DISTRIBUTION OF THE TRANSVERSE MOMENTUM OF SHOWER PARTICLES IN JETS

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Experimental data are presented on the distribution of the transverse momenta of secondary shower particles in jets produced by cosmic rays. Transverse momentum distributions that follow from various theories and also from various phenomenological descriptions of multiple production of mesons are analyzed and systematized. Comparison with the experiments narrows the possible choice of a scheme for description of the elementary process of multiple meson production.

THE theoretical formulas for the transverse-momentum distribution of mesons $p_\perp$, calculated in an arbitary system of coordinates, can be readily compared with the experimental data obtained in the laboratory system (l.s.). However, good agreement between the $p_\perp$ distribution with the experimental distribution is a necessary but insufficient condition for the correctness of a theoretical description, since it reflects simultaneously the angle and energy distributions of the generated particles. It is found, as follows from the analysis below, that the distribution of the transverse momenta becomes in many cases insensitive to the choice of different versions of the theory.

1. TRANSVERSE MOMENTUM DISTRIBUTION RESULTING FROM DIFFERENT VERSIONS OF THE THEORETICAL AND PHENOMENOLOGICAL DESCRIPTION OF MULTIPLE MESON PRODUCTION

a) One of the first theories of multiple production of mesons is the Heisenberg field theory,[1] based on the use of the nonlinear Lagrangian interaction. The energy spectrum ($\sim d\epsilon'/\epsilon'^2$) of the generated mesons in the center-of-mass system (c.m.s.), resulting from this theory, was experimentally confirmed by shower analysis.[2-9] The anisotropy in the angular distribution of the mesons generated at large energies (10^{11} ev) was explained by Heisenberg qualitatively by using the uncertainty principle. In a direction perpendicular to the motion of the colliding nucleons, the dimension of the generation region is $\hbar/\mu_\pi$, and consequently, $p_\perp \sim 1$.* In this case the mesons with momentum $p \gg \mu_\pi$ are scattered within an angle

$$\theta' \sim p_\perp / p \sim 1 / p.$$

As the energy increases, the degree of angular anisotropy of the generated particles increases. Later on, Symanzik* chose a function that reflects this law, and used this function to calculate the angular distribution in the laboratory system for the so-called "maximum-anisotropy" case.[10]

In the present paper we calculate the meson transverse-momentum distribution both in the "maximum anisotropy" assumption and in the assumption of angular isotropy in the c.m.s. The corresponding curves are shown in Fig. 1; the pertinent equations are

$$\frac{dN}{Ndp_\perp} = \frac{3p_\perp}{(p_\perp^2+1)^{5/2}}, \quad \frac{dN}{Ndp_\perp} = \frac{p_\perp}{(p_\perp^2+1)^{3/2}}; \tag{1}$$

$N^{-1}dN/dp_\perp$ is the relative differential intensity of the generated mesons.

b) In the Landau hydrodynamic theory[11] the c.m.s. energy of the generated mesons is uniquely related with the angle θ' between the directions of motion of the center of mass and of the primary particle. Assuming that all the secondary particles are pions, we can obtain the transverse-momentum distribution in the following parametric form:

$$\frac{dN}{Ndp_\perp} = \frac{C_2}{4C_1}\frac{\mu_\pi}{M}\frac{\exp[-L/3+2\sqrt{L^2-\lambda^2}/3](1+e^{-2\lambda})^2}{2e^{-2\lambda}-\lambda[1+\exp(-2\lambda)]/3\sqrt{L^2-\lambda^2}},$$

$$p_\perp = 2\frac{C_1 M}{\mu_\pi}\frac{\exp[-L/6+\sqrt{L^2-\lambda^2}/3]}{1+\exp(-2\lambda)}, \qquad \lambda < \left|\frac{\sqrt{3}}{2}L\right|, \tag{2}$$

$L = \ln \gamma_c$; γ_c is the c.m.s. Lorentz factor (nucleon-nucleon collision is assumed); C_1 and C_2 are constants of order unity, determined from the Landau

*The transverse momentum $p_\perp$ of the particles is measured in units of pion rest mass.

*We are very grateful to Mr. Symanzik for sending us the unpublished manuscript containing this calculation.

