POSSIBLE TRANSMISSION OF ELECTROMAGNETIC WAVES THROUGH A METAL IN A STRONG MAGNETIC FIELD

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It is shown that an electromagnetic wave propagating along a magnetic field can penetrate a metal plate perpendicular to the field if the Larmor frequency is higher than the frequency of the propagating wave and much higher than the collision frequency, and if the electron Larmor radius is smaller than the wavelength in the metal.

IT is well known that an electromagnetic wave characterized by a frequency ω which is smaller than the plasma frequency ω_0 cannot propagate through a plasma. Under these conditions the square of the index of refraction of the plasma is negative. If there is a magnetic field the refractive index for the wave which propagates along the field (if $|\omega_c^* \pm \omega|/\nu \gg 1$, where ν is the collision frequency) is given by:

$$N_{0\pm}^2 = 1 - \omega_0^2/\omega(\omega \pm \omega_c^*). \tag{1}$$

Here ω_c^* is the Larmor (cyclotron) frequency of the electron. The symbol $\pm$ denotes the two opposite senses of the circular polarization. It is apparent that when $\omega < \omega_c^*$ the wave that corresponds to N_{0-} can propagate through the plasma. Thus, it is reasonable to assume that a wave characterized by a frequency $\omega < \omega_c^*$, where $(\omega_c^* - \omega)/\nu \gg 1$, can propagate through a metal plate if there is a strong magnetic field perpendicular to the surface of the plate. If a plane-polarized wave strikes the plate at normal incidence (to the surface), a circularly polarized wave is transmitted through the plate; the refractive index (consequently the amplitude of the wave) depends on the magnetic field.

The expression for the refractive index given above holds for a classical electron gas in which spatial dispersion is neglected. For the electron concentrations typical of semi-metals, and even more so in metals, the refractive index for the transmitted wave can be very large, so that effects due to spatial dispersion may be quite significant.

In the present work we investigate the effect of spatial dispersion on transmission of the wave. The analysis is carried out for the simple case of a totally degenerate electron gas characterized by a quadratic isotropic dispersion relation. Collisions are taken into account in approximating the relaxation time; it is assumed that $(\omega_c^* - \omega) \gg \nu$. The analysis indicates that the spatial-dispersion correction is small if the following condition is satisfied:

$$\gamma = (r_0/\lambda)^2 (1 - \omega/\omega_c^*)^{-2} < 1/2.$$

Here r_0 is the Larmor radius of an electron at the Fermi surface and λ is the wavelength in the medium. When $\gamma > 2/3$ there is strong damping due to spatial dispersion and the usual expression for N_- is incorrect. In this case the transmitted wave is attenuated in a distance of several Larmor radii; hence the case $\gamma > 2/3$ is not of interest. When $\gamma < 1/2$ the amplitude of the transmitted wave is determined by the Fresnel formula and the boundary effects associated with the spatial dispersion are not important if the thickness of the plate is greater than several Larmor radii.

The dependence of log N_{0-} on log $(\omega_c/\omega - m^*/m)$ is shown in Fig. 1 for a vacuum wavelength of 2 cm and for different electron concentrations (n_0). In this figure m^* is the effective mass of the electron, m is the mass of the free electron* and $\omega_c = eH/mc$. The dashed line denotes $\gamma = 1/2$. The region in which transmission can take place lies below the dashed line. The values of the magnetic field are given on the lower abscissa axis. It is apparent that at concentrations corresponding to those in metals (10^{22} cm^{-3}) transmission can be observed only in extremely high (approximately 3×10^5 oe) magnetic fields; the conditions for transmission are much more favorable at lower concentrations.

The free electron mass m is introduced here only for convenience in making estimates; obviously the effect depends only on the effective mass m.

