RESONANCE ABSORPTION OF HIGH-FREQUENCY SOUND ENERGY BY SEMICONDUCTOR CURRENT CARRIERS IN A MAGNETIC FIELD

E. P. POKATILOV

Kishinev State University

Submitted to JETP editor July 27, 1959

J. Exptl. Theoret. Phys. (U.S.S.R.) 38, 1153-1159 (April, 1960)

The absorption coefficient of ultrasound in a semiconductor is estimated with account of quantization of the electron energy in a magnetic field.

In a number of papers published in recent years,[1-3] the results of experimental and theoretical investigations of the absorption of ultrasonic energy by current carriers in a magnetic field has been reported for the case of ultrasonic frequencies ω that are less than or equal to the collision frequency of the electrons $1/\tau$ (τ is the relaxation time).

The case of higher ultrasonic frequencies, for which the inequality

$$\omega \geqslant \omega_0 > 1/\tau, \tag{1}$$

where ω_0 = cyclotron frequency of the electron, is satisfied, has been investigated theoretically by the author.[4] This inequality is the inverse of that used in the papers mentioned above. However, the calculation[4] was made in the classical approximation $\lambda_s > \lambda_{el}$ for the special case $\kappa_z = 0$ (κ_z is the component of the wave vector of the ultrasonic wave in the direction of the magnetic field).

1. Inasmuch as the absorption of ultrasonic energy has a resonance character,[4] we shall make use of a variant of the Wigner-Weisskopf theory of quantum processes. The Schrödinger equation for electrons interacting in a magnetic field with phonons and lattice impurities has the form

$$i\hbar\partial\Psi/\partial t = (H_0 + H_{int})\Psi, \tag{2}$$

where

$$H_0 = (2m^*)^{-1}\left[\left(p_x + \frac{e\mathscr{H}}{c}y\right)^2 + p_y^2 + p_z^2\right] + H_{\text{phon}} \tag{3}$$

is the sum of Hamiltonian operators for the motion of an electron in a magnetic field, and of the acoustic phonons of the lattice, H_{phon};

$$H_{int} = H_{1\,int} + H_{2\,int} + H_{3\,int} \tag{4}$$

is the energy of interaction of the electron with the ultrasound ($H_{1\,int}$), the acoustic vibrations of the lattice ($H_{2\,int}$), and the impurities ($H_{3\,int}$).

At low temperatures, for which the resonance effects under consideration can be measured, the optical vibrations do not have an appreciable value. The interaction operators $H_{1\,int}$ and $H_{2\,int}$ are chosen in the forms

$$H_{1\,int} = a\,\mathrm{div}\,\mathbf{u} = V_\varkappa e^{i\varkappa\mathbf{r}} + V_\varkappa^* e^{-i\varkappa\mathbf{r}}, \qquad V_\varkappa = \tfrac{1}{2} i a \mathbf{u}_0 \varkappa e^{-i\omega t}; \tag{5}$$

$$H_{2\,int} = a\sum_{\varkappa' j}\mathrm{div}\,\mathbf{u}(\varkappa', j). \tag{6}$$

In Eqs. (5) and (6), the quantity a is the constant of interaction of the electron with the lattice vibrations, $\mathbf{u}_0$ is the vector of the amplitude of the ultrasonic wave, κ and ω are the wave vector and the frequency of the ultrasound, respectively, while κ' and j are the wave vector and the polarization of the acoustic vibrations. We have no need here for the explicit form of the operator $H_{3\,int}$.

