energy. The upper curves a' and a correspond to the calculations by Bhabha<sup>5</sup> as corrected by Block, King, and Wada<sup>4</sup> for the cases of no screening (a') and complete screening (a). The two lower curves b' and b (b' — no screening, b — complete screening) were calculated by us from the results of Murota, Ueda, and Tanaka,<sup>6</sup> whose calculation is more exact than Bhabha's.

As can be seen from the figure, the totality of the experimental results on the determination of  $\lambda$  for an energy interval of primary electrons 1-100 Bev is in satisfactory agreement with the theory of Murota et al. A certain disagreement between experiment and the predictions of the above mentioned theory for electrons in the energy interval 0.1-1 Bev is apparently due to an illegitimate extrapolation into the indicated energy region of the correction calculated by Koshiba and Kaplon<sup>7</sup> for the number of false tridents, which should lead to a substantial underestimate of the true number of tridents.

Thus the experimental results on the determination of the cross section for direct electron-positron pair production by electrons should apparently

## CYCLOTRON ABSORPTION OF ELECTRO-MAGNETIC WAVES IN A PLASMA

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THE propagation of electromagnetic waves in a magnetoactive plasma at frequencies  $\omega$ , close to  $m\omega_{\rm H}^{\rm e,i}$  (m = 1, 2, ....; where  $\omega_{\rm H}^{\rm e}$  is the gyromagnetic frequency of the electron and  $\omega_{\rm H}^{\rm i}$  is the gyromagnetic frequency of the ion) is characterized by strong absorption; this absorption is due to the thermal motion of the electrons and ions (cyclotron absorption)<sup>1-5</sup> and is of interest in connection with problems of microwave diagnostics and radio-frequency heating of plasmas.

The damping of waves characterized by  $\omega \approx m\omega_{\rm H}^{\rm e}$ , m = 2, 3, ..., is especially pronounced in the case of a double resonance, i.e., when  $m\omega_{\rm H} \approx \omega_{\star}$ , where  $\omega_{\star}$  is the frequency given by the condition

$$A = 1 - u_e - v_e + u_e v_e \cos^2 \theta = 0,$$
$$u_e = (\omega_H^e / \omega)^2, \quad v_e = (\Omega_e / \omega)^2,$$

be considered as being in agreement with the predictions of quantum electrodynamics up to 100 Bev energies for the primary electrons.

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\*These events were found by A. A. Varfolomeev's group.

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 $\Omega_{\rm e}$  is the electron Langmuir frequency, and  $\theta$  is the angle between the direction of propagation of the wave and the direction of the magnetic field. As is well known,<sup>2,6</sup> when  $\omega \approx \omega_{+}$  the index of refraction for the extraordinary wave  $n_{2}$  become very large and a plasma wave can appear. When  $\omega \approx m\omega_{\rm H}^{\rm e} \approx \omega_{+}$  and m = 3, 4 the complex indices of refraction for these waves, determined from the dispersion equation which has been reported earlier,<sup>2</sup> are

 $n' = n_{2,3} + i x_{2,3},$ 

where

$$\begin{split} n_{2,3}^{2} &= \{-A_{0} \pm (A_{0}^{2} - 4\beta_{e}^{2}B_{0}A_{1})^{1/2}\} / 2\beta_{e}^{2}A_{1} \ge 1, \\ \times_{2,3} &= \sigma_{m}^{e}\sin^{2}\theta \left(1 - u_{e}\right)n_{2,3}^{3}\left(2B_{0} + A_{0}n_{2,3}^{2}\right)^{-1}, \\ B_{0} &= (2 - v_{e})u_{e} - 2\left(1 - v_{e}\right)^{2} - u_{e}v_{e}\cos^{2}\theta, \\ A_{1} &= -v_{e}\left\{3\cos^{4}\theta \left(1 - u_{e}\right) + \cos^{2}\theta\sin^{2}\theta \left(6 - 3u_{e} + u_{e}^{2}\right)\right. \\ &\times \left(1 - u_{e}\right)^{-2} + 3\sin^{4}\theta \left(1 - 4u_{e}\right)^{-1}\right\}, \\ \sigma_{m}^{e} &= \frac{\sqrt{\pi}m^{2m-2}\sin^{2m-2}\theta\Omega_{e}^{2}}{2^{m+1/2}m!\cos\theta\omega_{H}^{e2}} \left(\beta_{e}n_{2,3}\right)^{2m-3}\exp\left(-z_{m}^{e2}\right), \\ z_{m}^{e} &= (1 - m\omega_{H}^{e}/\omega) \left(\sqrt{2}\beta_{e}n_{2,3}\cos\theta\right)^{-1}, \qquad \beta_{e} &= (T_{e}/m_{e}c^{2})^{1/2} \end{split}$$

