SECOND SOUND IN HELIUM II

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An investigation of second sound in helium has been conducted by the resonance method down to a temperature of 0.38°K. At temperatures below 0.5°K the attenuation of second sound is so great that resonances are not observed, and the second sound is transformed into rapidly-attenuated thermal waves. Experimental values of u_2 are presented from the λ-point to 0.55°K (1958 temperature scale). It is shown that the computed data for u_2 obtained by Bendt et al.[16] from experiments on the scattering of cold neutrons in helium agree well with the data from direct measurements of the second sound velocity.

THE velocity of second sound in superfluid helium has been measured at temperatures above 0.8°K and up to the λ-point by both resonance and pulse methods.[1-5] Its values as determined by various authors and by various methods agree among themselves to within the limits of experimental error. At temperatures below 0.8°K second sound has been studied by a number of authors[6-9] using the pulse method. In these papers, unfortunately, no account was taken of experimental peculiarities, and data are presented on the second sound velocity down to temperatures of 0.1 — 0.2°K. In pure He^4, however, the mean free path of the thermal quanta, for $T < 0.6°K$, becomes comparable with the dimensions of the containers used and with the second-sound wavelength, as noted in one of the papers cited.[8] The phenomenon of second sound itself therefore disappears, being transformed into ordinary, rapidly-attenuated thermal waves. The thermal effect observed in conjunction with the pulses is associated with the reception of acoustic waves of first sound, excited by thermal expansion of the liquid helium. In the work of Osborne,[10] measurements are performed on the phase of the temperature oscillations of a thermometer, placed at one end of a resonator, as related to the phase of the oscillations of a continuously operating radiator located at the other end of the resonator. The measurements are carried out over the temperature range 0.25 — 0.58°K. In this case, as in those preceding, it is hardly possible to speak of second sound.

The goal of the present work was the measurement of the velocity of second sound down to the very lowest temperatures at which second sound still exists. Such measurements may most reliably be made by the resonance method. Considerable difficulties naturally arise in this case, associated with the attainment and maintenance of temperatures of 0.5 — 0.8°K within a large volume of He^4, in the presence of appreciable heat inputs, and also with the necessity for receiving extremely weak second sound signals.

In order to produce and maintain temperatures in the range 0.4 — 1.2°K the He^3 cryostat described previously[11] was employed. Two resonators were used. The first was made from a glass tube 33 mm in internal diameter and 50 mm in length, to the ground ends of which plane glass disks were cemented with BF cement. To one of the disks was fixed a radiator of constantan wire 40 microns in diameter, with a resistance of 160 Ω; to the other, a thermometer of 30-micron phosphor bronze wire with an initial resistance ~ 100 Ω. Leads from heater and thermometer were passed separately through two one-millimeter apertures drilled in the tube at its center, between the disks. The resonator was placed within a receptacle filled with He^4 and cooled by He^3 vapor pumping. The second resonator was made of copper and was also a cylinder, but of greater size. Its internal diameter was 60 mm and its length 70 mm. Within, on a disk which was soldered to the He^3 bath container, was fixed a radiator of 30-micron constantan wire. The radiator was made in the form of a closely-wound single-layer bifilar spiral of 1000 Ω total resistance. At the opposite end of the resonator were placed two thermometers, of 30- and 40-micron bronze, each with ~ 100 Ω resistance, as well as samples of 50-micron aluminum wire and 67-micron cadmium wire.

The bronze thermometers were calibrated

against the He^3 vapor pressure in accordance with the data presented by Roberts and Sydoriak.[12] In addition, during the course of each experiment the operation of the thermometers was controlled according to the aluminum and cadmium superconducting transition points. Owing to insufficient purity of the aluminum, its transition was broadened over 0.02°. The temperature $T = 1.13°K$, at which the sample had lost half of its residual resistance, was taken as the aluminum transition point. The cadmium was adequately pure (99.999%); its transition was sharp and corresponded to $T = 0.52°K$. (The terrestrial magnetic field was not compensated for.) The thermometers and the super-conducting reference standards were connected in series and supplied with a stabilized measuring current of 1 ma. An apparatus developed by Vetchinkin[13] served as the stabilizer. The stabilizer guaranteed constancy of the current to 0.001% for a variation in the circuit resistance of 500 Ω. The current in the potentiometer circuit was likewise stabilized with an accuracy of 0.001%, with the aid of a simple system using a galvanometer and photo-resistors.[13] The use of stabilizing devices greatly simplified the work, since the necessity of controlling for constancy of the current was completely eliminated. During the measurements we determined the temperature from the direct deflection of a galvanometer, connected to the thermomenter as shown in the circuit of Fig. 1.