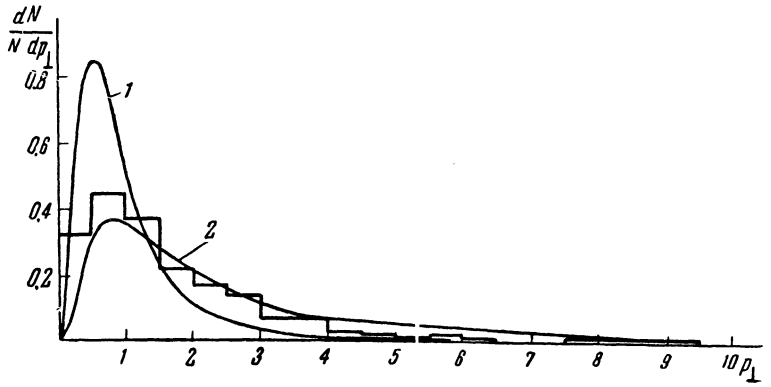

FIG. 1. Transverse-momentum distributions of the mesons according to Heisenberg's theory,[1,10] and the experimental distribution. Curves 1 and 2 are obtained under the assumption of maximum anistropy,[10] and isotropy of the angular distribution in the c.m.s., respectively.

integral conditions;[11] M is the nucleon mass.

The corresponding distributions for $\gamma_c = 20$ and 100 are given in Fig. 2 (curves 1 and 2). It follows from the form of the distribution that the majority of the particles have $p_\perp \gtrsim M/\mu_\pi$. Such a result does not agree with experiment.

Milekhin and Rozental'[12] have interpreted the occurence of so large transverse momenta as the consequence of the second stage of the expansion of an ideal liquid — conical scattering. They calculated the distribution of the transverse momentum of the generated mesons, starting with the assumption that such a distribution is determined exclusively by thermal motion in a one-dimensional relativistic current of a nucleonic liquid (see reference 13). The use of a Bose distribution for the pions and its integration over the longitudinal components[12] of the momentum $p_{||}$ makes the distribution of the mesons over $p_\perp$ depend on the critical temperature T_c at which the system begins to disintegrate. The corresponding distribution* for different values of the parameter $T_c/\mu_\pi = 1/3,\ 1/2,\ 1$, and 1.5 is shown in Fig. 2 (curves 3 — 6).

In a three-dimensional version of hydrodynamic theory,[14] Milekhin has shown that the transverse hydrodynamic velocity is much smaller than the thermal velocity. Therefore the transverse-momentum distribution of the generated mesons coincides with that obtained earlier.[12] (The disintegration temperature of the system is assumed to be $1\mu_\pi$.)

c) It is interesting to examine also the statistical theory, first employed by Fermi[15] to explain the Schein stars. At large energies, the thermodynamic approximation is applicable. Using Eq. (45) of reference 15 for the number of mesons dN generated in a phase-space element $d\tau$ we obtain a transverse-momentum distribution

$$N^{-1}dN/dp_\perp = \frac{\gamma^3 p_\perp^2}{af(\rho)} \int_{-1}^{+1} (1-y^2)\,dy \int_{-1}^{+1} (1-\eta^2)^{-3/2} \times \left\{ \exp\left[\frac{\gamma p_\perp}{(1-\eta^2)^{1/2}} (1 - \rho\eta y) \right] - 1 \right\}^{-1} d\eta,$$

$$f(\rho) = \left[\frac{1+\rho^2}{\rho^3} \ln \frac{1+\rho}{1-\rho} - \frac{2}{\rho^2} \right], \qquad (3)$$

where $a = 2.413$ and ρ is the nucleon collision parameter.* Calculations based on (3) were made[15] at an average value $\bar{\rho} = 0.959$.[15] The quantity $\gamma = \mu_\pi/T$ is expressed in terms of the energy of the colliding nucleons. Since $\gamma_c \lesssim 100$ for most of the compared showers, we have assumed, following Fermi, that equilibrium takes place only for the