 $T_e$  is the temperature of the electron gas and  $m_e$  is the mass of the electron. If, however,  $\omega \approx 2\omega_H^e \approx \omega_+$ ,

$$\operatorname{Re} n' \sim \operatorname{Im} n' \sim \beta_e^{-1/3} \text{ for } |1 \leftarrow 2\omega_H^e / \omega| \leqslant \beta_e^{3/4}, |A_0|^3 \leqslant \beta_e^2.$$

We now consider cases of ion cyclotron resonance. If  $\omega \approx \omega_H^i$ , the indices of refraction for the ordinary and extraordinary waves (when  $\beta_i c \ll V_A \ll c$ ) are given by the expressions

$$n_1^2 = N_+^2 = \frac{1 + \cos^2 \theta}{\cos^2 \theta} \frac{c^2 / V_A^2}{u_i - 1}, \quad n_2^2 = N_-^2 = \frac{c^2 / V_A^2}{1 + \cos^2 \theta}, \quad (2)$$

where  $c^2/V_A^2 = (\Omega_i / \omega_H^i)^2$  (the subscript i used in the quantity  $f_i$  denotes the quantity  $f_e$  with the electron mass replaced by the ion mass  $m_i$  and the temperature of electron gas replaced by the ion temperature  $T_i$ ). The expression for  $n_1$  given in (2) applies when  $|1 - \omega_H^i / \omega| \gg \beta_i N_+ \cos \theta$ ; in this case the cyclotron damping of the ordinary wave is exponentially small. When  $|1 - \omega_H^i / \omega| \ll \beta_i N_+ \cos \theta$  however, this wave is highly damped:

$$n'_{1} = n_{1} + i\varkappa_{1} = \frac{\sqrt{3} + i}{2} \left\{ \sqrt{\frac{\pi}{8}} \frac{c^{2} (1 + \cos^{2} \theta)}{V_{A}^{2} \beta_{i} \cos^{3} \theta} \right\}^{1/*}.$$
 (3)

The extraordinary wave also experiences cyclotron absorption:

where

$$z_1^i = (1 - \omega_H^i / \omega) (\sqrt{2} \beta_i N_- \cos \theta)^{-1},$$
$$w(z) = e^{-z^*} \left( 1 + \frac{2i}{\sqrt{\pi}} \int_0^z e^{t^*} dt \right).$$

The extraordinary wave is weakly damped:  $\kappa_2 \ll N_{-}$ since  $\beta_i N_{-} \ll 1$ . When  $\beta_i c \sim V_A$  propagation of both waves is impossible because of the strong damping:  $n_{1,2} \sim \kappa_{1,2} \sim 1/\beta_i$  if  $\omega \sim \omega_H^i$ .

