ON ELASTIC SCATTERING OF PHOTONS BY THE NUCLEAR COULOMB FIELD

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As is known, the only evaluations of the cross section for elastic scattering of photons by the nuclear Coulomb field have been made either at zero angle of incidence[1] or at very small angles (of the order of a degree) and at high energies (of the order of tens and hundreds of Mev).[2] On the other hand, experiments carried out to detect this effect (see reference 3), are concerned with angles greater than 15 — 20° and with energies of the order of a Mev. Thus, in order to compare theory and experiment, it is necessary to find an approximation which is valid within the experimental region. In what follows we shall describe two approximations whose region of validity is significantly better suited to this situation than are those of previously used methods.

As is known (see reference 2), the main difficulty in calculating the given effect consists of calculating expressions of the type

$$T = \frac{1}{\pi^2 i}\int d^4p \, \mathrm{Sp}\,\{\hat{e}(i\hat{p}+1)^{-1}\hat{e}'(i\hat{p}-i\hat{k}'+1)^{-1}\gamma_0(i\hat{p}-i\hat{k}' - i\hat{q}+1)^{-1}\gamma_0(i\hat{p}-i\hat{k}+1)^{-1}\}, \tag{1}$$

where $k \equiv (\mathbf{k}, \mathbf{w})$ and $k' \equiv (\mathbf{k}', \mathbf{w})$ indicate the initial and final photon 4-momenta e and e' are its initial and final polarization; $q \equiv (\mathbf{q}, 0)$ is the "recoil" momentum with respect to which the integration is carried out ($\hbar = c = m_e = 1$). The expression (1) leads in the usual manner to the equation

$$T = \frac{6}{\pi^2 i}\int_0^1 dx \int_0^x dy \int_0^y dz \int d^4p \, \frac{S(p;\, k,\, k',\, q;\, x,\, y,\, z)}{(p^2+L)^4}, \tag{2}$$

where S represents in principle the result of the calculation following (1), and

$$L = 1 - i\varepsilon + q^2x(1-x) + \Omega^2(k,\, k',\, q;\, x,\, y,\, z) \equiv \mu^2 + \Omega^2. \tag{3}$$

In order to reduce the number of different integrations and thus permit carrying the calculations to a conclusion, it is expedient to introduce into equation (1) the expansion

$$(i\hat{p}-i\hat{k}+1)^{-1} = (i\hat{p}-i\hat{k}'+1)^{-1} + (i\hat{p}-i\hat{k}'+1)^{-1} i\hat{\Delta}(i\hat{p}-i\hat{k}'+1)^{-1}+\ldots, \tag{4}$$

which leads to an expression for T in the form of an expansion in the terms of the parameter $\Delta = |\mathbf{k}-\mathbf{k}'|$.

We calculated the matrix element of the investigated process in this manner, retaining the first two terms of the expansion (4) (the results are given in reference 4). The integration with respect to $\mathbf{q}$ must then be carried out numerically.

Another possibility, which makes it easy to find an analytical expression for the cross section, comes out of the form of the expression (2). After integrating with respect to p in this expression, L appears in terms of $1/L$, $1/L^2$, and $\ln L$. If we expand these functions in powers Ω^2/μ^2, we can easily complete the integration with respect to x, y, z, and q. Such an expansion is permissible, provided

$$\omega^2 + 2\Delta^2 < 4. \tag{5}$$

After normalizing and summing over the polarizations, we obtain the final expression

$$\frac{d\sigma}{d\Omega} = (\alpha Z)^4 \frac{r_0^2}{32}\frac{\omega^4}{m^4}\Big\{\Big[c_1 - c_2\frac{\omega}{m}\sin\frac{\theta}{2} + c_3\sin^2\frac{\theta}{2}\Big]^2(1+\cos\theta) + \Big[c_4\frac{\omega}{m} - c_5\sin\frac{\theta}{2}\Big]^2\sin^2\frac{\theta}{2}\cos^4\frac{\theta}{2}\Big\}, \tag{6}$$

where $c_1, \ldots, c_5$ are numerical coefficients.[5] Coefficients for terms of the order of ω^2/m^2, $\omega\Delta/m^2$ and Δ^2/m^2 are calculated in reference 5. It is found that the coefficients of terms independent of ω and of those linear in ω become zero as a result of gauge invariance. Equation (6) shows that the cross section decreases with increasing scattering angle θ, and that this decrease is more rapid at higher energies. This fact agrees with experimental data. It is evident that $d\sigma/d\Omega\ (\omega, 180°) = 0$, as was to have been expected.

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