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TEMPERATURE DEPENDENCE OF MAGNETOSTRICTION

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The temperature dependence of linear (anisotropic) and volume (isotropic) magnetostriction in the low-temperature region is studied on the basis of the phenomenological method of the theory of spin waves.

1. In a recent work of the authors,[1] the temperature dependence of the ferromagnetic anisotropy constants was studied on the basis of the phenomenological theory of spin waves. The results obtained in that work can also be used for discussion of the problem of the temperature dependence of the constants of anisotropic magnetostriction; this will be done in the present article. In addition, the temperature dependence of the volume (isotropic) magnetostriction will also be considered here.

The magnetoelastic energy density of a ferromagnetic can be expressed in the general case, through terms linear in the components of the elastic stress tensor σ_{ls}, in the form

$$\mathcal{H}_{\mathrm{me}} = -\lambda_{ls;\, n_1 n_2 n_3} \sigma_{ls} m_x^{n_1} m_y^{n_2} m_z^{n_3} - G_{ijls} \sigma_{ls} \frac{\partial m_t}{\partial r_i} \frac{\partial m_t}{\partial r_j}. \qquad (1)$$

Here $\mathrm{m_t(r) = M_t(r)/M_0}$ are the components of the unit vector of the local magnetization; $\mathrm{M_0}$ is the absolute saturation; i, j, l, s, t ≡ x, y, z; $\mathrm{n_1}$, $\mathrm{n_2}$, $\mathrm{n_3}$ are integers such that $\mathrm{n_1 + n_2 + n_3 = 2N}$ is an even number; in (1), and generally in what follows, summation over twice occurring indices is understood.

The first term in (1) gives the anisotropic part of the magnetoelastic energy, expressed in the form of a series of ascending powers of the components of the magnetization. The second term represents the change of exchange energy produced by the stresses, so that

$$G_{ijls} = -\partial A_{ij} / \partial \sigma_{ls}, \qquad (2)$$

where $\mathrm{A_{ij}}$ are the exchange-interaction parameters. The specific form of the tensors λ and G is determined by the crystal symmetry.

We now note that the components of the elastic stress tensor σ_{ls}, in the problem under consideration, may be treated as parameters related through the elastic moduli to the equilibrium deformations of the crystal lattice, without allowance for thermal vibrations of the latter. For harmonic elastic vibrations of the lattice do not change the equilibrium deformations of the crystal and consequently do not affect the mean magnetoelastic energy of the ferromagnetic; and allowance for the anharmonicity of these vibrations leads to a thermal expansion, which in the first approximation combines additively with the temperature change of the magnetostrictive deformations. Thus the temperature dependence of the magnetostrictive deformations will be basically determined solely by the thermal oscillations of the magnetization, i.e., by the spin waves. Therefore our problem reduces to the calculation of the energy spectrum of the spin waves with, as compared with the previous work,[1] additional allowance for the magnetoelastic energy (1).

It is easy to see that addition to the Hamiltonian[1] of the magnetoelastic part (1) leads to the following expression for the energy of a spin wave:

$$\varepsilon_k = (\mu/M_0)\{2(A_{ij} - G_{ijls}\sigma_{ls})k_i k_j + M_0 H - \frac{1}{2}\sum_N [2N(2N+1) - \Delta_\alpha][f_N(0) - \sigma_{ls}\varphi_{ls;\,N}(0)]\}, \qquad (3)$$

where

$$\varphi_{ls,\,N} = \sum_{n_1+n_2+n_3=2N} \lambda_{ls,\, n_1 n_2 n_3} \alpha_x^{n_1} \alpha_y^{n_2} \alpha_z^{n_3} \qquad (4)$$

is a homogeneous polynomial of degree 2N in the direction cosines α_{i} of the equilibrium magnetization.* At the same time it is necessary to add to the energy $\mathcal{H}_0$ of the ground state a term of the form

$$-\sum_N \sigma_{ls}\varphi_{ls;\,N}(0). \qquad (5)$$

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*For the remaining symbols, see our previous paper.[1] In the expression (3) for ε_k, as compared with the corresponding formula of the other paper, the magnetostatic energy has been dropped as nonessential, and saturation ($\boldsymbol{\alpha}\mathbf{H} = \mathbf{H}$) has been assumed (see correction of a misprint, JETP **38**, 667 (1960), this issue, Soviet Phys. JETP **11**, 1326 (1960).

