SLOWING-DOWN TIME OF NONEQUILIBRIUM CURRENT CARRIERS IN SEMICONDUCTORS

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Slowing-down times are derived for electrons and holes which are not in thermal equilibrium with the semiconductor lattice. It is shown that account of degeneracy does not affect the slowing-down time appreciably.

A determination of the slowing-down time of nonequilibrium current carriers in semiconductors is of prime importance in the design of quantum-mechanical semiconductor generators and amplifiers.[1] Negative losses are possible in a semiconductor only if the time of production of a negative temperature in the semiconductor (slowing-down time) is shorter than the lifetime of the fast electrons and holes produced by the external excitation. We consider the nonequilibrium (fast) carriers to be electrons* (holes) having an initial energy ϵ_0 considerably in excess of the average thermal or degeneracy energy (if the gas is degenerate), but not exceeding the threshold energy ϵ_i of impact ionization of the valence band. In crystals that are sufficiently pure and regular, the slowing down of the electron is due to scattering by lattice vibrations, i.e., the impurities and imperfections can be disregarded in the slowing down.

We start with a derivation of the kinetic equation for the distribution function of the electrons in the crystal, with allowance for the Fermi degeneracy of the electron gas, since it becomes significant in the final stages of the slowing down. We leave out of the kinetic equation the term that describes collisions between electrons, since such a process does not lead directly to the process of interest to us, the reduction in the average electron energy.

The kinetic equation for $f(\mathbf{p})$ has the form (see reference 2)

$$\frac{\partial f(\mathbf{p})}{\partial t} = \sum_{\mathbf{q}} \{ -\rho_a(\mathbf{q}) f(\mathbf{p}) [1 - f(\mathbf{p}+\mathbf{q})] + \rho_e(\mathbf{q}) f(\mathbf{p}+\mathbf{q}) [1 - f(\mathbf{p})] - \rho_e(\mathbf{q}) f(\mathbf{p}) [1 - f(\mathbf{p}-\mathbf{q})] + \rho_a(\mathbf{q}) f(\mathbf{p}-\mathbf{q}) [1 - f(\mathbf{p})] \}. \quad (1)$$

*We shall discuss henceforth only conduction electrons, although all the results are equally valid for the holes of the valence band.

Here ρ_e and ρ_a are the probabilities of emission and absorption of a phonon with momentum $\mathbf{q}$ and energy $\hbar\omega_q$. We consider the slowing down of electrons in the absence of an electric field. Naturally, in this case $f(\mathbf{p}) \equiv f(\epsilon)$, where $\epsilon = p^2/2m$ is the electron energy and m the electron effective mass.

Following the usual procedure and changing from summation over $\mathbf{q}$ to integration over $d\mathbf{q}$, we expand the integrals in the right half of (1) in powers of the small quantity $\hbar\omega_q/\epsilon(p)$, and obtain*

$$\frac{\partial f(\varepsilon)}{\partial t} = \frac{1}{p}\frac{\partial}{\partial \varepsilon}\left\{ G(\varepsilon)\left[f(\varepsilon)(1 - f(\varepsilon)) + \eta_1(\varepsilon)\frac{\partial f(\varepsilon)}{\partial \varepsilon}\right]\right\},$$

$$G(\varepsilon) = \frac{Vm}{2\pi\hbar^4}\int_0^{2p} B(q)\,\hbar\omega_q q\,dq,$$

$$G(\varepsilon)\eta_1(\varepsilon) = \frac{Vm}{4\pi\hbar^4}\int_0^{2p} B(q)(\hbar\omega_q)^2(2n_q+1)\,q\,dq, \quad (2)$$

where $B(q)$ is the square of the electron-phonon interaction matrix element, n_q is the number of phonons of energy $\hbar\omega_q$, and V is the volume of the crystal.

The time of collision between the electron and the lattice, τ, is given by

$$\tau^{-1} = \frac{v}{l} = \frac{Vm}{4\pi\hbar^4 p^3}\int_0^{2p} B(q)(2n_q+1)\,q^3 dq, \quad (3)$$

where v is the electron velocity and l is the mean free path. Multiplying (2) by $\epsilon\rho(\epsilon)$, where $\rho(\epsilon)$ is the density of electron states with energy ϵ

$$\rho(\varepsilon) = 4\pi(2\pi\hbar)^{-3}V(2m)^{3/2}\varepsilon^{1/2},$$

and integrating over the energy of the electron, we obtain

$$\frac{dE}{dt} = -\int_0^{\infty}\frac{p(\varepsilon)}{p}G(\varepsilon)\left\{ f(\varepsilon)[1 - f(\varepsilon)] + \eta_1(\varepsilon)\frac{\partial f}{\partial \varepsilon}\right\} d\varepsilon. \quad (4)$$

*In the absence of degeneracy, (2) becomes identical with the equation obtained by Davydov, Stratton, and Keldysh.[3]

Here E is the average electron energy. The first term in the integral of (4) describes the energy lost by the electrons through spontaneous emission of phonons. The second term describes the diffusion of the electrons in energy space. As can be seen from what follows, in the scattering of an electron by the acoustic lattice vibrations, allowance for the second term in the integral of (4) leads to a logarithmic singularity in the time of establishment of equilibrium between the electrons and the lattice. Confining ourselves to an examination of the slowing down of electrons to energies exceeding the lattice temperature, we disregard for the time being the second term of (4).

