ELECTROMAGNETIC TRANSITION PROBABILITIES AND STATIC MOMENTS OF ODD-ODD ATOMIC NUCLEI

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Formulas are given relating the probabilities of gamma transitions with the static moments of odd-odd and neighboring odd spherical nuclei, assuming multiplet level structure. A rule for the relative intensities of transitions to levels of the same multiplet is given, which is analogous to the rule for relative intensities of transitions to levels of the same rotational band in deformed nuclei. This rule facilitates the determination of the spins and state configurations of odd-odd nuclei. Examples are discussed. The validity of the assumptions made is confirmed by the satisfactory agreement between the experimental and theoretical values of the magnetic dipole moments for a large group of odd-odd nuclei.

1. INTRODUCTION

THE collective model explains in a satisfactory way not only the static, but also the dynamic properties of atomic nuclei. The best quantitative agreement between the experimental and theoretical values of the γ transition probabilities is reached in the transitions associated with the rotational levels of deformed nuclei.[1] This is due to the fact that in the collective model the internal wave functions (describing the internal motion of the nucleons in the nucleus) of states of the same rotational band are identical. Therefore the relative intensities of transitions with the same multipole order to levels of the same rotational band do not depend on the exact form of the wave functions, and the transition probabilities within a given rotational band are determined directly by the static moments μ_1 and q_2.[2]

We shall show that one can obtain analogous relative transition probabilities for spherical nuclei with odd Z and N, since it is possible to separate out those states in the level scheme of these nuclei whose wave functions have in first approximation identical radial dependence and differ only in their angular parts.

The properties of odd-odd nuclei are mainly determined by the quantum states of the last unpaired proton, j_p^π, and neutron, j_n^π, which move in the spherical field of the even-even core. According to the independent particle model, the states (j_p^π, j_n^π) will be degenerate with respect to the total angular momentum $\mathbf{I} = \mathbf{j}_p + \mathbf{j}_n$. The remaining interaction between the unpaired nucleons, i.e., the interaction which does not enter in the self-consistent potential, removes the degeneracy by splitting each such state into $2j_i + 1$ levels $(j_i = \min\{j_p, j_n\})$, corresponding to all possible values of I. A system of levels having the same one-particle configuration (j_p^π, j_n^π) and the same parity $\pi = \pi_p\pi_n$, and differing only by the total spin I, is called a multiplet. The remaining interaction, however, leads to some configuration mixing. Each state $(j_p^\pi, j_n^\pi)\, I^\pi$, described by the wave function

$$\Phi(j_p j_n IM) = \sum_{m_p m_n} C^{IM}_{j_p m_p j_n m_n} \Psi_{j_p m_p}(\mathbf{r}_p) \Psi_{j_n m_n}(\mathbf{r}_n) \quad (1)$$

($\Psi_{j_p m_p}$ and $\Psi_{j_n m_n}$ are the wave functions of the unpaired proton and neutron, $C^{IM}_{j_p m_p j_n m_n}$ is a Clebsch-Gordan coefficient) has an admixture of states belonging to other multiplets but possessing the same total angular momentum and parity I^π:

$$\Psi(IM) = \alpha_0 \Phi_0 (j_p j_n IM) + \sum_i \alpha_i \Phi_i (j_p' j_n' IM). \quad (2)$$

The mixing of multiplets in odd-odd nuclei is similar to the mixing of rotational bands in deformed nuclei.[3]

We shall consider only those cases in which the residual interaction between p and n is small in comparison with the single-particle energies given by the self-consistent field of the nucleus. The admixture of other configurations is therefore small, $\alpha_i \ll \alpha_0$, and the levels of the odd-odd nuclei can be characterized by the quantum numbers j_p^π and j_n^π. In this classification of states we can divide the γ transitions in odd-odd nuclei into two groups: 1) one- and two-particle transitions between levels belonging to different multiplets, i.e., transitions in which the state of one or both unpaired nucleons is changed, and 2) transitions between levels of the same multiplet, i.e., transitions in which the states of the two unpaired nucleons remain unchanged.

Below we shall discuss transitions of both groups. The mixing of multiplets must be taken into account in those cases where the ground state configuration is such that the γ transition is forbidden or strongly retarded. The probability of allowed transitions is not appreciably affected by the mixing.