If we now calculate the thermodynamic potential Ω of the system according to the appropriate formula from reference 1 with allowance for (3) and (5), we can then find from it the components of the magnetostrictive strain tensor:

$$u_{ls} = -\frac{\partial\Omega}{\partial\sigma_{ls}} = \sum_N \left\{1 - \frac{M(0) - M(T, H)}{2M(0)}\left[2N(2N+1) - \Delta_\alpha\right]\right\}$$
$$\times \varphi_{ls;\,N}(0) + 2\frac{\mu}{M_0}\sum_{\mathbf{k}} G_{ijls}k_ik_j\left(e^{-\varepsilon_{\mathbf{k}}/\varkappa T} - 1\right)^{-1}, \qquad (6)$$

here M(T, H) is the mean magnetization of the crystal in field H at temperature T, and M(0) $\equiv M_0$ is the magnetization at T = 0°K. The first sum in (6) represents the anisotropic magnetostriction, which causes a dependence of the linear dimensions and shape of the ferromagnetic specimen on the direction of the magnetization. The second sum in (6) gives the isotropic magnetostriction, which leads to an additional temperature dependence of the volume of the ferromagnetic specimen as compared with a nonferromagnetic (spontaneous volume magnetostriction or thermostriction), and also to a dependence of the volume on the value of the magnetizing field in the paraprocess region (paraprocess magnetostriction).

2. We consider first the anisotropic magnetostriction. If for the polynomials $\varphi_{ls;N}$ we use harmonic polynomials, for which $\Delta\varphi_{ls;N} = 0$, then after substitution of (4) in (6) we can express the anisotropic part of the magnetostrictive strains in the form

$$u_{ls} = \sum_{n_1n_2n_3} \lambda_{ls;\,n_1n_2n_3}(T)\,\alpha_x^{n_1}\alpha_y^{n_2}\alpha_z^{n_3},$$

where the coefficients λ have the meaning of magnetostrictive constants at temperature T; here, independently of the symmetry class of the crystal, any magnetostriction constant of order $N = \frac{1}{2}(n_1 + n_2 + n_3)$ satisfies the general relation

$$[\lambda_N(0) - \lambda_N(T)]/\lambda_N(0)$$
$$= N(2N+1)[M(0) - M(T, H)]/M(0), \qquad (7)$$

which exactly coincides in form with the corresponding relation for the magnetocrystalline-anisotropy constants.*

As an example, we write the harmonic expansion of the magnetoelastic energy through terms of fourth degree in the direction cosines α_i of the magnetization (i.e., through N = 2) for crystals of cubic symmetry:

$$F_{\mathrm{me}} = \tfrac{1}{2}\lambda_1^{(1)}[\sigma_{xx}(2\alpha_x^2 - \alpha_y^2 - \alpha_z^2) + \sigma_{yy}(2\alpha_y^2 - \alpha_x^2 - \alpha_z^2)$$
$$+ \sigma_{zz}(2\alpha_z^2 - \alpha_x^2 - \alpha_y^2)] + 2\lambda_1^{(2)}[\sigma_{xy}\alpha_x\alpha_y + \sigma_{xz}\alpha_x\alpha_z + \sigma_{yz}\alpha_y\alpha_z]$$
$$+ \lambda_2^{(1)}(\sigma_{xx} + \sigma_{yy} + \sigma_{zz})[\alpha_x^4 + \alpha_y^4 + \alpha_z^4$$
$$- 3(\alpha_x^2\alpha_y^2 + \alpha_x^2\alpha_z^2 + \alpha_y^2\alpha_z^2)] + \lambda_2^{(2)}\{\sigma_{xx}[\alpha_x^4 - 3(\alpha_x^2\alpha_y^2$$
$$+ \alpha_x^2\alpha_z^2 - \alpha_y^2\alpha_z^2)] + \sigma_{yy}[\alpha_y^4 - 3(\alpha_x^2\alpha_y^2 + \alpha_y^2\alpha_z^2 - \alpha_x^2\alpha_z^2)]$$
$$+ \sigma_{zz}[\alpha_z^4 - 3(\alpha_x^2\alpha_z^2 + \alpha_y^2\alpha_z^2 - \alpha_x^2\alpha_y^2)]\}$$
$$+ 2\lambda_2^{(3)}\Big\{\sigma_{xy}\alpha_x\alpha_y\left[\alpha_z^2 - \tfrac{1}{6}(\alpha_x^2 + \alpha_y^2)\right]$$
$$+ \sigma_{xz}\alpha_x\alpha_z\left[\alpha_y^2 - \tfrac{1}{6}(\alpha_x^2 + \alpha_z^2)\right]$$
$$+ \sigma_{yz}\alpha_y\alpha_z\left[\alpha_x^2 - \tfrac{1}{6}(\alpha_y^2 + \alpha_z^2)\right]\Big\}. \qquad (8)$$