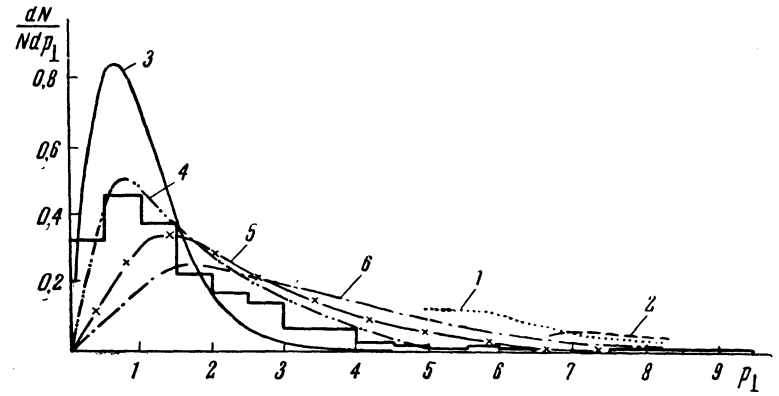

FIG. 2. Transverse-momentum distributions according to Landau's hydrodynamic theory.[11] Curves 1 and 2 are obtained for values of the Lorentz factor $\gamma_c = 20$ and 100. Curves 3 to 6 are obtained in the one-dimensional version[12,14] for various disintegration temperatures: $T_c/\mu_\pi = 1/3,\ 1/2$, and 1.5.

*We choose a system of units in which the Boltzmann constant and the velocity of light are equal to unity.

*Pomeranchuk, Feĭnberg, and Chernavskiĭ have indicated the difficulties connected in this theory with the choice of the meson-production volume and the introduction of the impact parameter.[16]

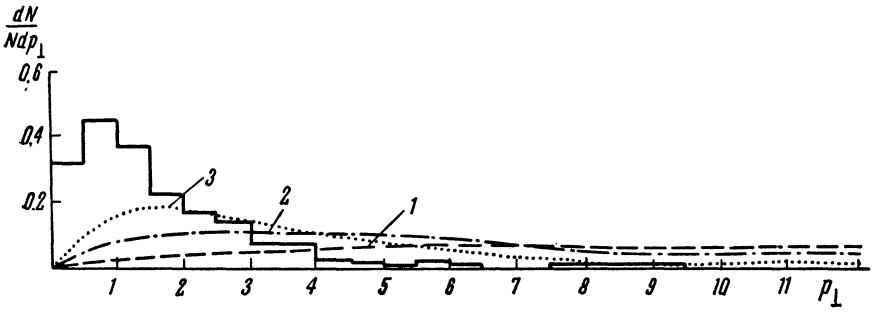

FIG. 3. Transverse-momentum distributions obtained from Fermi's theory,[15] taking account of the angular momentum in the c.m.s. Curves 1, 2, and 3 correspond to values $\alpha\gamma_c(\gamma_c - 1) = 100$, 10 and 0.5.

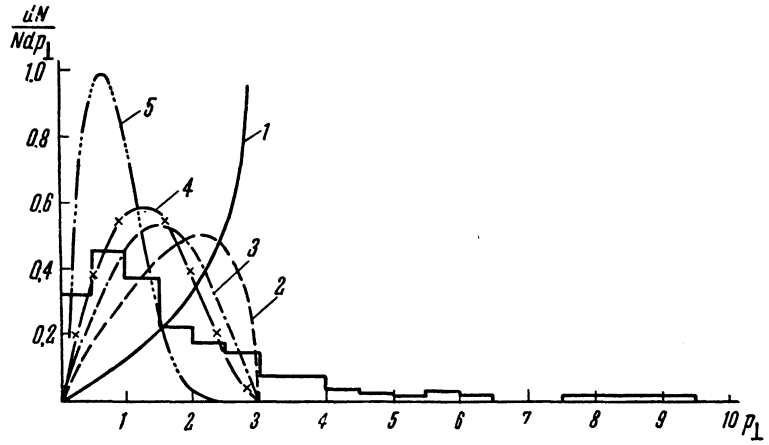

FIG. 4. Transverse-momentum distributions obtained under the assumption that the generated mesons are monoenergetic ($p_0 = 3$) in the c.m.s. Curves 1 to 5 correspond to various degrees of anisotropy: n = 0, 1, 2, 3, and 10.

pion gas; in other words, we have neglected the possible production of heavy particles.