2. TRANSITIONS BETWEEN LEVELS OF DIFFERENT MULTIPLETS

Among the transitions between levels belonging to different multiplets, the single-particle transitions play the most important role. The two-particle transitions are strongly retarded in comparison with the single-particle transitions. In most cases the corresponding states, therefore, do not decay directly, but in cascades via several single-particle transitions.

The reduced probability B of a single-particle transition of the type EL or ML in an odd-odd nucleus, $(j_1, j)\, I_1^\pi \rightarrow (j_2, j)\, I_2^\pi$, can be expressed in terms of the reduced probability of the corresponding single-proton or single-neutron transition $j_1 \rightarrow j_2$ in the neighboring odd nucleus. It is easily shown with the help of the Racah algebra that for pure configurations

$$B(\sigma L, I_1 \rightarrow I_2) = (2I_2+1)(2j_1+1) \times W^2(j_1 j_2 I_1 I_2; Lj)\, B(\sigma L, j_1 \rightarrow j_2), \tag{3}$$

where the W(abcd; ef) are Racah coefficients, whose numerical values are given in reference 4. It is seen from the expression (3) that the ratio of the reduced probabilities of single-particle transitions of identical multipole order within the same multiplet, $I_1 \rightarrow I_2$ and $I_1 \rightarrow I_3$, does not depend on the precise form of the wave functions, but is determined only by the angular momenta:

$$\frac{B(\sigma L, I_1 \rightarrow I_2)}{B(\sigma L, I_1 \rightarrow I_3)} = \frac{(2I_2+1)\, W^2(j_1 j_2 I_1 I_2; Lj)}{(2I_3+1)\, W^2(j_1 j_2 I_1 I_3; Lj)}. \tag{4}$$

Relation (4) is the analog of Alaga's rule for deformed nuclei, which determines the relative intensities of transitions to levels of the same rotational band.[2]

Transitions between levels of different rotational bands in deformed nuclei may be forbidden on account of the quantum number K (the projection of the total angular momentum on the axis of symmetry of the nucleus); in analogy to this, we find that in odd-odd spherical nuclei a considerable number of transitions between levels belonging to different multiplets must be retarded as a consequence of j — forbiddenness:

$$|j_2 - j_1| \leqslant L. \tag{5}$$

Formulas (3) and (4) determine the intensity of the γ transitions and facilitate the identification of the states of odd-odd nuclei. These relations were derived under the assumption of pure configurations. The mixing of multiplets leads to a considerable enhancement of the weak components of the γ spectrum, but has almost no effect on the intensive components. Appreciable deviations from the rule (4) may be caused by direct mixing of the initial and final states of the nucleus for I_1^π and I_2^π.

It should be noted that relations (3) and (4) permit us to estimate the probabilities not only of the allowed transitions, but also of transitions which are forbidden by the principal quantum number n or by the orbital angular momentum l, whereas the direct calculation using the single-particle model is impossible in these cases.

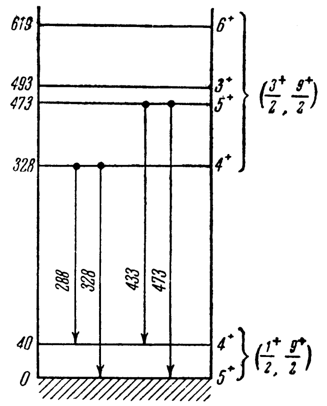

FIG. 1. Level scheme of $_{81}Tl^{208}_{127}$.

For example, in the nucleus $_{81}Tl^{208}$ (reference 5) the γ transitions of the type M1 with energies 288 and 328 kev and the transitions with energies 433 and 473 kev are forbidden by the quantum number l (Fig. 1), since the ground state and the first excited state at 40 kev belong to the configuration $(\frac{1}{2}^+, \frac{9}{2}^+)$ and the second (328 kev), third (473 kev), fourth (493 kev), and fifth (619 kev) excited states belong to the multiplet $(\frac{3}{2}^+, \frac{9}{2}^+)$. For the above-mentioned configurations we obtained the following theoretical ratios of the reduced transition probabilities of the type M1:

$$B(M1, 4 \rightarrow 4)/B(M1, 4 \rightarrow 5) = 2.75,$$
$$B(M1, 5 \rightarrow 4)/B(M1, 5 \rightarrow 5) = 0.67.$$

