polarization of the alternating electric field (direction of the high-frequency current) was not investigated for a fixed direction of **H**. Therefore, naturally, the previously predicted "resonance at a selected polarization" was not observed[24,25] at effective masses corresponding to a central section and elliptical reference points (at which the electron velocity **v** is parallel to **H**) of the Fermi surface. The result is modified for dR/dH in the following manner: resonance will be observed for any direction of polarization of the electric field **E** relative to **H**, with the exception of those directions, at which **E** is perpendicular to the direction of the electron velocity at the reference point ($\mathbf{E} \perp \mathbf{H}$) and at the point of the central section of the Fermi surface $p_z = 0$ for $v_n = 0$ (v_n is the projection of **v** on the normal to the surface of the metal).

8. Let us emphasize again that, generally speaking, only the ratio $\Omega \approx \omega/q$ is "stable," at sufficiently large $\omega\tau_0$ and in the absence of a noticeable superposition of the resonant frequencies, with respect to experimental errors and with respect to different assumptions of the theory. Thus, for example, the discrepancy between the experimental and theoretical values of the phase shift of the oscillation, pointed out by many authors,[7,10,11] can be caused by many factors: insufficiently large $\omega\tau_0$, the approximate character of Eq. (3.16) of reference 24 [used to determine the impedance and in which the complex terms of order $(\delta/r)^{1/2}$ have been discarded], the rotation of the principal axes of the surface-impedance tensor $R_{\mu\nu}$ with variation of the magnetic field (even near resonance — in the next higher approximations), **H** not parallel to the surface of the metal, etc.

9. To establish the form of the Fermi surface in metals and the electron velocities on it, it would be quite desirable to investigate further the anisotropy of the resonance, both in different crystallographic directions and in an inclined magnetic field, where the resonance may be due to diamagnetic quantum oscillations.[33-34] In particular, it is interesting to measure simultaneously the classical and the quantum parts of dR/dH. We note that cyclotron resonance is particularly useful in explaining the features of closed plane sections of the Fermi surface. Open surfaces affect little the form of the resonance curves, since for a given direction of **H**, they are encountered as a rule together with closed ones. It is possible to determine the direction of the open trajectories only from the reduction and vanishing (for a certain direction of **H**) of a given resonant frequency in an investigation of the anisotropy of the resonance. Data on galvanomagnetic phenomena can be used to obtain open trajectories.[30,35]

In conclusion, we take this opportunity to note that the solutions obtained by Mattis and Dresselhaus[17] and Rodriguez[18] for the kinetic equation of electrons in a magnetic field parallel to the surface of a metal do not satisfy the boundary conditions. Actually, owing to violation of the symmetry of the problem by the magnetic field, the continuation of the distribution function (even or zero) to the surface outside the metal does not correspond to the boundary conditions for any orientation of **H** (see references 20, 23, 26, 34, and 37), unlike the case $\mathbf{H} = 0$. The only exception is a perpendicular magnetic field with isotropic dispersion. In a parallel magnetic field, the problem becomes more complicated by the discontinuity of the distribution function.[36,37]

The situation is still more complicated in the quantum case, where the character of the reflection of the electrons from the surface of the metal changes the electron energy levels (see reference 33 for more details). The particular error made by these authors is that in solving the kinetic equation by Laplace transforms they did not account for the jump in the distribution function on the boundary between the metal and the vacuum. It was precisely an incorrect allowance for the boundary condition that has led to differences between the formulas of references 17 and 18 and those of references 22 — 24 and 33. Incidentally, in the case of a quadratic dispersion law, the reflection of the electrons from the surface of the metal does not influence greatly the results in either the classical or quantum case (with exception of the case of pure specular reflection, which is only of academic interest[26]), so that the results of Mattis and Dresselhaus and of Rodriguez are qualitatively correct.

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