In the case of multiple resonances

$$\omega \approx m \omega_{H}^{i}, \qquad m = 2, 3, ..., \qquad n_{1,2} = N_{\pm} + i \varkappa_{1,2},$$

where

$$N_{\pm}^{2} = \{\epsilon_{11} (1 + \cos^{2} \theta) \mp [\epsilon_{11}^{2} (1 + \cos^{2} \theta)^{2} - 4 \cos^{2} \theta (\epsilon_{11}^{2} + \epsilon_{12}^{2})]^{1/2} \} / 2 \cos^{2} \theta,$$
(5)

$$\kappa_{1,2} = \sigma_m^i N_{\pm} \{ (1 + \cos^2 \theta) N_{\pm}^2 - 2 \varepsilon_{11} - 2 i \varepsilon_{12} \} \{ 2 \cos^2 \theta N_{\pm}^4 - \varepsilon_{11} (1 + \cos^2 \theta) N_{\pm}^2 \}^{-1},$$
(6)

$$\sigma_{m}^{i} = \frac{\sqrt{\pi}m^{2m-2}\sin^{2m-2}\theta c^{2}}{2^{m+3/2}m!\cos\theta V_{A}^{2}} (\beta_{i}N_{\pm})^{2m-3}\exp(-z_{m}^{i2}),$$

$$z_{m}^{i} = (1 - m\omega_{H}^{i}/\omega) (\sqrt{2}\beta_{i}N_{\pm}\cos\theta)^{-1}, \quad \varepsilon_{11} = 1 - v_{i}/(1 - u_{i}),$$

$$\varepsilon_{12} = -iv_{i}/\sqrt{u_{i}}(1 - u_{i}).$$

If 
$$|z_m^i| \leq 1$$
, then  $\kappa_{1,2}/N_{\pm} \sim (\beta_i N_{\pm})^{2m-3}$ .  
Waves characterized by frequencies  $\omega \sim \omega_H^i$ 

( $\omega$  not necessarily close to  $m\omega_H^i$ ) are also damped as a consequence of absorption in the electron gas (Landau damping). The refractive indices for these waves are given by Eq. (5) and the damping coefficients are

$$\begin{aligned} & \kappa_{2,3} / N_{\pm} = \operatorname{Im} \left\{ \frac{1}{\varepsilon_{33}} \left[ \varepsilon_{11} \sin^2 \theta N_{\pm}^4 + (2\varepsilon_{12}\varepsilon_{23} \cos \theta \sin \theta - \varepsilon_{23}^2 \cos^2 \theta \right. \\ & \left. - \left( \varepsilon_{11}^2 + \varepsilon_{12}^2 \right) \sin^2 \theta \right) N_{\pm}^2 + \varepsilon_{11} \varepsilon_{23}^2 \right] + \left. \dot{\varepsilon_{22}} \left( \varepsilon_{11} - \cos^2 \theta N_{\pm}^2 \right) \right\} \\ & \times \left\{ 2\varepsilon_{11} \left( 1 + \cos^2 \theta \right) N_{\pm}^2 - 4\cos^2 \theta N_{\pm}^4 \right\}^{-1}, \end{aligned}$$
(7)

where

$$\begin{split} \varepsilon_{23} &= -i \tan \theta v_i \left( 1 + i \sqrt{\pi} z_0^e w \left( z_0^e \right) \right) / \sqrt{u_i}, \\ \varepsilon_{22}^{'} &= i 2 \sqrt{\pi} \left( m_e / m_i \right) \sin^2 \theta v_i z_0^e w \left( z_0^e \right) \beta_e^2 N_{\pm}^2 / u_i, \\ \varepsilon_{33} &= \left( 2 m_i / m_e \right) v_i \left( z_0^e \right)^2 \left( 1 + i \sqrt{\pi} z_0^e w \left( z_0^e \right) \right), \\ z_0^e &= \left( \sqrt{2} \beta_e N_{\pm} \cos \theta \right)^{-1}. \end{split}$$

The damping (7) is small;  $\kappa_{2,3} \ll N_{\pm}$ . Even if  $|z_0^e| \lesssim 1$ , i.e.,  $V_A \sim \beta_e c$ , we find  $\kappa_{2,3}/N_{\pm} \sim m_e/m_i$ .

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## CURVES FOR THE PHOTOPROTON YIELD FROM THE $C^{12}$ NUCLEUS

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A previously described<sup>1</sup> scintillation telescope was used to investigate the dependence of the photo-