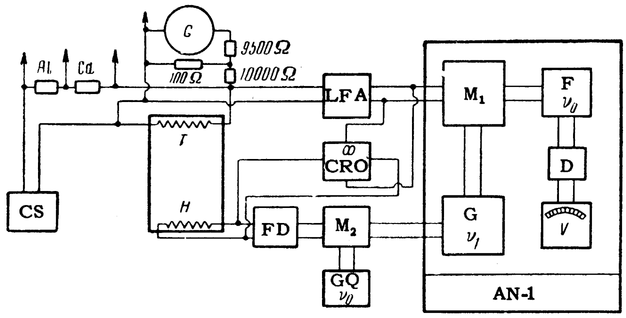

FIG. 1. Diagram of apparatus for observation of second sound (for symbols, cf. text).

For generation and measurement of the second sound oscillations an apparatus having high sensitivity and low noise level was prepared in the Institute's electronics shop. An AN-1 voltage analyzer was selected as a foundation, having its own internal generator G (cf. Fig. 1), working at frequencies from 56 to 76 Kc, a mixer M_1, and a narrow-band quartz filter F, tuned for a frequency of 56 Kc. Following the filter are a detector D and a voltmeter. The operating range of the AN-1 is from 100 to 20 kcs in frequency, and in voltage from 100 μv to 100 v; the passband width is 25 cps. An attachment was made for the AN-1 consisting of a generator, GQ, quartz-stabilized at the frequency of the filter F (56 kcs), a mixer M_2 with a filter passing frequencies from 100 cps to 10 kcs, and a divider FD to halve the frequency. In addition, the low-noise amplifier LFA described previously by Zinov'eva[14] was connected at the input of the AN-1.

The principle of operation of the circuit was as follows: a frequency $\nu = \nu_1 - \nu_0$ was produced (and high-frequency components filtered out) in the mixer M_2. In the divider FD this frequency was divided by two and applied to the heater H in the resonator. The heater radiated second sound of frequency ν, which was received by the thermometer T. From the thermometer the signal, amplified by the LFA, entered the mixer M_1 of the AN-1 analyzer. There it was mixed again with the frequency ν_1, and a signal of frequency $\nu_0 = \nu_1 - \nu$ passed through the quartz filter F, was detected by the detector D, and registered on the voltmeter V. The exact value of the frequency was determined from the number of periods recorded with the aid of a counting circuit during a known time interval.

The system described made it automatically possible to have in the receiving circuit, for any frequency of the generator, a narrow-band quartz filter set precisely at the frequency of the received signal. Together with the low-noise amplifier at the input, it assured positive registration of signals exceeding by a few times the noise level, which was $\sim 0.03\,\mu$v. In addition, the voltage supplied to the heater was simultaneously applied to the horizontal sweep of the cathode-ray oscilloscope CRO, to the vertical plates of which was fed the amplified signal from the thermometer. With the aid of the oscilloscope it was possible to determine the phase of the second sound oscillations relative to the phase of the radiator. Inasmuch as it was very difficult to avoid pickup from the circuit at frequencies which were multiples of 50, especially at the uneven harmonics, the measurements in the low temperature region were carried out in the following manner: a frequency was selected at which the pickup was at a minimum. Near 1°K, where the resonance peaks are extremely narrow, the temperature was adjusted to correspond exactly to resonance, and a Lissajous figure in the form of a symmetrical figure-eight was produced on the oscilloscope CRO by adjusting the LFA amplifier. Then with slow cooling or warming-up of the resonator, which had previously been cooled to 0.4°K, the temperatures corresponding to resonances were recorded, according to the readings of the circuit of the cathode-ray oscilloscope.