The experimental values for the ratios of these transitions are

$$B(288\text{ kev})/B(328\text{ kev}) = 3.30,$$
$$B(433\text{ kev})/B(473\text{ kev}) = 0.34.$$

The discrepancy between these figures is due to two factors: firstly, to the presence of an admixture of E2 to the basic radiation of the type M1, and, secondly, to the mixing of states with the same spin and parity. In the case of the first ratio these factors act in opposite directions and compensate each other partially, so that the experimental ratio differs from the theoretical one only by 20%. In the second case both factors lead to a decrease in the ratio, so that the experimental value is about one half of the theoretical value. The comparison of the experimental and theoretical ratios of the intensity of the γ transitions including the effects of the configuration mixing leads to the assignment of the spin 5^+ to the level 328 kev and the spin 4^+ to the level 473 kev.

The rule of relative intensities (4) may be very useful in the analysis of the γ spectra of nuclear reactions with single-particle character, such as the radiative capture processes (n, γ) and (p, γ), and also of the stripping reactions (d, p) and (d, n), since it is easy to determine the configurations of the states of the odd-odd nuclei in these processes.

For example, the capturing state in the capture of a thermal neutron by an odd-proton nucleus has the configuration $(j_p^\pi, \frac{1}{2}^+)\, I_1^\pi$, where j_p^π characterizes the ground state of the target nucleus; the total angular momentum I_1 can have only the two values $j_p \pm \frac{1}{2}$. All intensive γ transitions are single-particle transitions. They are caused by the direct or successive transitions of the neutron captured by the nucleus into the lowest unfilled state. If the configuration and spin of the ground state only of the odd-odd nucleus are known, one can determine which of the two possible values of the spin characterize the capturing state by comparing the intensities of the γ transitions to the ground state and to the neighboring states within the interval of 1 Mev. In many cases it is also possible to identify other intensive transitions. The comparison with the spectrum of the radiative capture of thermal neutrons by the neighboring even-even nucleus is here of great help. In an odd-neutron nucleus the spin and parity of the capturing state are $\frac{1}{2}^+$, so that intensive γ transitions of the type E1 and M1 are possible only to the levels $\frac{1}{2}^\mp$ and $\frac{3}{2}^\mp$. In the neighboring odd-odd nucleus each of these levels is split up (see Fig. 2). To each intensive component in the spectrum of the odd-neutron nucleus there will thus correspond two or three components of the same multipole order in the spectrum of the odd-odd nucleus. The ratios of the intensities of these components can be calculated theoretically for both possible values of the spin of the odd-odd nucleus in the capturing state $I_1 = j_p \pm \frac{1}{2}$ and all values of j_p. These ratios are listed in the table.

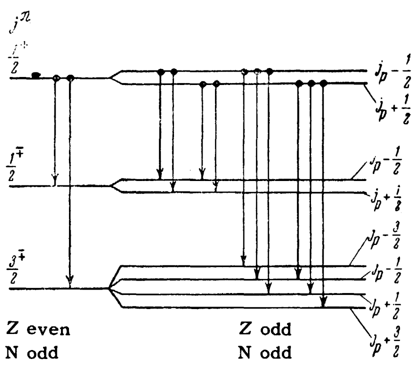

FIG. 2. Scheme of transitions of the type E1(M1) accompanying the capture of thermal neutrons. To each transition in the odd-neutron nucleus correspond several transitions in the neighboring odd-odd nucleus.

