the probability of this effect:

$$W_{00}^{ee'} = \frac{2}{9}\frac{1}{192\pi^3} Z^3\alpha^5\mu^5\left(\frac{7}{20} - \omega\right)|\mu^2 Q_0'|^2 g^2. \quad (9)$$

The ratio of the probabilities for the two effects is

$$W_{00}^{ee'}/W_{00}^{\mu e} = 2.36\cdot 10^{-3}\left(1 - \frac{6}{7}\omega\right)/\eta. \quad (10)$$

(Here we neglected the small difference between Q_0 and Q_0'.) It is seen from (10) that this ratio becomes comparable to unity for $\eta \approx 3 \times 10^{-6}$, which is about 10^3 times smaller than the upper limit for η calculated by Weinberg and Feinberg from the data of Steinberger and Wolfe ($\eta \leq 4 \times 10^{-3}$).

I express my gratitude to M. Ya. Balats and Ya. B. Zel'dovich for an interesting discussion.

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POLARIZATION OF NUCLEI IN NONMETALLIC FERROMAGNETIC SUBSTANCES

G. R. KHUTSISHVILI

Institute of Physics, Academy of Sciences, Georgian S.S.R.

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IN 1955 a method for polarizing the nuclei of ferromagnetic atoms was proposed by us[1] and simultaneously by Kurti and his co-workers.[2] The corresponding experiments on the anisotropy of γ radiation have been made by a number of authors.[1-4] All of these experiments were made on ferromagnetic metals.

It would be interesting to make experiments of this type also in the case of nonmetallic ferromagnetic substances. As an example let us consider the case of a ferrite, i.e., a compound of the type $XO \cdot Fe_2O_3$, where X is a divalent metal ion. The ferrite is ferromagnetic if X is the ion of a divalent metal of the iron transition group (Fe, Co, Ni, Mn, Ti, Cu).* These ferrites have the reciprocal spinel structure. It is well known that in the spinel lattice there are one tetrahedral cation position A and two octahedral cation positions B for each molecule. In the case of the reciprocal spinel structure the X^{++} ions occupy half the positions B, and the Fe^{+++} ions occupy all the positions A and half the positions B.

We assume that the external magnetic field is larger than the saturation field. Then the specimen is a single domain. If furthermore the temperature is sufficiently low, the state will be close to the state of absolute saturation. We proceed further to apply the Néel model,[5] according to which each of the sublattices A and B is magnetized to saturation, with their magnetizations oppositely directed. Therefore the resultant magnetization is determined by the X^{++} ions whose magnetic moments are directed along the external field. It follows that the fields exerted by the shells of X^{++} ions on their nuclei will all have the same direction (parallel or antiparallel to the external field, depending on the sign of the hyperfine structure constant). At an extremely low temperature we get a considerable polarization of the X nuclei. The limiting degree of polarization (obtained if the Zeeman energy of the nuclear spin in the internal field caused by the ion shell is much larger than kT) will be equal to unity.

These considerations apply to the ferrites of Co, Ni, Mn, Ti, and Cu. It is easy to see that in these ferrites the iron nuclei will not be polarized, but only aligned. It is also obvious that in the ferrite of iron (i.e., in magnetite, Fe_3O_4) the limiting degree of polarization of the iron nuclei will be $\frac{1}{3}$.

As a second example let us consider the case of a ferromagnetic garnet, i.e., a compound of the type $3X_2O_3 \cdot 5Fe_2O_3$, where X is the trivalent ion of a rare-earth element. A model analogous to the Néel model has been proposed to explain the properties of garnets[6,7] (cf. also reference 8). According to this model there are three types of positions for cations in a garnet, namely, in each molecule there are six so-called d positions, four a positions, and six c positions. The d and a positions are occupied by Fe^{+++} ions, and the c positions by X^{+++} ions. At low temperatures all the sublattices are magnetized to saturation, with the c and a sublattices magnetized

parallel to each other, and the d sublattice magnetized in the opposite direction (with the total magnetization parallel to the external field). It is clear from this that at ultralow temperatures both the X nuclei and the iron nuclei will be polarized, and that the limiting degrees of polarization will be unity for the X nuclei and $-\frac{1}{5}$ for the iron nuclei.