We express the solution of (2) as a superposition of wave functions that are solutions of the equation

$$i\hbar\partial\Psi/\partial t = H_0\Psi, \tag{7}$$

$$\Psi = \sum_\lambda b_\lambda \exp(-iE_\lambda t/\hbar)\,\psi_{n,k_x,k_z}\,\Phi_{N_\varkappa}\prod\Phi_{N_{\varkappa'}}, \tag{8}$$

where

$$\psi_{n,k_x,k_z} = C_n \exp\{i(k_x x + k_z z) - \tfrac{1}{2}\alpha(y - y_0)^2\}\,H_n(\sqrt{\alpha}(y - y_0)) \tag{9}$$

is the normalized wave function of the free electron in a magnetic field, $\alpha = |e|\mathcal{H}/c\hbar$ and the remaining notation is the same as that in the book by Landau and Lifshitz;[5] Φ_{N_κ} is the oscillator wave function describing the ultrasonic vibration of the lattice; $\Pi\Phi_{N_{\kappa'}}$ is the product of the oscillator wave functions of the acoustical vibrations of the lattice

$$E_\lambda = \varepsilon_\parallel(k_z) + \hbar\omega_0(n + \tfrac{1}{2}) + \hbar\omega(N_\varkappa + \tfrac{1}{2}) + \sum\hbar\omega_{\varkappa'}(N_{\varkappa'} + \tfrac{1}{2}). \tag{10}$$

In (10), $\epsilon_\parallel$ is the kinetic energy of motion of the electron along the direction of the magnetic field; $\hbar\omega_0(n+\frac{1}{2})$ is the energy of motion in the plane x, y; $\hbar\omega(N_K+\frac{1}{2})$ is the ultrasonic energy; $\sum_{K'}\hbar\omega_{K'}(N_{K'}+\frac{1}{2})$ is the energy of the acoustic vibration; λ is an index denoting the total set of quantum numbers of the system. In the sum (8), one of the amplitudes b_{λ_0} is equal to unity at the initial instant of time while the other is equal to zero. The indices λ and λ_0 differ in the quantum numbers of the electron states, and also by the occupation numbers of the phonon levels; in the state λ these occupation numbers differ from their initial values by unity or by zero.

Applying the Wigner-Weisskopf method,[6] we find the equation for b_λ, the transition-probability amplitude, and for $\gamma_n(k_z)$, the reciprocal of the mean lifetime of the electron in the state n, k_z:

$$b_\lambda = -H_{int}(\lambda, \lambda_0)[\exp\{i(E_\lambda - E_{\lambda_0}) \times t/\hbar - \gamma t/2\}\,\hbar/(E_\lambda - E_{\lambda_0}) + i\gamma/2], \quad (11)$$

$$\gamma_n = -\frac{2}{\hbar^2}\sum_\lambda |H_{int}|^2[1-\exp\{i(E_{\lambda_0}-E_\lambda)t/\hbar + \gamma t/2\}] \times [i(E_{\lambda_0}-E_\lambda)/\hbar + \gamma/2]^{-1}. \quad (12)$$

To find the transition probabilities under the action of ultrasound, it is necessary to compute $H_{1\,int}(\lambda, \lambda_0)$. The phonon part of the wave function (8) contributes the following factor to the square of the modulus of the matrix element:

$$\left|\int \Phi_{N_\varkappa -1} V_\varkappa \Phi_{N_\varkappa} du\right|^2 = \left|\int \Phi_{N_\varkappa +1} V_\varkappa^* \Phi_{N_\varkappa} du\right|^2 \approx \frac{1}{16}a^2\varkappa^2 u_0^2. \quad (13)$$

In (13) the number of ultrasonic phonons N_K, which is assumed to be much larger than unity, is approximately expressed by the ultrasonic amplitude.