Ratios of the reduced probabilities of transitions of the type E1 or M1 to levels of the same multiplet of an odd-odd nucleus in the capture of thermal neutrons*

Initial state	$I_1 = j_p - 1/2$						$I_1 = j_p + 1/2$					
	$j_n = 1/2$		$j_n = 3/2$				$j_n = 1/2$		$j_n = 3/2$			
Final state	$j_p - 1/2$	$j_p + 1/2$	$j_p - 3/2$	$j_p - 1/2$	$j_p + 1/2$	$j_p + 3/2$	$j_p - 1/2$	$j_p + 1/2$	$j_p - 3/2$	$j_p - 1/2$	$j_p + 1/2$	$j_p + 3/2$
$j_p = 1/2$	0	1	—	—	1	0	0.50	1	—	—	0.20	1
$j_p = 3/2$	0.20	1	0.40	1	1	0	1	1	0	0.07	0.36	1
$j_p = 5/2$	0.29	1	0.77	1	0.80	0	1	0.80	0	0.12	0.43	1
$j_p = 7/2$	0.33	1	0.95	1	0.71	0	1	0.71	0	0.16	0.48	1
$j_p = 9/2$	0.36	1	1	0.94	0.63	0	1	0.67	0	0.19	0.51	1
$j_p = 11/2$	0.39	1	1	0.88	0.56	0	1	0.64	0	0.20	0.53	1

*For large energy differences in heavy nuclei the E2 transitions may be more intensive than the transitions of type M1.

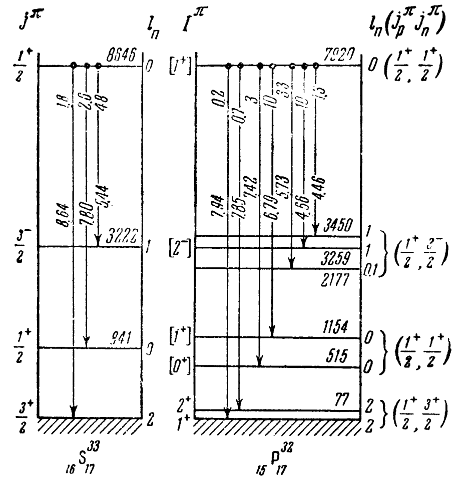

FIG. 3. Scheme of the principal transitions in the reactions $S^{32}(n_{therm}, \gamma)S^{33}$ and $P^{31}(n_{therm}, \gamma)P^{32}$. The values of the spins determined by the relative intensities of the transitions are quoted in square brackets.

Let us consider the spectrum of the γ rays emitted in the reaction $P^{31}(n, \gamma)P^{32}$ and let us compare it with the spectrum of the γ rays in the reaction $S^{32}(n, \gamma)S^{33}$ (reference 6). The nuclei P^{32} and S^{33} have the same number of neutrons. To each intensive transition in the S^{33} nucleus there correspond several transitions in the P^{32} nucleus (Fig. 3). The capturing state of P^{32} has spin and parity 0^+ or 1^+, since for a thermal neutron $j_n^\pi = \frac{1}{2}^+$ and the ground state of P^{31} is characterized by $j_p^\pi = \frac{1}{2}^+$. It follows from the level scheme of P^{31} and S^{33}, that the configuration of the ground state and the first excited state at 77 kev of the P^{32} nucleus is $(\frac{1}{2}^+, \frac{3}{2}^+)$. This is confirmed by the fact that for both states the orbital angular momentum of the neutron is $l_n = 2$, and the spins and parities are 1^+ and 2^+. To the transition 8.64 Mev in the S^{33} nucleus there correspond the transitions 7.94 and 7.85 Mev in the P^{32} nucleus. The experimental ratio of the reduced probabilities of these transitions is $B(7.94) : B(7.85) = 0.30 : 1.00$. It follows from the table that this ratio should be equal to $0.20 : 1.00$ if $I_1 = I^+$ and to $1.00 : 0.00$ if $I_1 = 0^+$ (only transitions of the type M1 are considered). The comparison of the above-mentioned ratios indicates that $I_1 = 1^+$. The slight enhancement of the 7.94 Mev transition is due to the mixing of the initial and final states, since $I_1^\pi = I_2^\pi = 1^+$.