We consider the slowing down of an individual electron, without allowance for degeneracy. In this case

$$f(\varepsilon)\,\rho(\varepsilon) = \delta(\varepsilon - E). \tag{5}$$

1. For acoustic phonons we have[4]

$$G(\varepsilon) = 16m^3C^2\varepsilon^2/9\pi\hbar^4Mn_0, \quad 1/\tau = 4C^2m^2kTv/9\pi\hbar^4Mn_0u^2, \tag{6}$$

where C^2 is the interaction constant, n the density of the lattice atoms, M the mass of the lattice atom, u the velocity of sound, T the lattice temperature, and k Boltzmann's constant. Using (5) and (6), we obtain from (4)

$$dE/dt = -a_0E^{3/2}. \tag{7}$$

expressed in terms of the free path l_{ac} in scattering by acoustic lattice vibrations, a_0 is given by

$$a_0 = 2(2m)^{1/2}u^2/l_{ac}(T)\,kT. \tag{8}$$

The time to slow down from an energy E_0 to E is

$$t = (2/a_0)(E^{-1/2} - E_0^{-1/2}). \tag{9}$$

We note that the slowing-down time t depends on two constants of the semiconductor — mobility and velocity of sound.

When $E_0 \gg E$, the time to slow down to energies corresponding to $T \sim 300°K$ is $t \sim 4.5 \times 10^{-10}$ sec for n-type germanium ($w \sim 3600$ cm^2. $^{-1}$sec^{-1} at room temperature and $u \sim 4.94 \times 10^5$ cm/sec), and $t \sim 5.1 \times 10^{-11}$ sec for n-type silicon ($w \sim 1200$ cm^2 v^{-1} sec^{-1}, $u \sim 8.5 \times 10^5$ cm/sec).

The ratio of the time to slow down from E_0 to E, with $E_0 \gg E$, in two differenct semiconductors x and y is

$$t_x/t_y = w_xu_y^2/w_yu_x^2,$$

where w is the mobility.

2. We proceed to examine the slowing down of a fast electron by the optical lattice vibrations of a valence simiconductor. In this case (2) and (4) hold when the electron energy $E \gg \hbar\omega_0$, where $\hbar\omega_0$ is the energy of the optical phonon ($\hbar\omega_0 \approx kT_D$, where T_D is the Debye temperature).

Then[4]

$$G(\varepsilon) = D^2K^2m^2\varepsilon/\pi\hbar^2Mn_0, \quad \tau^{-1} = D^2K^2m^2v/2\pi\hbar^4Mn_0\hbar\omega_0, \tag{10}$$
$$dE/dt = -(K^2D^2m^{3/2}/\sqrt{2\pi n_0M\hbar^2})\,E^{1/2},$$

where K is the reciprocal-lattice vector, and D^2 is the constant of interaction with the optical phonons, analogous to C^2. The time to slow down from E_0 to E is

$$t_{op} = (2^{3/2}\pi Mn_0\hbar^2/K^2D^2m^{3/2})(E_0^{1/2} - E^{1/2}). \tag{11}$$

Comparison of the time of slowing down by optical and acoustical phonons yields, for $C^2 \approx D^2$,

$$t_{op}/t_{ac} \approx mu^2/kT_D \approx 10^{-2}.$$

Negative temperatures can be produced in the semiconductor only when the electrons of the conduction band and the holes of the valence band become degenerate as they are slowed down by the crystal lattice. As is well known, the degeneracy is connected with the temperature (average energy) and density of the electron gas. Reduction of the average electron-gas energy to low values increases greatly the slowing-down time, so that the electrons can recombine prior to production of the negative temperature. It is therefore sensible to confine oneself to not too low average electron energies, but to increase their density.* As shown by Fröhlich and Paranjape,[2] the exchange energy is more intense in inter-electron collisions than in collisions with acoustic phonons, even at an electron density $n \approx 10^{14}$cm^{-3} (when $T < T_D$ and $\bar{\epsilon} < kT_D$, the interaction with the optical phonon is insignificant).