We now compute the electron part of the matrix element. In accord with the work of Zil'berman,[7]

$$\int \exp\{-\tfrac{1}{2}\alpha(y-y_0')^2\} H_{n'}(\sqrt{\alpha}(y-y_0')) \times \exp\{i\varkappa_y y - \tfrac{1}{2}\alpha(y-y_0)^2\} H_n(\sqrt{\alpha}(y-y_0))\,dy = \exp\{i\varkappa_y y_0 + i\beta(\varkappa_y, y_0-y_0')\}\int \exp\{-\alpha y'^2 + i\varkappa_\rho y'\} H_{n'}(\sqrt{\alpha}y')H_n(\sqrt{\alpha}y')\,dy'. \quad (14)$$

The form of the phase $\beta(\kappa_y, y_0-y_0')$ is given in reference 7. We use here the notation $\kappa_\rho^2 = (k_x'-k_x)^2 + \kappa_y^2$, $y' = y - y_0$. The integral in (14) is given by

$$\sqrt{\pi/\alpha}\,2^{n_1}(i\varkappa_\rho/\sqrt{\alpha})^{n_2-n_1}n_1!\,L_{n_1}^{n_2-n_1}(\varkappa_\rho^2/2\alpha)\exp\{-\varkappa_\rho^2/2\alpha\}; \quad (15)$$

n_2 and n_1 are the largest and smallest of the numbers n and n′, respectively, and L_n^m is the Laguerre polynomial.

As a result of these calculations we get for the probability of transition of an electron (under the action of ultrasound) from the state n, k_z, k_x into the state n′, k_z', k_x',

$$|b_\lambda|^2 = (2\pi)^2\frac{1}{16}(a\varkappa u_0)^2\lambda_\rho^{n_2-n_1}e^{-\lambda_\rho}[L_{n_1}^{n_2-n_1}(\lambda_\rho)]^2 \times [(n_1+1)\ldots n_2]^{-1}|[\exp\{i(E_\lambda-E_{\lambda_0})t/\hbar - \gamma t/2\} - 1]/[(E_\lambda - E_{\lambda_0})/\hbar + i\gamma/2]|^2\,\delta(-k_z' + k_z \pm \varkappa_z), \quad (16)$$

where $\lambda_\rho = \kappa_\rho^2/2\alpha$ is a dimensionless parameter that determines the ratio of the radius of the orbit of the electron in the state n = 0 to the projection of the ultrasonic wave in the direction perpendicular to the magnetic field. In obtaining (16), $|H_{1\,int}|^2$ was integrated over k_x'; as a result, κ_ρ^2 became equal to $\kappa_x^2 + \kappa_y^2$. The plus sign in the δ function in front of κ_z corresponds to a transition with absorption of sound energy, while the minus sign corresponds to a transition with radiation.

2. The ultrasonic absorption coefficient Γ is obtained from the formula

$$\Gamma = U/c_0 E_{so}, \quad (17)$$

where c_0 is the sound velocity, $E_{so} = \rho u_0^2\omega^2/2$ is the sound energy density, and U is the energy absorbed per second per unit volume by the current carriers of the semiconductor. Calculation of Γ thus reduces to calculation of U. Below we shall introduce formulas for U under different assumptions relative to the state of the electron gas.

It is easy to obtain, using the Pauli principle, a formula for the mean number of transitions from the level n, k_z to the level n′, k_z',

$$\gamma_n(k_z)f_n(k_z)(1-f_{n'}(k_z'))|b_\lambda(\infty)|^2; \quad (18)$$

$f_n(k_z)$ is the distribution function of the electrons. If we substitute $|b_\lambda|^2$ from (16) in (18), taking the case of absorption of a phonon, and then integrate over k_z', k_z, sum over n, n′, and multiply the resultant expression by $2g = |e|\mathcal{H}/\pi\hbar c$ (the density of states with the given quantum numbers n and k_z), then we obtain the number of electron transitions with absorption of ultrasonic phonons per unit time. In similar fashion, we find the number of transitions with radiation of ultrasonic phonons. Multiplying these numbers of transitions by $\hbar\omega$ and $-\hbar\omega$, respectively, and adding, we obtain, finally, the unknown value of U:

$$U = 2g\hbar\omega\left\{\sum_{n,n'}\int\gamma_n f_n(k_z) \times [1-f_{n'}(k_z+\varkappa_z)]\,|b_\lambda(k_z+\varkappa_z, \infty)|^2 dk_z - \sum_{n',n}\int\gamma_n f_n(k_z)[1-f_{n'}(k_z-\varkappa_z)]\,|b_\lambda(k_z-\varkappa_z, \infty)|^2 dk_z\right\}. \quad (19)$$