Using the expression[15] for the energy density ξ, we can write for the energy balance

$$\frac{4}{3}\pi\frac{\hbar^3}{\mu_\pi^3}\frac{1}{\gamma_c}\xi = 2M\alpha(\gamma_c - 1), \qquad (4)$$

hence

$$\gamma = [\pi\alpha\gamma_c(\gamma_c - 1)]^{-1/4}. \qquad (5)$$

The quantity α can be treated in Eq. (4) either as the factor by which the production volume is increased[16]

$$\Omega = \Omega_1/\alpha \qquad (\Omega_1 \approx \hbar^3/\mu_\pi^3\gamma_c),$$

or as the fraction of energy transferred by the primary nucleons to the equilibrium system.[17]

The distributions shown in Fig. 3 have been obtained by numerically integrating (3) for $\alpha(\gamma_c - 1)\gamma_c = 100$, 10, and 0.5.

It is appropriate to note here that the choice of the form of the phase volume is important in the calculation of the distribution over the transverse momenta, and incidentally also in the calculation of the angular distribution and the total number of particles. Various qualitative models frequently encountered in the literature make use of the assumption that the mesons are monoenergetic. Let us see how this assumption influences the distribution over the transverse momenta at different degrees of angular anisotropy in the system of meson emission [this may be both the center of mass, as well as the system connected with each

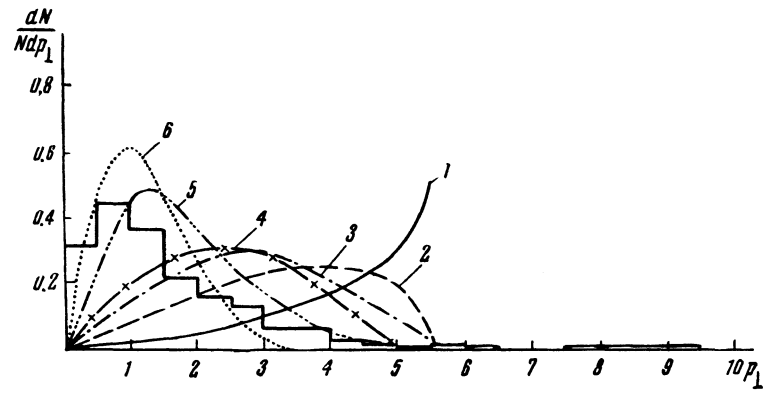

FIG. 5. Transverse-momentum distributions obtained under the assumption that the mesons are monoenergetic ($p_0 = 5.7$). Curves 1 to 6 correspond to n = 0, 1, 2, 3, 10, and 16.

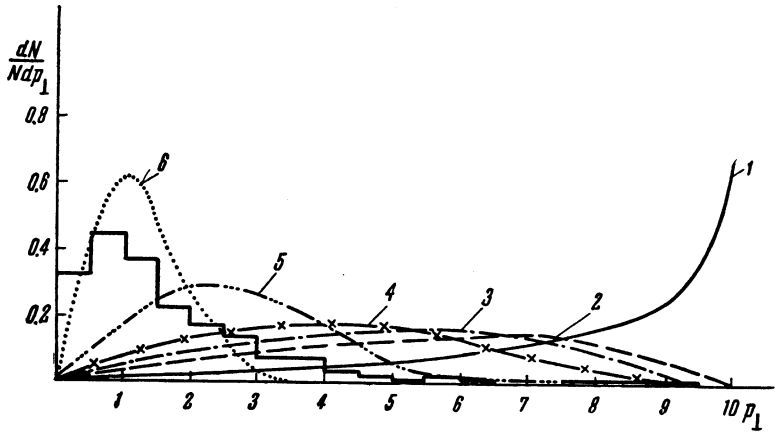

FIG. 6. Transverse-momentum distributions obtained under the assumption that the mesons are monoenergetic ($p_0 = 10$). Curves 1 to 6 correspond to n = 0, 1, 2, 3, 10, and 50.