In addition, a more detailed investigation of the dependence of the amplitude of the second sound oscillations within the resonator upon temperature was carried out in the copper resonator. Curves of this sort, obtained in the copper resonator for frequencies of 633.6, 730 and 816.8 cps, are presented in Fig. 2. As can be seen from Fig. 2, the second sound resonances are strongly washed out as the temperature is lowered. For the frequency of 816.8 cps at T = 0.535°K it is evident from the phase of the oscillations on the oscilloscope that the conditions for resonance are present; due to the great damping, however, the amplitude maximum is no longer observed.

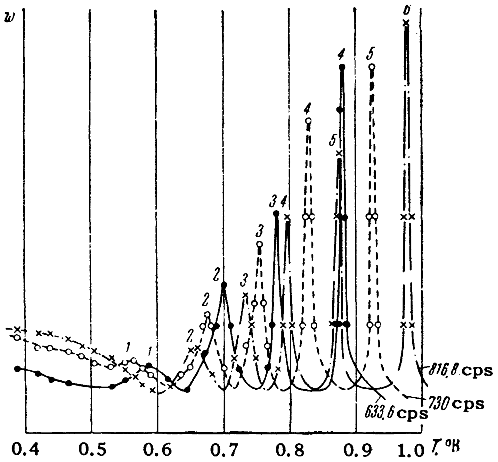

FIG. 2. Dependence of amplitude of second sound oscillations in resonator upon temperature. Number of resonances indicated by numbers on curves. Frequency and current supplied to heater held constant.

In determining the velocity u_2 from observation of the amplitude maxima, it is necessary to introduce for the washed-out peaks a correction to the value of u_2. If the length of the resonator is designated by l, and the law for propagation of second sound in the resonator is taken in the form

$$w = w_1 e^{-\gamma l} \sin(\omega t - \omega x / u_2),$$

then near resonance $\omega l/u_2 = n\pi + \delta$, and the amplitude of the oscillations will be[15]

$$w_1 = w_0 / 2\sqrt{\delta^2 + (\gamma l)^2}.$$

The condition for a maximum in w_1 will be a minimum value for the expression under the radical; i.e.,

$$\delta\, \partial\delta / \partial T + \gamma l^2\, \partial\gamma / \partial T = 0.$$

But

$$\partial\delta / \partial T = -(\omega l / u_2^2)\, \partial u_2 / \partial T$$

and the maximum will be observed at

$$\frac{\omega l}{u_2} = n\pi + \frac{u_2^2 \gamma l\, \partial\gamma / \partial T}{\omega\, \partial u_2 / \partial T} \quad \text{or} \quad u_2 = \frac{\omega l}{n\pi}\left(1 - \frac{u_2^3 \gamma\, \partial\gamma / \partial T}{\omega^2\, \partial u_2 / \partial T}\right).$$

This correction amounts, at low temperatures, to approximately 3%. If the observation is carried out using the cathode-ray oscilloscope — i.e., according to the phase of the oscillations — then it is not necessary to introduce the correction. The correction corresponding to the change in l from room temperature down to that of the helium is, in all, 0.3% for copper, and even less for glass.

Results of the measurement of u_2 at temperatures below 1°K are presented in Fig. 3. The curve of du_2/dT is also given in the figure, at

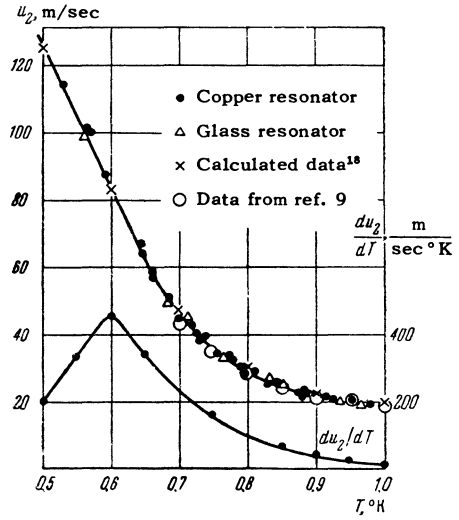

FIG. 3. Dependence of second sound velocity upon temperature.