The results of calculation of γ_n by (12) will be published separately; in the present paper we shall assume γ to be constant. We introduce in (19) the new summation indices $\Delta n = n_2 - n_1$ and $n = n_1$, and substitute the expression for $|b_\lambda|^2$; we get, finally,

$$U = g\left(\frac{a\varkappa u_0}{4}\right)^2 \frac{\gamma\omega}{\pi\hbar} \sum_{\Delta n,\, n=0}^{\infty} \lambda_\rho^{\Delta n} e^{-\lambda_\rho}$$
$$\times [(n+1)\ldots(n+\Delta n)]^{-1} [L_n^{\Delta n}(\lambda_\rho)]^2$$
$$\times \left\{ \int \frac{[f_n(k_z) - f_{n+\Delta n}(k_z + \varkappa_z)]\, dk_z}{(\Delta n\omega_0 - \omega + [\varepsilon_\parallel(k_z+\varkappa_z) - \varepsilon_\parallel(k_z)]/\hbar)^2 + \gamma^2/4} \right.$$
$$\left. + \int \frac{[f_{n+\Delta n}(k_z) - f_n(k_z+\varkappa_z)]\, dk_z}{(\Delta n\omega_0 + \omega + [\varepsilon_\parallel(k_z) - \varepsilon_\parallel(k_z+\varkappa_z)]/\hbar)^2 + \gamma^2/4} \right\}. \quad (20)$$

We shall now consider certain special cases of Eq. (20).

Nondegenerate Semiconductor

a) $\kappa_z = 0$. As a consequence of the large value of the denominator of the second integral in (20), this term can be neglected. Substituting in (20)

$$f \sim \exp\{-[\varepsilon_\parallel + \hbar\omega_0(n + \tfrac{1}{2}) - \mu]/\Theta\}, \quad (21)$$

We can sum in it over n. Eliminating $e^{\mu/\Theta}$ from (20) by means of the formula for the electron concentration

$$N = \frac{g}{\pi} e^{\mu/\Theta} [1 - e^{-\hbar\omega_0/\Theta}]^{-1} \int e^{-\varepsilon_\parallel/\Theta}\, dk_z \quad (22)$$

we get an expression for U containing only a summation over Δn:

$$U = 2N\left(\frac{a\varkappa u_0}{4}\right)^2 \frac{\omega}{\hbar} \sum_{\Delta n=0}^{\infty} \frac{\operatorname{sh}(\Delta n\hbar\omega_0/2\Theta)}{(\Delta n\omega_0 - \omega)^2 + \gamma^2/4}$$
$$\times \exp\{-\lambda_\rho(1 + e^{-\hbar\omega_0/\Theta})(1 - e^{-\hbar\omega_0/\Theta})^{-1}\}$$
$$\times I_{\Delta n}(2\lambda_\rho e^{-\hbar\omega_0/2\Theta}[1 - e^{-\hbar\omega_0/\Theta}]^{-1}). \quad (23)$$

Here $I_{\Delta n}$ is the Bessel function of imaginary argument.

