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CERENKOV RADIATION FROM A CHARGED PARTICLE WITH INTRINSIC MAGNETIC MOMENT

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The effect of spin on the intensity and polarization of the radiation produced in a ferrite by a charged particle with intrinsic magnetic moment is considered by quantum electrodynamic methods. The radiation is analyzed near threshold.

THE Cerenkov radiation produced by electrons, and its polarization, have been studied by a number of authors;[1-5] a number of other authors have considered the radiation due to a "superluminal" magnetic moment.[6,7] In this note we consider the "mixed" case, i.e., the radiation from a charged particle with intrinsic magnetic moment.

As is well known, the operator associated with the energy of interaction between a charged particle, having intrinsic magnetic moment, and an electromagnetic field in a medium, is given by

$$W = e(\boldsymbol{\alpha}\mathbf{A}) + \mu_0\{\rho_3(\boldsymbol{\sigma}\mathbf{B}) + \rho_2(\boldsymbol{\sigma}\mathbf{E})\}, \qquad (1)$$

where ρ_3, ρ_2, $\boldsymbol{\alpha}$, and $\boldsymbol{\sigma}$ are Dirac matrices. The vector potential $\mathbf{A}$ for the quantized transverse electromagnetic field in a medium characterized by $\epsilon(\omega)$ and $\mu(\omega)$ can be written in the form[8]

$$\mathbf{A} = L^{-3/2}\sum_{\varkappa}(2\pi c''\hbar/\varkappa)^{1/2}[\mathbf{a}\exp(-ic'\varkappa t + i\boldsymbol{\varkappa}\mathbf{r})$$
$$+ \mathbf{a}^{+}\exp(ic'\varkappa t - i\boldsymbol{\varkappa}\mathbf{r})], \qquad (2)$$

where $c'' = c'\mu$, $c' = c/n$, $n = \sqrt{\epsilon\mu}$ and $\hbar\boldsymbol{\kappa}$ is the momentum of the photon.

In order to investigate the polarization of the radiation, we separate the amplitude of the vector potential into two components:[9]

$$\mathbf{a} = \mathbf{a}_2 + \mathbf{a}_3 = \boldsymbol{\beta}_2 q_2 + \boldsymbol{\beta}_3 q_3,$$
$$\boldsymbol{\beta}_2 = [\boldsymbol{\varkappa}^0\times\mathbf{k}^0]/\sqrt{1-(\boldsymbol{\varkappa}^0\mathbf{k}^0)^2}, \quad \boldsymbol{\beta}_3 = [\boldsymbol{\varkappa}^0\times\boldsymbol{\beta}_2] \qquad (3)$$

for linear polarization and

$$\mathbf{a} = \mathbf{a}_1 + \mathbf{a}_{-1} = \boldsymbol{\beta}_1 q_1 + \boldsymbol{\beta}_{-1} q_{-1},$$
$$\sqrt{2}\boldsymbol{\beta}_\lambda = \boldsymbol{\beta}_2 + i\lambda\boldsymbol{\beta}_3 \quad (\lambda = 1, -1) \qquad (4)$$

for circular polarization. Here $\boldsymbol{\kappa}^0 = \boldsymbol{\kappa}/\kappa$, $\mathbf{k}^0 = \mathbf{k}/k$, and the q_j $(j = 2, 3, 1, -1)$ are the quantum-mechanical operators of the vector potential $\mathbf{A}$.

If there are no photons at the initial time, the interaction-energy operator (cf. Eq. 1) becomes

$$W_j^{+} = L^{-3/2}\sum_{\varkappa}(2\pi c''\hbar/\varkappa)^{1/2}\exp\{i(c'\varkappa t - \boldsymbol{\varkappa}\mathbf{r})\}$$
$$\times\{e(\boldsymbol{\alpha}\mathbf{a}_j^{+}) - i\mu_0[\rho_3(\boldsymbol{\sigma}[\boldsymbol{\varkappa}\times\mathbf{a}_j^{+}]) + \varkappa n^{-1}\rho_2(\boldsymbol{\sigma}\mathbf{a}_j^{+})]\}, \qquad (5)$$

where multiple photon production has been neglected.

Using perturbation-theory methods, we can find the radiation probability for a charged particle with intrinsic magnetic moment:[4]

$$W_j = \frac{2\pi}{c\hbar^2}\sum_{\mathbf{k}',\boldsymbol{\varkappa}} R_j^{+}R_j\delta_{\mathbf{k},\mathbf{k}'+\boldsymbol{\varkappa}}\delta(K' + \varkappa/n - K),$$

$$R_j = L^{-3/2}(2\pi c''\hbar/\varkappa)^{1/2}b'^{+}\{e(\boldsymbol{\alpha}\mathbf{a}_j^{+})$$
$$- i\mu_0[\rho_3(\boldsymbol{\sigma}[\boldsymbol{\varkappa}\times\mathbf{a}_j^{+}]) + n^{-1}\varkappa\rho_2(\boldsymbol{\sigma}\mathbf{a}_j^{+})]\}b, \qquad (6)$$

where $\hbar\mathbf{k}$, $c\hbar K$, and $\hbar\mathbf{k}'$, $c\hbar K'$ are the momentum and energy of the particle before and after it radiates, and $\hbar k_0/c$ is the particle mass. Computing the necessary matrix elements in Eq. (6),[10] we can obtain an expression for the radiation intensity per unit path for the case of linear polarization $(j = 2, 3)$:

$$W_{j\,\mathrm{tot}}^{s,s'} = W_{j\,\mathrm{ch}}^{s,s'} + W_{j\,\mathrm{m}}^{s,s'} + W_{j\,\mathrm{cr}}^{s,s'}. \qquad (7)$$