Thus we see that by applying a magnetic field to a ferrite or garnet cooled to an ultralow temperature one can obtain polarization of the nuclei not only of ferromagnetic atoms, but also of many paramagnetic atoms.

Measurements of the nuclear polarization produced can be made by means of nuclear magnetic resonance, with the resonance frequency corresponding to the internal field.[9] In the case of radioactive nuclei the polarization can be observed by studying the angular anisotropy of the γ radiation or the angular asymmetry of the β radiation. The study of β radiation is more advantageous, since it gives a possibility for direct determination of the degree of polarization of the nuclei.

Experiments of this kind can be made not only for the purpose of obtaining polarized nuclei, but also in order to study the properties of ferrites and garnets and of other nonmetallic ferromagnetic materials. In particular, such experiments can be made with so-called mixed ferrites and garnets, for which the structure problem has not yet been finally solved.

*This class also includes the ferrite of magnesium, which is ferromagnetic in spite of the fact that the Mg^{++} ion is nonmagnetic.

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ON THE PROBLEM OF THE DIRECT RECONSTRUCTION OF THE ELASTIC-SCATTERING AMPLITUDE

G. I. KOPYLOV and Z. D. LOMAKINA*

Joint Institute for Nuclear Research

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A paper by Puzikov, Ryndin, and Smorodinskiĭ[1] has dealt with the problem of the reconstruction of the scattering matrix. In particular, it contains a suggestion that phase-shift analysis be replaced by direct solution of the system of equations that follows from the unitarity condition.

We have made an attempt to find a method for solving this system for the simplest case of the scattering of spinless particles by a center of force. In this case one gets the following system of equations for the real part $R(\mu)$ and the imaginary part $I(\mu)$ ($\mu = \cos\vartheta$) of the scattering amplitude (σ is the scattering cross section):

$$R^2(\mu) + I^2(\mu) = \sigma(\mu), \tag{1}$$

$$I(\mu) = \frac{k}{4\pi}\int_{-1}^{1}\int_{0}^{2\pi}[I(\mu'')I(\mu') + R(\mu')R(\mu'')]\,d\mu'd\varphi, \tag{2}$$

where

$$\mu'' = \mu\mu' + \sqrt{(1-\mu^2)(1-\mu'^2)}\cos\varphi. \tag{3}$$

It must be remarked that the choice of $I(\mu)$ and $R(\mu)$ as the unknown functions is to be preferred to the description of the complex amplitude by its absolute value and phase, which was suggested in reference 1. The latter description is unstable under small changes of the amplitude.

The system (1) — (2) was solved by Newton's method for functional equations.[2] Instead of the nonlinear equation (2) one gets for the correction $\xi(\mu)$ to the n-th approximation $I_n(\mu)$ the linear integral equation (k = 1)

$$\xi(\mu) + \frac{1}{2\pi}\iint\xi(\mu')\frac{R_n(\mu'')I_n(\mu') - R_n(\mu')I_n(\mu'')}{R_n(\mu')}\,d\mu'd\varphi$$
$$= -I_n(\mu) + \frac{1}{4\pi}\iint\Big[R_n(\mu'')R_n(\mu') + I_n(\mu')I_n(\mu'')$$
$$+ R_n(\mu'')\frac{\sigma(\mu') - R_n^2(\mu') - I_n^2(\mu')}{R_n(\mu')}\Big]d\mu'd\varphi. \tag{4}$$

The successive approximations for $I(\mu)$ and $R(\mu)$ are obtained from the formulas

$$I_{n+1} = I_n + \xi, \quad |R_{n+1}| = |\sigma - I_{n+1}^2|^{1/2}.$$