In the derivation of Eq. (23), nondegeneracy was assumed; therefore we can assume that $\hbar\omega_0/\Theta < 1$ and

$$\exp\{-\hbar\omega_0/\Theta\} - 1 \sim \hbar\omega_0/\Theta. \quad (24)$$

To go to the classical limit, we must assume $\lambda_\rho \ll 1$. Substituting (24) in (22), and setting $(\omega/\omega_0)^2\Theta/m^*c_0^2 = x$, we find an approximate expression for U:

$$U \approx \frac{N}{\Theta}\left(\frac{a\varkappa u_0}{4}\right)^2 \gamma\omega \sum_{\Delta n=1}^{\infty} \Delta n\omega_0 e^{-x} I_{\Delta n}(x)/[(\Delta n\omega_0 - \omega)^2 + \gamma^2/4], \quad (25)$$

which is very close to Eq. (10) of reference 4, obtained by classical means. The fundamental difference between (10) of reference 4 and (25) is that instead of $1/\tau^2$ in the denominator of the classical formula, we here have $\gamma^2/4$. A formula that agrees exactly with the classical can be obtained if we use, in place of the probability (16), the square of the modulus of the matrix element of the Dirac theory of quantum transitions, averaged over time by means of the factor $e^{-t/\tau}$. The numerical calculations and the consequences of (25) were discussed in reference 4. A graph is given there, too, from which the resonance character of the ultrasonic absorption is clearly evident.

b) $\kappa_z \neq 0$. We substitute f from (21) in (20) and sum over n:

$$U = g\left(\frac{a\varkappa u_0}{4}\right)^2 \frac{\omega\gamma}{\pi\hbar} \sum_{\Delta n} (1 - e^{-\hbar\omega_0/\Theta})^{-1}$$
$$\times \exp\left\{-\lambda_\rho \frac{1 + e^{-\hbar\omega_0/\Theta}}{1 - e^{-\hbar\omega_0/\Theta}} + \frac{\hbar\omega_0\Delta n}{2\Theta}\right\}$$
$$\times I_{\Delta n}(2\lambda_\rho e^{-\hbar\omega_0/\Theta}/[1 - e^{-\hbar\omega_0/\Theta}])\, e^{\mu/\Theta}$$
$$\times \left\{ \int \frac{\exp(-\varepsilon_\parallel/\Theta) - \exp\{-\Delta n\hbar\omega_0/\Theta - \varepsilon_\parallel(k_z+\varkappa_z)/\Theta\}}{[\Delta n\omega_0 - \omega + (\varepsilon_\parallel(k_z+\varkappa_z) - \varepsilon_\parallel(k_z))/\hbar]^2 + \gamma^2/4} dk_z \right.$$
$$\left. + \int \frac{\exp(-\varepsilon_\parallel/\Theta) - \exp\{\Delta n\hbar\omega_0/\Theta - \varepsilon_\parallel(k_z+\varkappa_z)/\Theta\}}{[\Delta n\omega_0 + \omega + (\varepsilon_\parallel(k_z) - \varepsilon_\parallel(k_z+\varkappa_z))/\hbar]^2 + \gamma^2/4} dk_z \right\}. \quad (26)$$

It is of interest to note that in both cases a) and b) of the nondegenerate state of the current carrier, the energy U is proportional to the number of particles N.

Metal

We shall make an approximate estimate of U for $\kappa_z = 0$. We neglect the second integral in (20) and take the mean value of the positive expression $[L_n^{\Delta n}]^2 \lambda_\rho^{\Delta n} e^{-\lambda_\rho} [(n+1)\ldots(n+\Delta n)]^{-1}$ outside the summation over n. We obtain

$$U \sim (g/m^*)(a\varkappa u_0/4)^2 (\gamma\omega\Delta n\hbar\omega_0 N/\omega_0\mu); \quad (27)$$

μ is the chemical potential of the electron gas. Equation (27) shows that only a comparatively small part $\Delta n\hbar\omega_0/\mu$ of all of the electrons plays a role in the process of absorption of ultrasonic energy in metals.

