parallel to each other, and the d sublattice magnetized in the opposite direction (with the total magnetization parallel to the external field). It is clear from this that at ultralow temperatures both the X nuclei and the iron nuclei will be polarized, and that the limiting degrees of polarization will be unity for the X nuclei and $-1/5$ for the iron nuclei.

Thus we see that by applying a magnetic field to a ferrite or garnet cooled to an ultralow temperature one can obtain polarization of the nuclei not only of ferromagnetic atoms, but also of many paramagnetic atoms.

Measurements of the nuclear polarization produced can be made by means of nuclear magnetic resonance, with the resonance frequency corresponding to the internal field.[9] In the case of radioactive nuclei the polarization can be observed by studying the angular anisotropy of the γ radiation or the angular asymmetry of the β radiation. The study of β radiation is more advantageous, since it gives a possibility for direct determination of the degree of polarization of the nuclei.

Experiments of this kind can be made not only for the purpose of obtaining polarized nuclei, but also in order to study the properties of ferrites and garnets and of other nonmetallic ferromagnetic materials. In particular, such experiments can be made with so-called mixed ferrites and garnets, for which the structure problem has not yet been finally solved.

*This class also includes the ferrite of magnesium, which is ferromagnetic in spite of the fact that the Mg^{++} ion is nonmagnetic.

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ON THE PROBLEM OF THE DIRECT RECONSTRUCTION OF THE ELASTIC-SCATTERING AMPLITUDE

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A paper by Puzikov, Ryndin, and Smorodinskiĭ[1] has dealt with the problem of the reconstruction of the scattering matrix. In particular, it contains a suggestion that phase-shift analysis be replaced by direct solution of the system of equations that follows from the unitarity condition.

We have made an attempt to find a method for solving this system for the simplest case of the scattering of spinless particles by a center of force. In this case one gets the following system of equations for the real part $R(\mu)$ and the imaginary part $I(\mu)$ ($\mu = \cos\vartheta$) of the scattering amplitude (σ is the scattering cross section):

$$R^2(\mu) + I^2(\mu) = \sigma(\mu), \tag{1}$$

$$I(\mu) = \frac{k}{4\pi}\int_{-1}^{1}\int_{0}^{2\pi}[I(\mu'')I(\mu') + R(\mu')R(\mu'')]\,d\mu' d\varphi, \tag{2}$$

where

$$\mu'' = \mu\mu' + \sqrt{(1-\mu^2)(1-\mu'^2)}\cos\varphi. \tag{3}$$

It must be remarked that the choice of $I(\mu)$ and $R(\mu)$ as the unknown functions is to be preferred to the description of the complex amplitude by its absolute value and phase, which was suggested in reference 1. The latter description is unstable under small changes of the amplitude.

The system (1) — (2) was solved by Newton's method for functional equations.[2] Instead of the nonlinear equation (2) one gets for the correction $\xi(\mu)$ to the n-th approximation $I_n(\mu)$ the linear integral equation ($k = 1$)

$$\xi(\mu) + \frac{1}{2\pi}\iint \xi(\mu')\frac{R_n(\mu'')I_n(\mu') - R_n(\mu')I_n(\mu'')}{R_n(\mu')}\,d\mu' d\varphi$$
$$= -I_n(\mu) + \frac{1}{4\pi}\iint\Big[R_n(\mu'')R_n(\mu') + I_n(\mu')I_n(\mu'')$$
$$+ R_n(\mu'')\frac{\sigma(\mu') - R_n^2(\mu') - I_n^2(\mu')}{R_n(\mu')}\Big]\,d\mu' d\varphi. \tag{4}$$

The successive approximations for $I(\mu)$ and $R(\mu)$ are obtained from the formulas

$$I_{n+1} = I_n + \xi, \quad |R_{n+1}| = |\sigma - I_{n+1}^2|^{1/2}.$$

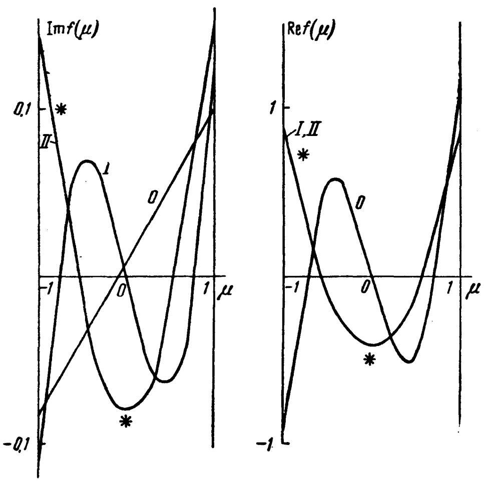
When $R_{n+1}(\mu)$ passes through zero the sign of $R_{n+1}(\mu)$ is fixed by the requirement of continuity of the derivative.

The integrals in Eq. (4) lose their meaning if $R_n(\mu')$ has zeroes μ_s in the interval $(-1, 1)$. To regularize the integrals we can use their principal values:

$$\Phi(R(\mu)) = \int_{-1}^{1} \frac{\varphi(\mu)}{R(\mu)} d\mu = \int_{-1}^{1} \left[\frac{\varphi(\mu)}{R(\mu)} - \sum_s \frac{\varphi(\mu_s)}{(\mu - \mu_s) R'(\mu_s)} \right] d\mu + \sum_s \frac{\varphi(\mu_s)}{R'(\mu_s)} \ln \frac{1-\mu_s}{1+\mu_s}. \quad (5)$$

It was found that such applications of the theory of generalized functions assure convergence even in cases in which the numbers of zeroes of the solution and of the zeroth approximation are not the same (see diagram). Equation (4) was solved with an electronic computer by reducing it to an algebraic system of linear equations. The number of equations is then equal to the number of points used in the numerical-integration formula (that of Gauss) plus the number of zeroes of the real part of the scattering amplitude [cf. the factor $\varphi(\mu_s)$ in Eq. (5)]. In the examples solved the number of zeroes does not exceed three, and the number of points for the integration formula was six. As a rule the functions chosen as zeroth approximations were extremely far removed from the actual solutions of the system.

In spite of this, rapid convergence of the iteration process was found in seven out of the ten examples solved. The correction to the zeroth approximation, which was often larger in magnitude than $I_0(\mu)$ and $R_0(\mu)$ themselves, at once gave almost the correct value of $|R(\mu)|$. The next approximation led to the correct value of $I(\mu)$. Three or four approximations were usually enough to get three-figure accuracy (see diagram). The convergence is slower in the three examples in which either R or R_0 has no zeroes, i.e., in which the regularization by Eq. (5) either is unnecessary ($R_0 > 0$) or leads to a $\Phi[R_0(\mu)]$ that differs sharply from $\Phi[R(\mu)]$ ($R > 0$, R_0 changes sign). In all ten examples the scattering amplitude in forward directions ($\mu > 0$) was reconstructed with particularly high accuracy; the reconstruction for R was better than that for I.



Convergence of the iteration process. Curves 0, I, II are respectively the zeroth, first, and second approximations, and * is the exact solution.

The numerical experiment which we have made allows us to hope that also in the more complicated case of scattering of particles with spin direct solution of the system of nonlinear equations can give the desired scattering matrix, bypassing phase-shift analysis.[1]

For a detailed exposition of the results of the present work, see reference 3.

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ELECTRONIC PARAMAGNETIC RESONANCE OF THE Ti^{3+} ION IN CORUNDUM

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Up to recently, electronic paramagnetic resonance (e.p.r.) of the Ti^{3+} ion was observed only in cesium-titanium alum.[1,2] The authors of these