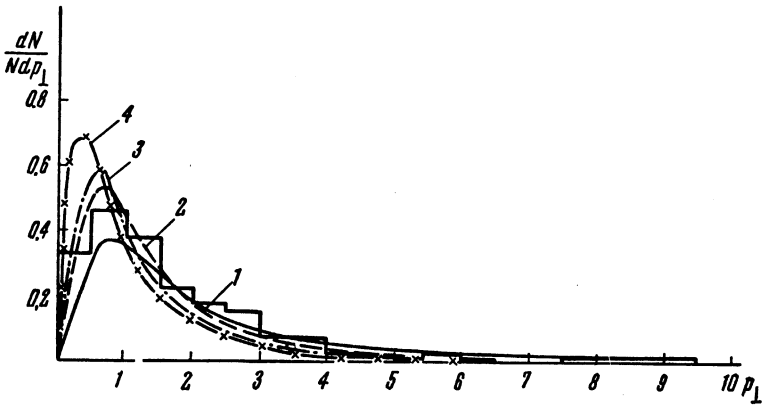

FIG. 7. Transverse-momentum distribution of the mesons obtained under the assumption of angular anisotropy ($\sim\cos^{2n}\theta'$) and of an energy spectrum from Heisenberg's theory.[1] Curves 1 to 4 correspond to n = 0, 1, 2, and 3. The histogram is the same as in the preceding figures.

of the meson emission centers (fire ball).[18-20] If the angular distribution is approximated by the function $\cos^{2n}\theta'$, the distribution by transverse momenta will have the form

$$\frac{dN}{Ndp_\perp} = (2n+1)\left(1-\frac{p_\perp^2}{p_0^2}\right)^{n-1/2}\frac{p_\perp}{p_0^2}, \qquad 0 \leqslant p_\perp \leqslant p_0, \quad (6)$$

where p_0 is the proposed value of the meson momentum. The calculation was made for different values of n and p_0, and more will be said below regarding the specific choice of p_0. Results of the calculations are illustrated in Figs. 4 — 6.

d) In conclusion, let us consider the frequently encountered version of the theory,[21-22] in which the Heisenberg energy spectrum is combined with an isotropic angular distribution. In the center-of-mass system, such a distribution has the form

$$dN = C\frac{p^2dp}{(1+p^2)^2}\cos^{2n}\theta' d\Omega, \qquad d\Omega = -d\cos\theta'. \quad (7)$$

Making the substitution $p = p_\perp/\sin\theta'$ and integrating over the angles, we arrive at a transverse-momentum distribution dependent on n:

$$\frac{dN}{Ndp_\perp} = 3\frac{p_\perp(\sqrt{1+p_\perp^2}-p_\perp)}{\sqrt{1+p_\perp^2}\{(1+p_\perp^2)^{1/2}+p_\perp\}}, \qquad n=1, \quad (8)$$

$$\frac{dN}{Ndp_\perp} = 5\frac{p_\perp(\sqrt{1+p_\perp^2}-p_\perp)}{(\sqrt{1+p_\perp^2}+p_\perp)}, \qquad n=2, \quad (9)$$

$$\frac{dN}{Ndp_\perp} = \frac{7}{4}\frac{p_\perp(p_\perp+4\sqrt{p_\perp^2+1})}{(p_\perp+\sqrt{1+p_\perp^2})^4}, \qquad n=3. \quad (10)$$

The corresponding distributions for different values of n are shown in Fig. 7.

2. COMPARISON WITH EXPERIMENTAL DATA

The experimental distribution of the transverse momenta of the generated particles was obtained from showers registered in emulsions.[2-6,23,24] We selected stars with energies $E > 10^{11}$ ev, which can be considered with high degree of probability, as being produced in nucleon-nucleon collisions. The momentum of the secondary shower particles was determined from multiple-scattering measurements. The total number of particles was 161. The histogram obtained is shown in Figs. 1—7. The maximum in the distribution of the transverse momenta is located near $p_\perp = 1$.

a) It is seen from Fig. 1 that the experimental distribution of $p_\perp$ lies between curves 1 and 2, which are obtained from the Heisenberg theory for two limiting cases, and that the positions of the maxima of the curves are in good agreement with experiment. It follows from the comparison that the assumed limiting values of the c.m.s. angular distribution of the mesons are correct. The

actual angular distributions lie apparently between these limits.

