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EFFECT OF CONDUCTIVITY ANISOTROPY IN A MAGNETIC FIELD ON THE STRUCTURE OF A SHOCK WAVE IN MAGNETOGASDYNAMICS

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Submitted to JETP editor June 27, 1959

J. Exptl. Theoret. Phys. (U.S.S.R.) **38**, 252-253 (January, 1960)

A systematic study of shock waves in a magnetized plasma must also take account of its anisotropic properties. Such a study is very difficult because of the cumbersome character of the initial equations, and can be carried out only by numerical integration. However, one can obtain a quantitative estimate of the thickness of the shock front in the plasma in a quite elementary way, and study the qualitative peculiarities of the effect of conduction anisotropy on the structure of shock waves.

We shall begin with the induction equation for an anisotropic plasma, which for the stationary case can be written in the form[1]

$$-\mathrm{curl}\,[\mathbf{u}\times\mathbf{H}] = \frac{c^2}{4\pi\sigma}\left\{\nabla^2\mathbf{H} - \frac{\omega\tau}{H}\,\mathrm{curl}\,[\mathrm{curl}\,\mathbf{H}\times\mathbf{H}]\right\} \quad (1)$$

Here $\sigma = \tau ne^2/m$ is the electrical conductivity, $\omega = eH/mc$ is the Larmor frequency for electrons, and τ is the time of free flight; the remaining notation is standard. We assume that the plane of the front coincides with the coordinate plane $x = 0$. In passage through the front of an oblique shock, the conditions of conservation of mass flow $j = \rho u_x = \mathrm{const}$, of momentum flux (we need only the tangential components $u_{y,z} = H_x H_{y,z}/4\pi j$), and also $H_x = \mathrm{const}$. Taking it into account that all the parameters in our case change only with x, Eq. (1), written in components with account of the conservation conditions, can be integrated once. We get

$$H_z(u_x - H_x^2/4\pi j) = \frac{c^2}{4\pi\sigma}(H_z' - \omega\tau H_x H_y'/H) + \mathrm{const},$$
$$H_y(u_x - H_x^2/4\pi j) = \frac{c^2}{4\pi\sigma}(H_y' + \omega\tau H_x H_z'/H) + \mathrm{const}, \quad (2)$$

and the x component of Eq. (1) vanishing identically. If the dependence of u_x and $\omega\tau H_x/H$ on the coordinates is not taken into account, integration of (2) gives

$$H_y \pm iH_z \sim \exp\left\{x\,\frac{4\pi\sigma}{c^2}\left(u_x - \frac{H_x^2}{4\pi j}\right)\frac{1 \pm i\omega\tau H_x/H}{1 + (\omega\tau H_x/H)^2}\right\}. \quad (3)$$

In the perpendicular wave $H_x = 0$, (2) reduces to equations already considered (see, for example, reference 2). The thickness of the front here is of the order $\Delta x = c^2/4\pi\sigma u_x$. Thus the conduction anisotropy has no effect on the structure of the perpendicular shock wave.

In an oblique gasmagnetic shock wave, $H_x \neq 0$. It then follows from (3) that the vector $\{0, H_y, H_z\}$ inside the front rotates about the x axis with period

$$p \approx \frac{2\pi c^2}{4\pi\sigma}\,\frac{1 + (\omega\tau H_x/H)^2}{(u_x - H_x^2/4\pi j)\,\omega\tau H_x/H} \simeq \frac{c^2}{2\sigma}\,\frac{\omega\tau\cos\varphi}{u_x - H_x^2/4\pi j}.$$

This effect can be considered as a generalization to nonlinear motion of the well known rotation of the plane of polarization of waves in an isotropic medium. An appreciable change in the absolute value takes place at distances of the order

$$\Delta x \approx \frac{c^2}{4\pi\sigma}\,\frac{1 + (\omega\tau H_x/H)^2}{u_x - H_x^2/4\pi j} \approx \frac{c^2}{4\pi\sigma}\,\frac{(\omega\tau\cos\varphi)^2}{u_x - H_x^2/4\pi j},$$

which must be considered in the thickness of an oblique shock front in an anisotropic magnetized plasma. Here φ is the angle of inclination of the front to the magnetic field. Thus the conduction anisotropy leads to a very great increase in the front thickness of oblique gasdynamic shocks. It should be noted that the front thickness of oblique gasmagnetic shock waves is proportional to the square of the magnetic field intensity.

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