The 7.80-Mev transition in the S^{33} nucleus corresponds to the 7.42 and 6.79 Mev γ transitions to the multiplet $(\frac{1}{2}^+, \frac{1}{2}^+)$ in the P^{32} nucleus (see Fig. 3). This configuration is confirmed by the fact that for both levels the orbital angular momentum of the neutron is $l_n = 0$. The experimental ratio of the reduced probabilities of these transitions is $B(7.42) : B(6.79) = 0.23 : 1.00$. It is seen from the table that $B(1^+ \to 0^+) : B(1^+ \to 1^+) = 0.50 : 1.00$, i.e., the transition to the level $(\frac{1}{2}^+, \frac{1}{2}^+)\,1^+$ should be more intensive than the transition to the level $(\frac{1}{2}^+, \frac{1}{2}^+)\,0^+$. The configuration mixing further enhances the transition $1^+ \to 1^+$. The comparison of the theoretical ratio with the experimental one leads to the assignment of the spin and parity 1^+ to the level 1154 kev of the P^{32} nucleus, and of the spin and parity 0^+ to the 515 kev level.

To the transition of the type E1 with the energy 5.44 Mev in the S^{33} nucleus there corresponds the transition 4.66 Mev or one of the transitions 5.73 or 4.46 Mev in the P^{32} nucleus. It is seen from the table that the most intensive transition for the multiplet $(\frac{1}{2}^+, \frac{3}{2}^-)$ should be to the level $I_2^\pi = 2^-$. This permits us to assign the spin 2^- to the level 3259 kev of the P^{32} nucleus.

It is evident from the examples of Tl^{208} and P^{32} how the rule of relative intensities (4) allows us to assign spins and parities to the levels of odd-odd nuclei. This relation can be used in an analogous way for the analysis of the γ spectra of the processes (p, γ), (d, p), (d, n), etc. The application of this rule to β decay is less justified, since in the β decay of even-even nuclei to levels of odd-odd nuclei one must take into account the effects of the pair interaction.

3. TRANSITIONS BETWEEN LEVELS OF THE SAME MULTIPLET

Just like the levels of the same rotational band in deformed nuclei, the levels of the same multiplet in odd-odd nuclei have identical parity. Only transitions of the type M1, E2, M3, E4, etc. are possible between such levels. In the case of the rotational band the regular, monotonic sequence of spins has as a consequence that the transitions M3 and of higher multipolarity are suppressed by the transitions of the type M1 and E2. In odd-odd nuclei the sequence of spins is not that simple. It is determined by the spin-dependent part of the residual potential and is different for different quantum states of the proton and the neutron.[7] In a number of cases one therefore observes transitions of the type M3, E4, and even M5 between levels of the same multiplet.

The reduced probabilities B of γ transitions $I_1 \to I_2$ of the type EL or ML between levels of the same multiplet (j_p^π, j_n^π) can be expressed in terms of the static electric or magnetic moments of the same multipolarity m_L^p, m_L^n of the neighboring odd-proton or odd-neutron nuclei. It can be shown with the help of the Racah algebra that, under the assumption of pure configurations,

$$B(\sigma L, I_1 \to I_2) = |am_L^p + bm_L^n|^2. \quad (6)$$

The coefficients a and b have the following form

$$a = \nu_L \sqrt{\frac{(2L+1)(2I_2+1)(2j_p+1)(j_p+1)}{4\pi j_p}}\, W(j_p j_p I_1 I_2; L j_n) \quad (7a)$$

$$b = \nu_L \sqrt{\frac{(2L+1)(2I_2+1)(2j_n+1)(j_n+1)}{4\pi j_n}}\, W(j_n j_n I_1 I_2; L j_p), \quad (7b)$$

where $\nu_1 = 1$, $\nu_2 = 1/2$, $\nu_3 = -1$, and $\nu_4 = 1$ are chosen such that the definitions of m_L^p and m_L^n agree with the generally accepted definitions of the static magnetic dipole (μ_1), electric quadrupole (q_2), magnetic octupole (μ_3), and electric hexadecapole (q_4) moments of the atomic nuclei.[8]

The static electric and magnetic moments M_L of odd-odd nuclei can also be expressed in terms of the m_L^p and m_L^n of the neighboring odd nuclei:

$$M_L(I) = a' m_L^p + b' m_L^n, \quad (8)$$

where

$$a' = (-1)^{j_p - j_n + I + L} \sqrt{\frac{I(2I+1)(2j_p+1)(j_p+1)}{(I+1)j_p}} \times W(j_p j_p II; L j_n), \quad (9a)$$

$$b' = (-1)^{j_n - j_p + I + L} \sqrt{\frac{I(2I+1)(2j_n+1)(j_n+1)}{(I+1)j_n}} \times W(j_n j_n II; L j_p). \quad (9b)$$