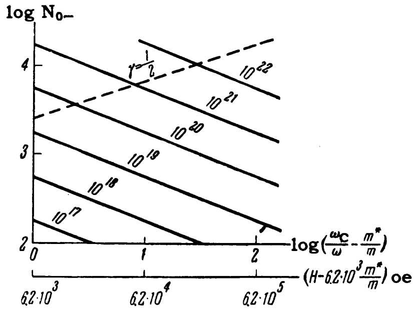

FIG. 1

The imaginary part of the refractive index is given by

$$N'_- = N_{0-}\nu / 2\,(\omega_c^* - \omega), \qquad \omega_c^* = eH / m^* c.$$

A ray transmitted at normal incidence through a plate of thickness l, which is perpendicular to a magnetic field, will be attenuated (in power) by a factor $16 N_{0-}^{-2} \exp\{-2N'_- \omega l/c\}$. If the attenuation is not to be excessive the thickness of the plate must be of order

$$l_0 = \left(\frac{\nu}{\omega_c^* - \omega}\frac{\omega}{c}N_{0-}\right)^{-1} = \lambda\,\frac{\omega_c^* - \omega}{\nu},$$

where λ is the wavelength in the medium.

Taking the values $n_0 = 10^{22}\ \mathrm{cm}^{-3}$, $H = 3 \times 10^5$ oe, and $\nu = 3 \times 10^9\ \mathrm{sec}^{-1}$, we find $l_0 = 3 \times 10^{-2}$ m/m^* cm. Thus, there are no serious difficulties from this point of view.

The expression for the transmission coefficient given above applies when $l \gtrsim l_0$. If however, $l \ll l_0$, it is necessary to take account of effects due to the interference of waves which are multiply reflected from the boundaries; in this case the expression for the transmission coefficient becomes more complicated. We shall neglect these effects. When the condition $l \gtrsim l_0$ is satisfied the transmission of the wave through each boundary can be considered separately.

We now demonstrate the validity of the statements made above. If an electromagnetic wave is normally incident on the surface of a metal in the presence of a magnetic field perpendicular to the surface, the electric field $E(z)$ associated with the wave inside the metal is given by[1,2]

$$E_-(z) = \frac{E'_-(0)}{\pi}\int_{-\infty}^{\infty}\frac{e^{ikz}dk}{k^2 - \omega^2 c^{-2}\varepsilon_-(k, \omega)}. \qquad (2)$$

Here, for simplicity it is assumed that the reflection of electrons from the surface of the metal is specular. The z axis is parallel to the magnetic field inside the metal.

$$E_-(z) = E_x(z) - iE_y(z), \quad \varepsilon_- = \varepsilon_{xx} - i\varepsilon_{xy},$$

and $E'_-(0)$ is the field derivative at the boundary of the metal

$$\varepsilon_-(k, \omega) = 1 + (\varepsilon_0 - 1)F(q),$$

$$F(q) = \frac{3}{4}\left[\frac{2}{q^2} + \frac{1}{q}\left(\frac{1}{q^2} - 1\right)\ln\frac{1-q}{1+q}\right];$$

$$q = kv_F / \Omega, \qquad \Omega = \omega_c^* - \omega - i\nu, \quad \varepsilon_0 = 1 + \omega_0^2/\omega\Omega, \quad (3)$$

where v_F is the electron velocity at the Fermi surface. Using the new variable of integration in (2), we have

$$E_-(z) = \frac{E'_-(0)}{\pi}\int_C\frac{\exp(iq\Omega z / v_F)}{q^2 - \gamma F(q)}\,dq\,\frac{v_F}{\Omega}. \qquad (4)$$

In this treatment the displacement current is neglected. The parameter $\gamma = \epsilon_0(\omega/c)^2(v_F/\Omega)^2$; when $\omega_c^* \gg \omega$, $\gamma = (r_0/\lambda)^2$, where r_0 is the electron Larmor radius and λ is the wavelength in the medium.

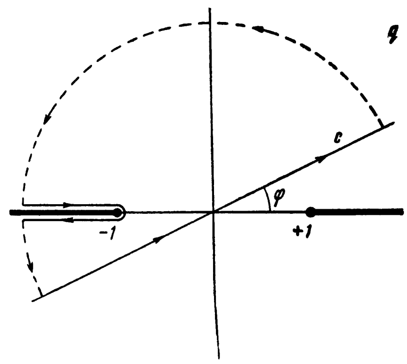

FIG. 2. Path of integration.