Here the lower index on λ corresponds to the order N of the constant, and the upper index enumerates the different constants of a single order. Since the generally used form of expression for F_{me} for cubic crystals differs from (8) and is nonharmonic, the relation (7) need not be satisfied by the magnetostriction constants that enter into that expression. Instead of (7) there are obtained more complicated formulas, in which "entanglement" of the magnetostriction constants of different orders occurs. For example, for the first constants $\lambda_1^{(1,2)}$ we should have

$$\Delta\lambda_1^{(1,2)}(T)/\lambda_1^{(1,2)}(0) = P_1^{(1,2)}\Delta M(T, H)/M(0), \qquad (9)$$

where $P_1^{(1)} = 3[1 - 2\lambda_2^{(2)}(0)/\lambda_1^{(1)}(0)]$ and $P_1^{(2)} = 3[1 - \lambda_2^{(3)}(0)/3\lambda_1^{(2)}(0)]$; whereas for the harmonic form of expression (8), according to (7), when N = 1 there is obtained the universal value of the coefficients $P_1^{(1)} = P_1^{(2)} = 3$.

Experiment shows that the constants λ_2 often have the same orders of magnitude as the constants λ_1. Therefore in the nonharmonic analysis of experimental data, it is hardly possible to obtain any general law for the temperature dependence of the "nonharmonic" constants λ_1: from formula (9) it is evident that, depending on the relation between $\lambda_1(0)$ and $\lambda_2(0)$, different ferromagnetics may differ not only in the magnitude but even in the sign of the temperature rate of change of the constants $\lambda_1(T)$. The advantage of the harmonic form

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*As is known, magnetostrictive strains lead to a certain change of the magnetic anisotropy constants: to the purely crystallographic anistropy constants K_N of order N there are added magnetostrictive corrections $\delta K_N = C\lambda_{N_1}\lambda_{N_2}$, where the C's are definite linear combinations of the elastic moduli and where $N_1 + N_2 = N$.[2] By virtue of the relation (7), the temperature dependence of the magnetostrictive part of the anisotropy constants is determined by the formula

$$[\delta K_N(0) - \delta K_N(T)]/\delta K_N(0) = P_N[M(0) - M(T, H)]/M(0),$$

where $P_N = N(2N+1) - 4N_1N_2$, whereas for the crystallographic constants K_N the same formula holds but with $P_N = N(2N+1)$. For example, for a cubic crystal, to the first crystallographic anisotropy constant K_2, for which $P_2 = 10$, there is added a magnetostrictive part δK_2 with $P_2 = 6$. Since δK_N increases with decreasing temperature more slowly than does K_N, the relative contribution of the magnetostrictive strains to the anisotropy is smaller, the lower the temperature.

of expression for the magnetoelastic energy consists precisely in the fact that because it eliminates this random factor, it permits exhibition of the true law for the temperature dependence of magnetistriction.

It should be remarked that, just as for the anisotropy constants,[3] the applicability of the relation (7) is apparently limited to a narrower range of low temperatures than the range of applicability of the "$T^{3/2}$ law" for the temperature dependence of the spontaneous magnetization. Unfortunately, at the present time there exist almost no experimental investigations of the temperature dependence of the magnetostriction in the region of sufficiently low temperatures.

A theoretical calculation of the temperature dependence of the magnetostriction constants on the basis of spin-wave theory was first achieved by Gusev,[4] who showed that in the low-temperature range, always, $\Delta\lambda(T) \sim T^{3/2}$; but he did not investigate the relation between the temperature dependences of the magnetostriction and of the magnetization. A number of earlier works,[5] based on the molecular-filed approximation, led to the following relation between the first constants of magnetostriction and of spontaneous magnetization:

$$\lambda_1(T)/\lambda_1(0) = [M(T)/M(0)]^{P_1}, \tag{10}$$

where $P_1 = 2$. We remark that formula (7) for $N = 1$ in the low-temperature region, where it has its only applicability, can also be expressed approximately in the form (10), but with the difference that $P_1 = 3$.