Knowing the moments μ_1 and q_2 of the ground states of the odd nuclei, we can thus calculate the probabilities of the transitions M1 and E2 between the levels of the first multiplet of the neighboring odd-odd nucleus (see, for example, reference 9) and its static moments M_1 and Q_2 not only for the ground state, but also for the excited states belonging to this multiplet. On the other hand, knowing two experimental values of $B(\sigma L)$ or M_L which characterize one multiplet of the odd-odd nucleus, one could determine the moments m_L^p and m_L^n, and thus not only μ_1 and q_2, but also μ_3 and q_4. However, in the majority of cases, only one value of $B(\sigma L)$ is known, and the solution of the problem is not unique. We can, therefore, estimate the static moments by this method only in those cases when $a \gg b$ (or $b \gg a$) and the contribution of m_n (or m_p) can be neglected, or when $j_p = j_n$, so that $a = b$ and $B(\sigma L)$ is given directly by the static moment $M_L(I)$ of the same odd-odd nucleus. For $j_p = j_n$ we have

$$B(\sigma L, I_1 \to I_2) = \frac{(2L+1)(I+1)(2I_2+1)\nu_L^2}{4\pi I(2I+1)} \frac{W^2(jjI_1I_2; Lj)}{W^2(jjII; Lj)} |M_L(I)|^2 \quad (10)$$

For example, in the B^{10} nucleus the ground state $I^\pi = 3^+$ and the first excited state at 720 kev, $I^\pi = 1^+$, belong to the multiplet $(3/2^-, 3/2^-)$. The lifetime of the first excited state is $\tau_{1/2} = 1.05 \times 10^{-9}$ sec (reference 5), which corresponds to $B(E2) = 3.94 \times 10^{-4} e^2$ barns^2. With this information we can determine the quadrupole moment for an arbitrary level of the given multiplet. For the first excited state of B^{10} we have $Q_2(1^+) = \pm 0.047\,e$ barns, and for the ground state $Q_2(3^+) = \pm 0.070\,e$ barns. The quadrupole moment of the ground state has been measured experimentally and is equal to $Q_2 = +0.074\,e$ barns (reference 10).

In the same way we can estimate, for example, the moment Q_4 of the Sc^{44} nucleus. The ground state $I^\pi = 3^+$, and the first excited state at 270 kev, $I^\pi = 7^+$, belong to the multiplet $(7/2^-, 7/2^-)$. The lifetime of the first excited state is $\tau_{1/2} = 2.46$ days (reference 5), which corresponds to $B(E4) = 2.1 \times 10^{-5} e^2$ (barns)^4. From this we find for the ground state of Sc^{44}, $Q_4(3^+) = \pm 2.5 \times 10^{-3}$ e (barns)^2, and for the isomeric state $Q_4(7^+) = \pm 4.5 \times 10^{-2}$ e (barns)^2. These values are 100 to 1000 times smaller than the values of Q_4 determined from the intensities of the decays to levels of the rotational bands in heavy deformed nuclei.[11]

An extensive comparison of the experimental and theoretical values is possible only for the M_1, the static magnetic dipole moments of odd-odd nuclei. Formula (8) for this special case was given in the paper of Schwartz;[12] in the review article of Blin-Stoyle[13] M_1^{exp} and M_1^{theor} were compared in 17 cases. Until now it has been possible to make this comparison for 45 nuclei. Out of the 45 considered cases, 36 are in agreement with experiment with an accuracy of 25%; 4 cases give a discrepancy, but here the sensitivity to small changes in μ_1^p and μ_1^n is very great, so that agreement with experiment could be easily achieved. There is serious disagreement (5 cases) only for those nuclei for which the assumed model is manifestly wrong, that is, for deformed nuclei and for nuclei with configuration levels which cannot be explained by the single-particle scheme of Mayer.[14] The satisfactory agreement between M_1^{exp} and M_1^{theor} indicates that the hypothesis of a multiplet structure of the levels is valid for a large group of odd-odd nuclei.

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