b) The Landau hydrodynamic theory[11] (see curves 1 and 2 in Fig. 2) leads to excessive transverse momenta. This is the consequence of the exceedingly hard energy distribution of the generated mesons, inasmuch as the angular distribution obtained from the Landau theory is in satisfactory agreement with the experimental data.[3,4,18] The modernization of the hydrodynamic theory[12,13] is based on the idea that the transverse hydrodynamic velocity of the particles is insignificant compared with the thermal velocities determined from the condition of statistical equilibrium of the elements of the system. As far as the distribution of the transverse momenta goes, this idea leads to good agreement with experiment at a temperature $T_c = (0.5 \text{ to } 1)\,\mu_\pi$ (curves 5 and 6 of Fig. 2). On the other hand, the mean value of the transverse momentum $\bar{p}_\perp$ in the three-dimensional version of the hydrodynamic theory, according to reference 14, is also of order μ_π. If we use this value of $\bar{p}_\perp$ and the energy dependence of the multiplicity ($n \sim \gamma_c^{1/2}$), which follows from the hydrodynamic theory, it is easy to estimate the order of magnitude of the average c.m.s. angle of emission of shower particles, $\bar{\theta}'$:

$$n \sim \gamma_c^{1/2}, \qquad \bar{p} \sim M\gamma_c^{1/2}, \qquad \bar{\theta}' \sim \bar{p}_\perp/\bar{p} \sim (\mu_\pi/M)\,\gamma_c^{-1/2}.$$

The value of $\bar{\theta}'$ estimated in this manner is much less than the average value of the angle ($\sim 2\sqrt{2/\pi L}$), estimated from the angular distribution given in the paper by Milekhin.[14] This lack of agreement may indicate that in the three-dimensional version of the hydrodynamic theory[14] it is either necessary to forego the dependence $n \sim \gamma_c^{1/2}$, or the order of magnitude claimed for the transverse momentum is incorrect.

c) From a comparison of the experimental distribution with the curves calculated from the Fermi theory (see Fig. 3), it follows that for no reasonable values of the quantity $\alpha\gamma_c(\gamma_c - 1)$ is agreement with experiment reached. If the quantity α is taken to mean an inelasticity coefficient, then for $\gamma_c = 10$ the values of α corresponding to curves 1, 2, and 3 of Fig. 3 are 1, 0.1, and 0.05, respectively. It is easy to see that with increasing γ_c the discrepancy with experiment increases. The quantity α can be estimated by stipulating that the energy spectrum of the theory agree with experiment. This calculation, carried out by Baktybaev for the showers considered, yields $\alpha \simeq 0.01$.

d) The distribution over the transverse momenta corresponding to the assumption of monoenergetic generated mesons contains the momentum p_0 as a parameter. In the calculations we used a quantity p_0, equal to the average value of the meson momentum. This value depends on the system of coordinates in which the analysis is made. For the center-of-mass system and the excited-volume systems, respectively,[19,20] the values obtained for p_0 were 5.7 and 3 (Figs. 4 and 5).

Inasmuch as values of $p_\perp$ up to 10 are seen on the histogram, we decided to plot these curves for $p_0 = 10$ (Fig. 6).

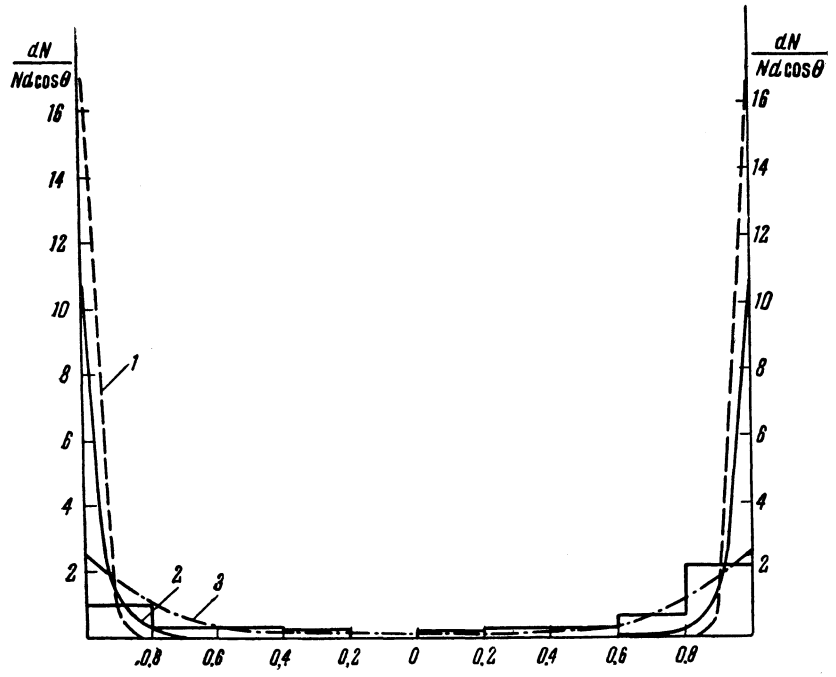

FIG. 8. Histogram of the angular distribution of the particles in the c.m.s., and differential angular distributions obtained under the assumption of an angular anisotropy $\sim\cos^{2n}\theta'$. Curves 1, 2, and 3 correspond to n = 16, 10, and 2.