Among the early researches it is necessary to mention also the work of Vonsovskiĭ,[6] who, within the framework of a special microscopic model, calculated the spin-orbit interaction of the electrons responsible for ferromagnetism; this enabled him to give an explanation in principle of the nonmonotonic temperature dependence of the magnetostriction constants that is sometimes observed experimentally. In this theory, however, there enter a number of undetermined parameters, so that comparison of it with experiment is difficult. In all the works enumerated, no attention was directed to the role of the form of expression for the magnetoelastic energy.

The present theory, being based on the most general modern ideas about the nature of ferromagnetic and magnetoelastic phenomena, will presumably, after its detailed verification, make possible a number of definite conclusions regarding the limits of applicability of the existing spin-wave description of these phenomena.

3. In conclusion, we give a few relations for the isotropic volume magnetostriction. From the second sum in formula (6) the relative change of volume of the ferromagnetic can be found by calculating the value of $w = \delta V/V_0 = \Sigma u_{ll}$ corresponding to this sum. For temperatures $\kappa T \gg \mu H$ we have approximately

$$w = w_0 + H\partial w/\partial H,$$

where

$$w_0 = \frac{3\zeta(^5/_2)\,\Gamma(^5/_2)\,\mu G}{2\pi^2 M_0}\left(\frac{M_0\varkappa T}{2\mu A}\right)^{5/2} \tag{11}$$

is the spontaneous magnetostriction (thermostriction), and

$$\frac{\partial w}{\partial H} = -\frac{3\zeta(^5/_2)\,\Gamma(^5/_2)\,\mu^2 G}{2\pi^2 M_0\varkappa T}\left(\frac{M_0\varkappa T}{2\mu A}\right)^{5/2} \tag{12}$$

is the coefficient of paraprocess magnetostriction. Here we have for simplicity taken $A_{ij} = A\delta_{ij}$ and $G_{ijll} = G\delta_{ij}$ (cubic crystal); ζ and Γ are, respectively, the Riemann zeta function and the gamma function. We remark furthermore that according to (2) the parameter G can be defined as $G = \partial A/\partial p$, where p is hydrostatic pressure.

By comparing expressions (11) and (12) with each other and also with the corresponding expressions for the spin heat-capacity C_S and for the temperature change of spontaneous magnetization ΔM,* it is not difficult to establish the following simple relations among all these quantities:

$$\frac{1}{3}\frac{\partial w_0}{\partial T} = -\frac{5}{6}\frac{\varkappa}{\mu}\frac{\partial w}{\partial H} = \frac{1}{\Theta_c}\frac{\partial \Theta_c}{\partial p} C_S \approx 2\frac{\varkappa}{\mu}\frac{1}{\Theta_c}\frac{\partial\Theta_c}{\partial p}\Delta M. \tag{13}$$

Instead of the exchange-coupling parameter A, we have introduced here the Curie temperature Θ_C, assuming a direct proportionality between them.

The quantity $\frac{1}{3}\partial w_0/\partial T = \Delta\alpha_M$ represents an additional linear coefficient of thermal expansion, connected with thermostriction. As was to be expected in accordance with Grüneisen's law,[8] it is proportional to the spin part of the heat capacity. A quantitative estimate of $\Delta\alpha_M$ can be made by using experimental data on the paraprocess magnetostriction.[9] For the majority of magnetic materials in the room-temperature range, $\partial w/\partial H = 10^{-10}$ to 10^{-9} oe^{-1}. Hence, according to (13), $\Delta\alpha_M = 10^{-6}$ to 10^{-5}. Experimental investigations

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*See, for example, the work of Kaganov and Tsukernik.[7] In our notation, we have

$$C_S = \frac{5\zeta(^5/_2)\,\Gamma(^5/_2)\,\varkappa}{2\zeta(^3/_2)\,\Gamma(^3/_2)\,\mu}\Delta M = \frac{5\Gamma(^5/_2)\,\zeta(^5/_2)}{8\pi^2}\varkappa\left(\frac{M_0\varkappa T}{2\mu A}\right)^{3/2}.$$

of the volume magnetostriction at lower temperatures are unfortunately lacking.

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