Semiconductor in which the Current Carriers are in a Degenerate State

For those comparatively high magnetic field intensities that guarantee satisfaction of the inequality $\omega_0 > 1/\tau$, the states of the electron with energy $\epsilon_\parallel + \hbar\omega_0(n + \frac{1}{2})$ are strongly degenerate. The multiplicity of the degeneracy 2g at $\mathcal{H} \sim 10^3$ oe is very large (of the order 10^{10}). Such strong degeneracy causes only a small number of the low-lying quantum levels to be filled. For example, for $N \sim 10^{17}\ \mathrm{cm}^{-3}$ and $\omega_0 \sim 10^{10}$, the number of the highest filled discrete level is of the order of 10. Therefore, the coefficient of absorption can be computed by direct summation of several non-vanishing terms in (20). As before, we shall consider two cases.

a) $\kappa_z = 0$. We introduce a new integration variable

$$(\hbar/2\pi)\, dk_z = [(2m^*/\varepsilon_\parallel)^{1/2}/2\pi\hbar]\, d\varepsilon_\parallel . \qquad (28)$$

We substitute (28) in (20) and, using the notation

$$F_{-1/2}\left(\mu - \hbar\omega_0\left(n+\frac{1}{2}\right)\right) = \int \frac{d\varepsilon_\parallel}{\varepsilon_\parallel^{1/2}\,[\exp\{(\varepsilon_\parallel + \hbar\omega_0(n+1/2) - \mu)/\Theta\} + 1]}$$

— the Fermi integral with index $-\frac{1}{2}$, we obtain

$$U = (g/2\pi^2\hbar^2)(a\varkappa u_0/4)^2 \gamma\omega (2m^*)^{1/2} \sum_{\Delta n, n} \frac{\lambda_\rho^{\Delta n}[L_n^{\Delta n}]^2 e^{-\lambda_\rho}\{F_{-1/2}(\mu - (n+1/2)\hbar\omega_0) - F_{-1/2}(\mu - (n+\Delta n + 1/2)\hbar\omega_0)\}}{(n+1)\ldots(n+\Delta n)[(\Delta n\omega_0 - \omega)^2 + \gamma^2/4]} . \qquad (29)$$

The numerical value of Γ can be found from the tables for $F_{-1/2}$ (see, for example, reference 8).

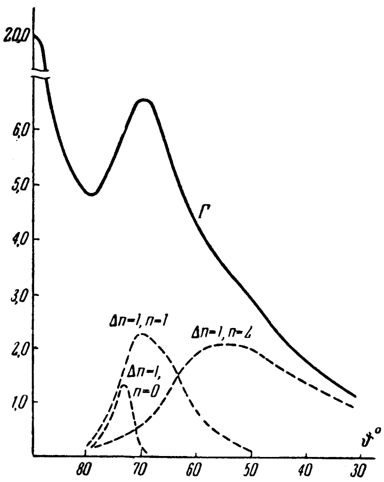
b) $\kappa_z \neq 0$. In this case, too, there will be comparatively few non-vanishing terms in the sum over n. If $\hbar\omega_0 \gg \Theta$, the gas can be considered strongly degenerate. Calculation of the integrals in (20) leads to the following formula for the ultrasonic absorption:

$$\Gamma = \frac{gm^*a^2}{16\pi\rho c_0^2\hbar^2} \sum_{\Delta n} (\lambda_\rho^0)^{\Delta n} \frac{\sin^{2\Delta n}\vartheta}{\cos\vartheta} \sum_n \frac{2e^{-\lambda_\rho}}{(n+1)\ldots(n+\Delta n)} \left[L_n^{\Delta n}\right]^2$$
$$\times \Big\{\tan^{-1}\Big[\frac{\omega_0}{\gamma}\Big(\lambda_\rho^0\Big[\cos^2\vartheta + 2k_z \times\Big(\mu - \hbar\omega_0\Big(n+\frac{1}{2}\Big)\Big)\frac{\cos\vartheta}{\varkappa}\Big] + 2\Big(\Delta n - \frac{\omega}{\omega_0}\Big)\Big)\Big]$$
$$- \tan^{-1}\Big[\frac{\omega_0}{\gamma}\Big(\lambda_\rho^0\Big[\cos^2\vartheta - 2k_z \times\Big(\mu - \hbar\omega_0\Big(n+\frac{1}{2}\Big)\Big)\frac{\cos\vartheta}{\varkappa}\Big] + 2\Big(\Delta n - \frac{\omega}{\omega_0}\Big)\Big)\Big]$$
$$- \tan^{-1}\Big[\frac{\omega_0}{\gamma}\Big(\lambda_\rho^0\Big[\cos^2\vartheta + 2k_z \times\Big(\mu - \hbar\omega_0\Big(n+\Delta n+\frac{1}{2}\Big)\Big)\frac{\cos\vartheta}{\varkappa}\Big] + 2\Big(\Delta n - \frac{\omega}{\omega_0}\Big)\Big)\Big]$$
$$+ \tan^{-1}\Big[\frac{\omega_0}{\gamma}\Big(\lambda_\rho^0\Big[\cos^2\vartheta - 2k_z \times\Big(\mu - \hbar\omega_0\Big(n+\Delta n+\frac{1}{2}\Big)\Big)\frac{\cos\vartheta}{\varkappa}\Big] + 2\Big(\Delta n - \frac{\omega}{\omega_0}\Big)\Big)\Big] + \ldots\Big\}. \qquad (30)$$

Here $\lambda_\rho^0 = \hbar\omega^2/2m^*\omega_0 c_0^2$, $k_z[\mu - \hbar\omega_0(n+\frac{1}{2})]$ is the Fermi value of the wave number for an electron in the n-th discrete level. The dots take the place of the sum of an additional four arctangents similar to those written down, with the parameter $2(\Delta n + \omega/\omega_0)$ in place of $2(\Delta n - \omega/\omega_0)$. In (30), ϑ is the angle between the direction of propagation of the ultrasound and the intensity of the magnetic field.

The calculation of Γ has been carried out according to Eq. (30) for ϑ in the interval from 0 — 90°, and a graph has been constructed of the dimensionless quantity $\Gamma \cdot 16\pi\rho c_0^2\hbar^2/gm^*a^2$. The following numerical values of parameters were chosen for the calculation: $\omega/\omega_0 = 2$; $\lambda_\rho^0 = 0.2$, $c_0 = 5 \times 10^5$ cm/sec. The chemical potential μ we set equal to $3.75\hbar\omega$, which corresponds to an electron concentration $N \approx 5 \times 10^{16}\ \text{cm}^{-3}$. Summation in (30) was carried out over n from 0 to 3 for two values $\Delta n = 1, 2$. For $\Delta n > 2$, the term $(\lambda_\rho^0)^{\Delta n}$ becomes a very small quantity in our example.

The dashed lines in the drawing indicate graphs of the individual terms of the sum in (30) with $\Delta n = 1$ and $n = 0, 1, 2$. Each of these has a maximum for intermediate values of ϑ, which describes the resonance of electrons found in the discrete level with $n = 0, 1, 2$. In this resonance process, part of the energy of the phonon (in the given example,



Dependence of the sound absorption Γ (in units of $gm^*a^2/16\pi\rho c_0^2\hbar^2$) on angle ϑ. The contribution of the different terms of the sum (30) corresponding to different values of n and Δn is shown by the dashed lines.

$\frac{1}{2}$) is used up to change the translational motion of the electron in the direction of the magnetic field. From the laws of conservation of energy and momentum, it is easy to obtain an equation for ϑ for which a particular type of resonance sets in. This resonance is determined by the zeros of the arctangents in (30):

$$\lambda_\rho^0(\cos^2\vartheta \pm 2k_z\cos\vartheta/\varkappa) + 2(\Delta n - \omega/\omega_0) = 0. \qquad (31)$$

The deeper the level n, the larger the resonance angle ϑ. This circumstance is excellently illustrated by the given graphs.