the bottom. With the copper resonator the measurements were carried out at frequencies of 634, 730, 817, 930 and 1440 cps (all indicated by the same symbol), and with the glass resonator, at frequencies of 990 and 1352 cps. In Fig. 3 are shown the data of Bendt, Cowan and Yarnell[16] obtained for u_2, by calculation, from the spectrum of the thermal excitations in liquid helium, using the spectrum established experimentally[17] from measurements of the scattering of cold neutrons in helium II. The data of Kramers and co-authors,[9] from measurements of u_2 by the pulse method, are also presented. Inasmuch as at T = 0.7°K and below there is observed a considerable dispersion of second sound, due to the great attenuation, the pulses are strongly distorted, and the method does not yield satisfactory results. Kramers and his co-authors succeeded in obtaining more or less reasonable results, thanks to a special method for reducing the experiments, but these only at temperatures above 0.7°K. Data for u_2 by other authors[6-8] at temperatures below 0.8°K have not had dispersion taken into account in the proper

T, °K	u_2, m/sec experiment	u_2, m/sec calculation[16]	T, °K	u_2, m/sec experiment	u_2, m/sec calculation[16]	T, °K	u_2, m/sec experiment	u_2, m/sec calculation[16]	T, °K	u_2, m/sec experiment
0.50		125	1.12	18.30		1.65	20.32		2.025	15.7
0.55	105		1.15	18.33		1.70	20.3	20.0	2.05	14.7
0.60	83	83.2	1.20	18.4	18.9	1.75	20.15		2.075	13.5
0.65	62		1.25	18.7		1.80	19.85	19.2	2.10	12.2
0.70	46.5	47.2	1.30	19.1	19.2	1.825	19.55		2.11	11.5
0.75	36.0		1.35	19.4		1.850	19.3		2.12	10.8
0.80	29.0	29.9	1.40	19.7	19.7	1.875	19.0		2.13	10.0
0.85	24.4		1.45	19.9		1.90	18.7	17.8	2.14	9.1
0.90	21.6	22.6	1.50	20.1	20.0	1.925	18.3		2.15	8.1
0.95	19.8		1.55	20.25		1.950	17.8		2.16	6.7
1.00	18.9	19.8	1.60	20.3	20.1	1.975	17.1		2.17	4.7
1.05	18.5		1.63	20.33		2.00	16.4	15.2	2.175	2.5
1.10	18.33	18.9								

way, and differ even more from those obtained in the present experiment.

In the copper resonator, the measurement of u_2 was carried out in the region of higher temperatures as well, up to T = 2.175°K (T_λ = 2.176°K). The combined results from the measurement of the second sound velocity u_2 under saturated vapor pressure over the whole temperature range are presented in the table. These data are obtained by averaging the most trustworthy measurements performed at the Institute for Physics Problems, and are referred to the 1958 temperature scale.[18] For comparison, the calculated data of Cowan et al.[16] are also given in the table. As is evident, the two sets of data are very close to one another, and practically agree to within the limits of error for the calculated and experimental data.

Measurement of the width of the resonance peaks makes it possible to determine the second sound attenuation. The attenuation at low temperatures is large; therefore it is no longer possible to take the quantity γl as equal to the half-width δ of the resonance curve. It is necessary to use the following approximation. With an accuracy up to terms in δ^7

$$\gamma l = \delta (1 - \delta^2 / 3).$$

At low temperatures, the correction reached 20%. For weak signals, moreover, it was necessary to introduce a correction for background. If a is the background amplitude, w_0 the amplitude of the signal at maximum, w_1 the whole amplitude at maximum, and w_2 the amplitude at which the half-width is equal to δ, then

$$w_1 = \sqrt{w_0^2 + a^2}, \quad w_2 = \sqrt{w_0^2 / 2 + a^2} = \sqrt{(w_1^2 + a^2)/2}.$$