Thus, at the electron densities necessary to obtain negative temperatures, the form of the distribution function can be assumed known:

$$f(\varepsilon) = (e^{(\varepsilon-\mu)/k\Theta} + 1)^{-1}, \tag{12}$$

where μ is the Fermi level and Θ the temperature of the electron gas. The slowing-down process becomes a reduction in the temperature Θ to the lattice temperature T. We have determined earlier the electron slowing-down time without account of degeneracy or of the second term of the right half of (4). We now obtain from (4) the time required for the electron gas temperature to change from Θ_0 to Θ, taking interaction with acoustical phonons only into account.

Using (12), we obtain in the usual manner from (4)

*The electron density need, however, not yield a negative dielectric constant in the frequency range of interest.

$$\partial E/\partial t = -2a_0(\Theta - T)\overline{E^{1/2}},$$
$$\overline{E^{1/2}} = \int_0^\infty \varepsilon^{1/2}\rho(\varepsilon) f(\varepsilon)\, d\varepsilon. \quad (13)$$

Equation (13) is a differential equation for $\Theta(t)$ and can be integrated in terms of elementary functions in the case of sufficiently strong degeneracy, when we can put $\mu/k\Theta \gg 1$. Applying in this case the usual procedure of expanding the integrals[5] for E, $E^{1/2}$ and μ, we obtain

$$\Theta\partial\Theta/\partial t = -(3a_0/k\pi^2\mu_0^{1/2})(\Theta - T)\left[\mu_0^2 + \tfrac{1}{6}\pi^2k^2\Theta^2\right],$$
$$2m\mu_0 = (\pi\hbar)^2(3n/\pi)^{2/3}. \quad (14)$$

Integrating (14) we obtain

$$t = \frac{k\pi^2}{3a_0\mu_0^{3/2}}\left(1 + \frac{\pi^2k^2T^2}{6\mu_0^2}\right)^{-1}\left\{T\ln\frac{(\Theta_0 - T)(1' + \pi^2k^2\Theta^2/6\mu_0^2)^{1/2}}{(\Theta - T)(1 + \pi^2k^2\Theta_0^2/6\mu_0^2)^{1/2}}\right.$$
$$\left. + \frac{\sqrt{6}\mu_0}{\pi k}\left[\tan^{-1}\left(\frac{\pi k}{\sqrt{6}\mu_0}\Theta_0\right) - \tan^{-1}\left(\frac{\pi k}{\sqrt{6}\mu_0}\Theta\right)\right]\right\}. \quad (15)$$

The first term in the bracket of (15), which results from allowance for the second term in the right half of (4), does logarithmically to infinity as $\Theta \to T$. This is equivalent to an infinite time to establish equilibrium between the electrons and the lattice. When $\Theta \gg T$, the logarithm in (15) can be neglected, and the slowing-down time becomes

$$t \approx (k\pi^2/3a_0\mu_0^{3/2})(\Theta_0 - \Theta). \quad (16)$$

In comparing the slowing-down time with and without allowance for degeneracy [see Eq. (9)], it is convenient to replace the temperature in (16) by the average electron energy ϵ, hence ($\Theta_0 \gg \Theta$)

$$t \approx (2\pi/5a_0\varepsilon^{1/2})(\varepsilon_0/\varepsilon - 1)^{1/2},$$
$$\varepsilon \approx \tfrac{3}{5}\mu_0, \quad \varepsilon_0 \approx \tfrac{3}{5}\mu_0\left(1 + \tfrac{5}{12}\pi^2k^2\Theta_0^2/\mu_0^2\right).$$

In this case, in spite of the different forms of (9) and (16), the times are of the same order of magnitude.

Let us point out that there is no need for considering the slowing down of a degenerate electron gas by optical phonons, for in the cases of interest to us the average electron energy at which degeneracy becomes important is less than $\hbar\omega_0$. Thus, the electron gas slows down in a semiconductor in two stages: first by the optical lattice vibrations to energies $\sim\hbar\omega_0$, in a short time, given by (11), and then by the acoustic vibrations in a considerably longer times, in accordance with (9) and (16). Consequently, the time required to produce negative temperatures is determined essentially by the slowing-down of the electrons from energies $\hbar\omega_0$ to energies corresponding to the necessary degree of degeneracy with allowance for interactions with acoustic lattice vibrations only.

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[1] Basov, Vul, and Popov, JETP **37**, 587 (1959), Soviet Phys. JETP **10**, 416 (1960).

[2] H. Frohlich and B. V. Paranjape, Proc. Phys. Soc. **B69**, 21 (1956).

[3] B. I. Davydov, JETP **7**, 1069 (1937). R. Stratton, Proc. Roy. Soc. **242**, 355 (1957). L. V. Keldysh, JETP **37**, 713 (1959), Soviet Phys. JETP **10**, 509 (1960).

[4] F. Seitz, Phys. Rev. **73**, 549 (1948).

[5] L. D. Landau and E. M. Lifshitz, Статистическая физика (Statistical Physics), M-L., Gostekhizdat, 1951.

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