The resonance effect under consideration can be of interest in the following connection. The absorption of phonons with $\kappa_z \neq 0$ leads to a change of the momentum of the electrons by $\hbar\kappa_z$. In unit time, $U/\hbar\omega$ phonons are absorbed, and, consequently, the rate of change of the projection of the momentum of the electrons on the z axis is equal to $\dot{P}_z = \kappa_z U/\omega$. It is easy to estimate the current flowing in the direction of the magnetic field,

$$j_z \approx e\tau\varkappa_z E_{so}\Gamma / m^* \omega, \tag{32}$$

or the intensity of the electric field inside the bounded semiconductor:

$$E_z \sim e\tau\varkappa_z E_{so}\Gamma / m^* \omega\sigma, \tag{33}$$

σ is the electrical conductivity of the semiconductor.r

3. Experiments on ultrasonic resonance of current carriers required low temperatures of the order of 1°K and high ultrasonic frequencies $\sim 10^{10}$, 10^{11} cps. At the present time, ultrasonic technology has come close to this range of frequencies. Thus, for example, Baranskiĭ[9], and Bömmel and Dransfeld[10] have obtained frequencies $\sim 2.5 \times 10^9$ cps, while Jacobson[11] has reached 10^{10} cps. Recently results have been published of the measurement of ultrasonic absorption in silicon, germanium, etc, for frequencies close to 10^9, and at temperatures of about −150° C.[12] If the results of experiments are roughly extrapolated to the region of much lower temperatures ~1°K and much higher frequencies $\omega \sim 10^{10}, 10^{11}$, then one can assume that the lattice absorption should under these conditions be of the order of 10^2 cm^{-1}, while the electronic part estimated by Eq. (25) is $\Gamma_{el} \sim 10$ cm^{-1} (for values of the parameters $N = 10^{12}$ cm^{-3}, $a = 15 \times 10^{-12}$ ev, $c_0 \sim 10^5$ cm/sec, $x \sim 1$, $\omega \sim 10^{11}$, $\gamma^{-1}\omega_0 \sim 1$) although an order lower is sufficiently large for experimental observation. It is also necessary to remark that the measurement of the electric field produced by the effect of "amplification" of the electrons along the magnetic lines of force [which, by (31), (32), is proportional to the electronic part of the absorption coefficient], can be shown to be a much easier problem than the separation of the contribution of the electronic absorption from the general absorption of the semiconductor.

In conclusion, I want to express my gratitude to Yu. E. Perlin for useful discussions and to V. L. Gurevich for a discussion of the fundamental aspects of the problem and valuable comments.

[1] H. Bömmel, Phys. Rev. **108**, 1094 (1957).

[2] Morse, Bohm, and Gavenda, Phys. Rev. **109**, 1394 (1958).

[3] A. Pippard, Phil. Mag. **2**, 1147 (1957).

[4] E. P. Pokatilov, JETP **36**, 5 (1959), Soviet Phys. JETP **9**, 1037 (1959).

[5] L. D. Landau and E. M. Lifshitz, Квантовая механика, (Quantum Mechanics), Gostekhizdat, 1948 [Engl. Pergamon, 1958].

[6] W. Heitler, Quantum Theory of Radiation, Oxford, 1959.

[7] A. E. Zil'berman, JETP **23**, 49 (1952).

[8] P. Rhodes, Proc. Roy. Soc. (London) **A204**, 396 (1950).

[9] K. N. Baranskiĭ, Dokl. Akad. Nauk SSSR **114**, 517 (1957), Soviet Phys.-Doklady **2**, 237 (1957).

[10] H. Bömmel and K. Dransfeld, Phys. Rev. Lett. **1**, 234 (1958).

[11] D. Jacobson, Phys. Rev. Lett. **2**, 249 (1959).

[12] Lamb, Redwood, and Steinshleifer, Phys. Rev. Lett. **3**, 28 (1959).

Translated by R. T. Beyer
226