A monoenergetic and isotropic meson distribution ($n = 0$) leads to a transverse-momentum distribution (curves 1 of Figs. 4, 5, and 6) which does not agree at all with the experimental data. It follows from Fig. 4 that for no value of n do the curves agree with the histogram of the distribution of transverse momenta. A considerable fraction ($\sim 15\%$) of the particles has a value $p_\perp > 3$. For large values of p_0, agreement with experiment is reached only for $n > 10$ (when $p_0 = 5.7$ and 10, and accordingly when $n = 16$ and 50).

Figure 8 shows the histogram of the overall c.m.s. angular distribution of the shower particles.[2-4,6,9,23] The ordinates are the relative differential meson densities, $N^{-1}dN/d\cos\theta'$, as functions of $\cos\theta'$. Curves 1 to 3 correspond to $n = 16$, 10, and 2.

From a comparison of Figs. 5, 6, and 8 it follows that the values $n > 10$ lead to a sharp angular anisotropy, which does not agree with the observed c.m.s. angular distribution. Thus, the assumption that the generated mesons are monoenergetic does not lead, for an anisotropic angular distribution ($\sim \cos^{2n}\theta'$), to an agreement between the distribution over $p_\perp$ and the experimental distribution.

e) The distribution of the transverse meson momenta obtained by assuming an anisotropic angular distribution ($\sim \cos^{2n}\theta'$) and an energy spectrum from the Heisenberg theory, is compared with the histogram on Fig. 7. Unlike the preceding case, the curves agree with the experimental distribution for considerably lower values of n, which is in agreement with the c.m.s. angular distribution of the mesons, shown in Fig. 8.

CONCLUSION

A comparison between different versions of the theory and experiment leads to the following conclusions:

1) It is impossible to explain the observed distribution of the transverse momenta by assuming the generated mesons to be monoenergetic, since a condition of sharp anistropy is imposed on the angular distribution. It is natural to assume that the anisotropy in the angular distribution of the mesons is greater than in the c.m.s. than in the system of excited volumes. A direct comparison with the experimental c.m.s. angular distribution shows that the experimental angular distribution is much less anisotropic even in this system. This contradiction disappears if it is assumed that the energy spectrum of the generated mesons is similar to the spectrum that follows from the Heisenberg theory.[1]

2) The Landau hydrodynamic theory[11] shifts the distribution of $p_\perp$ towards the larger transverse momenta, owing to the exceedingly hard energy spectrum predicted by this theory for the generated mesons. In the revised version of the theory,[14] as in the one-dimensional version,[12] the distribution of the transverse momenta is in good agreement with experiment, but it does not follow from the hydrodynamics, and is introduced by superposing the thermal motion of the particles on the hydrodynamic motion, which is assumed to be less developed in the transverse direction.

3) The Fermi theory[15] in the thermodynamic approximation leads to a distribution of transverse momenta which does not agree with experiment.

4) In the Heisenberg theory[1,10] the distribution over the transverse momenta is in satisfactory agreement with experiment. The generated-particle energy spectrum derived from this theory has found experimental verification.[2-9] The angular distribution does not follow directly from the theory, but is qualitatively explained by Heisenberg, starting with a correct representation of the order of magnitude of the mean transverse momenta. The c.m.s. angular distribution function, introduced by Symanzik on the basis of these representations, is confirmed both by the distribution over $p_\perp$ and by direct comparison[3,4,25] with experiment.

5) Analysis shows that distribution of the transverse momenta of the generated particles is described satisfactorily both by the hydrodynamic theory, in which only thermal motion of the particles is important in the transverse direction, and by the Heisenberg field theory. The experimentally observed distribution over the transverse momenta thus does not allow us to give preference to either of the foregoing versions of the theory of multiple production of mesons.

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