Here

$$W_{j\,\mathrm{cm}}^{s,s'} = \frac{e\mu_0}{2}\int\limits_{\cos\theta\leqslant 1}\mu(\omega)\frac{\hbar n^2\omega^3}{pc^4\beta}\sqrt{1-\beta^2}\{(1-n^{-2})$$
$$+ ss'\xi[1 + n^{-2} - (2/\beta n)\cos\theta - 2(j-2)n^{-2}\sin^2\theta]\}d\omega;$$

and $W_{j\,\mathrm{ch}}^{s,s'}$ and $W_{j\,\mathrm{m}}^{s,s'}$ are the radiation intensities for the charge and magnetic moment respectively; these have been found earlier;[7,11]

$$\xi = \left[1 + \frac{\hbar^2 n^2\omega^2}{p^2c^2} - \frac{2\hbar n\omega}{pc}\cos\theta\right]^{-1/2}, \quad \cos\theta = \frac{1}{\beta n} + \frac{\hbar n\omega}{2pc}(1 - n^{-2}).$$

In what follows we will consider only the cross term $W_{j\,\mathrm{cr}}^{s,s'}$.

If the spin orientation is unknown before and after emission of the photon and the total radiation intensity is measured, then we must average over initial spin states and sum over final spin states in Eq. (7); this procedure yields

$$W_{j\,\mathrm{cr}} = e\mu_0\int\limits_{\cos\theta\leqslant 1}\mu(\omega)\frac{\hbar n^2\omega^3}{pc^4\beta}\sqrt{1-\beta^2}(1-n^{-2})d\omega, \qquad (8)$$

whence it is apparent that the polarization is not linear. In the relativistic or classical approximations, the cross term $W_{j\,cr}$ tends toward zero, in complete agreement with the classical theory.

It follows from Eq. (7) that close to threshold ($\cos\theta \approx 1$) the intensity is given by

$$W_{j\,cr}^{s,s'} = \frac{e\mu_0}{2} \int\limits_{\cos\theta\leqslant 1} \mu(\omega) \frac{\hbar n^2\omega^3}{pc^4\beta} \sqrt{1-\beta^2}(1-n^{-2}) \times (1-ss')\,d\omega \quad (j = 2, 3). \tag{9}$$

Whence, $W_{j\,cr}^{s,s'} = 0$ for $ss' = +1$; for $ss' = -1$, however,

$$W_{j\,cr}^{s,s'} = e\mu_0 \int\limits_{\cos\theta\leqslant 1} \mu(\omega) \frac{\hbar n^2\omega^3}{pc^4\beta} \sqrt{1-\beta^2}(1-n^{-2})\,d\omega, \tag{10}$$

that is to say, the radiation near threshold is associated with a spin flip, a pure quantum effect. The fact that the threshold radiation is finite is a consequence of this quantum effect.

Using similar techniques we can obtain an expression for the radiation intensity per unit path in the case of circular polarization

$$W_{j\,tot}^{s,s'} = W_{j\,ch}^{s,s'} + W_{j\,m}^{s,s'} + W_{j\,cr}^{s,s'} \qquad (j = 1, -1), \tag{11}$$

where

$$W_{j\,cr}^{s,s'} = \frac{e\mu_0}{2} \int\limits_{\cos\theta\leqslant 1} \mu(\omega) \frac{\hbar n^2\omega^3}{pc^4\beta} \sqrt{1-\beta^2} \Big\{(1-n^{-2}) \times \Big[1 + js\cos\theta - js'\xi\Big(\cos\theta - \frac{\hbar n\omega}{pc}\Big)\Big] + ss'\xi\Big[1 + n^{-2} - \frac{2}{\beta n}\cos\theta - n^{-2}\sin^2\theta\Big]\Big\}\,d\omega.$$

Using Eq. (11), we can determine the threshold radiation for a particle with oriented spin:

$$W_{j\,cr}^{s,s'} = \frac{e\mu_0}{2} \int\limits_{\cos\theta\leqslant 1} \mu(\omega) \frac{\hbar n^2\omega^3}{pc^4\beta} \sqrt{1-\beta^2} \times (1-n^{-2})(1+js)(1-ss')\,d\omega. \tag{12}$$

It will be apparent that near threshold the radiation is associated with a spin flip; for a flux of particles characterized by spin $s = +1$, the radiation near threshold exhibits right-circular polarization ($j = +1$) while if $s = -1$ it exhibits left-circular polarization ($j = -1$), since $W_{j\,cr}^{s,s'} \neq 0$ only when $js = +1$.

It should be noted that the radiation described by Eq. (8) is not polarized, because we have averaged over initial spin states so that particles characterized by $s = +1$ and $s = -1$ make equal contributions.

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