The contour C passes through the origin and is inclined to the real axis at an angle $\varphi = \tan^{-1}[\nu/(\omega_c^* - \omega)]$. The function $F(q)$ has branching points at $q = \pm 1$. Hence we take cuts along the real axis from $-\infty$ to -1 and from 1 to ∞. It is evident from Fig. 2 that the integral of interest to us can be reduced to the sum of the residues at the poles inside the closed contour which has been indicated and the integral which encloses the left cut. A calculation shows that $|F(q)| < 2$ on the real axis for $q \geq 1$. Hence, when $|\gamma| < 1/2$, in accordance with the Rouché theorem[3], the equation $q^2 - \gamma F(q) = 0$ has two roots in the entire complex plane taken with the cuts. When $|\gamma| < 1/2$ these roots can be found by expanding $F(q)$ in powers of q. These roots are $q_{1,2} = \pm\sqrt{\gamma}$. The root $q_1 = +\sqrt{\gamma}$ lies inside the closed contour. Thus

$$E_-(z) = N^{-1}H_-(0)\exp(iN\omega z / c) + A(z), \qquad (5)$$

where $N = (\omega_0^2/\omega\Omega)^{1/2}$ is the refractive index, $H_-(0) = (ic/\omega)E'_-(0)$ is the magnetic field at the boundary and A(z) is the integral over the contour which goes around the left cut.

When $|\gamma| < 1/2$

$$A(z) \approx \frac{1}{N} H_-(0) \gamma^{3/2} \frac{3}{2} \int_1^\infty \exp\left(iq \frac{\Omega}{v_F} z\right) \frac{1}{q^5} (1 - q^2)\, dq.$$

When $z \gg v_F/(\omega_c^* - \omega)$

$$A(z) = \frac{3}{N} H_-(0) \gamma^{3/2} \left(\frac{v_F}{z\Omega}\right)^2 \exp\left(i \frac{\Omega}{v_F} z\right) + O\left[\left(\frac{v_F}{z\Omega}\right)^3\right].$$

Thus, the term A(z), which describes the effect of the boundary, is small when $z \gg v_F/|\Omega|$, i.e., at distances that are much larger than the Larmor radius of the electron (for $\omega_c^* \gg \omega$). The transmitted wave is attenuated by a factor of e in a distance $2v_F/\nu\sqrt{\gamma}$. We may note that the amplitude of the transmitted wave is the same as that given by the Fresnel formulas, while the refractive index is determined by the root of the dispersion equation $k^2 - \omega^2 c^{-2} \epsilon_-(k, \omega) = 0$, as though the medium were homogeneous.

A numerical solution of this equation for $|\gamma| < 2/3$ shows that there are two roots which are in very good agreement with the values of $q_{1,2}$ used above. When $|\gamma| > 2/3$ there are no real roots; the attenuation is large and the wave is not transmitted through the metal.

The above analysis indicates that when the Larmor radius is less than a half wavelength in the medium the effect of spatial dispersion is to distort the field only in the boundary layer, which is several Larmor radii in thickness. Hence, one can neglect spatial dispersion in estimating the effect of deviations of the magnetic field (from the normal to the surface) on transmission. If the wave propagates at an angle φ to the magnetic field (normal to the surface), the refractive index for the transmitted wave is given by the expression $N_- = \omega_0^2/\omega_c^*\omega \cos\varphi$ where $\omega_0 \gg \omega_c \gg \omega$ and $\varphi \ll \pi/2$.

Thus, the accuracy of orientation of the magnetic field is not of great importance.

We may note that an expression for the refractive index which takes account of the anisotropy of the electron mass has been obtained by Gurevich and Ipatova.[4]

The expression for $\epsilon_-(k, \omega)$ in (3) applies when $\hbar\omega_c^* \ll m^* v_F^2/2$. However, the quantum-mechanical expression for $\epsilon_-(k, \omega)$ obtained by the method which has been used by us earlier[5] leads to the expression in (1) for the refractive index for all cases which can be realized in practice, even when $\hbar\omega_c^* \gg m^* v_F^2/2$.

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