Inasmuch as in our case the resonators were broad and short, the surface losses were incomparably smaller than the volume losses, and it was possible to neglect them. Data on the second sound attenuation are represented in Fig. 4. The y axis represents $\log(\gamma/\omega^2)$; this quantity is frequency-independent, since the volume losses are proportional to the square of the frequency. The data obtained by Kramers and co-authors,[9] and a curve constructed from the measurements of Zinov'eva,[19] are also shown in the figure. It should be noted that according to the data of Zinov'eva and Kramers the attenuation at temperatures above 0.8°K is much smaller than in the present work. This is explained by the fact that in such broad, short resonators, beginning as early as the third resonance, several types of oscillations arise at once. In our case, at the higher temperatures, resonances beyond the second were the ones observed, and it was impossible to determine from them the attenuation of plane waves of second sound. The attenuation as determined from the first and second resonances, however, lies on a prolongation of Zinov'eva's curve, and close to the data of Kramers. In several papers,[9,10] a mean free path for the phonons is determined from the second sound attenuation. At low temperatures, however, there are few rotons, and a fundamental role is played by the scattering of phonons on phonons. In this case the mean free path depends very strongly upon the angle of incidence of the phonons, and is a maximum when the directions coincide. At the same time, collisions of phonons

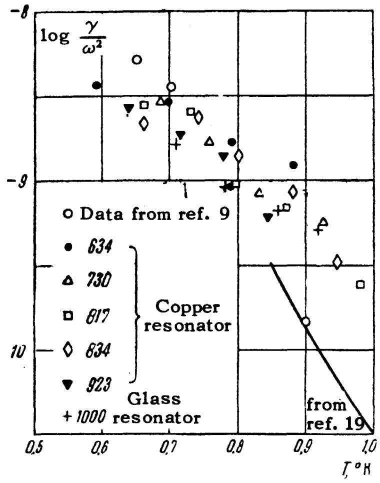

FIG. 4. Dependence of $\log(\gamma/\omega^2)$ upon temperature (numbers following symbols in list are frequencies in cps).

having the same direction cannot lead to the generation of second sound. The problem of the mean free path at low temperatures must therefore be investigated separately; in any case a model in the form of elastic spheres is not suitable for them.

Second sound might be observed at temperatures below 0.55° by using lower frequencies; in this case, however, the dimensions of the apparatus will grow considerably. Thus for observation of a resonance of identical form — i.e., maintaining the same $\gamma l = \gamma_0 l_0$ — taking $u_2 = l\omega/\pi$ and the increase in attenuation $d \log(\gamma/\gamma_0)/dT = -4$, for $T = 0.5°K$ as compared with $T = 0.55°K$, it is necessary to use a resonator with $l = 2l_0$, and by comparison with $T = 0.6°K$, a resonator with $l = 6l_0$, or, in the present case, a length ~ 0.4 m. Under the usual experimental conditions it is thus impossible to observe second sound at temperatures below 0.5°K.

Osborne[10] has carried out an investigation of the propagation of periodically-radiated heat, at temperatures between 0.25 and 0.58°K and frequencies from 60 to 2000 cps. The radiating heater was located at one end of a closed tube 2 cm in diameter. At the other closed end was placed a receiving thermometer. The phases of the oscillation were compared for various lengths of the tube (0.38, 3.16, 9.26, and 12.35 cm). The conditions in Osborne's experiment were close to those described in the present work; in the overlapping region the values should therefore agree, but Osborne's $u_2 = 145$ m/sec for the point at 0.54°K, while at 0.58°K, the quantity $u_2 = 115$ m/sec. The considerable increase in Osborne's values for u_2 can be explained by the fact that, in reducing the experiment, the author tacitly assumes the phonon-phonon scattering process to be analogous to the scattering of elastic spheres. The cardinal difference between these processes, however, lies in the fact that for elastic spheres any other sort of collision has a much greater probability than a collision in closely similar directions. The greatest contribution to the generation and propagation of second sound oscillations, though, is provided by collisions at large angles. However, in collisions of phonons the opposite picture prevails. Proceeding from these considerations, he obtained the extremely strange result that the phonon mean free path increases by a factor of three, in all from 0.58 to 0.25°K. From Osborne's experiments it is thus possible to affirm only the considerable likelihood of the conclusion that the concept of the phonon mean free path as applied to the propagation of heat in helium at temperatures below 0.7°K must be investigated in greater detail. In any case, to speak of second sound at temperatures below 0.5°K under presently attainable experimental conditions seems scarcely expedient. Here, phenomena will appear completely different.

In conclusion, I take this opportunity to thank Academician P. L. Kapitza for his interest in the work, Prof. I. M. Khalatnikov for valued discussions, and also A. I. Filimonov for his aid in carrying out the experiments.

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