[3]A. I. Nikishov, JETP 36, 1323 (1959), Soviet Phys. JETP 9, 937 (1959).

Translated by W. H. Furry
306

ON THE INFLUENCE OF THE EXCHANGE INTERACTION ON THE TRANSITION TEMPERATURE OF SUPERCONDUCTORS

S. V. VONSOVSKIĬ and M. S. SVIRSKIĬ

Institute for Metal Physics, Academy of Sciences, U.S.S.R.

Submitted to JETP editor July 30, 1959

J. Exptl. Theoret. Phys. (U.S.S.R.) 37, 1494-1496 (November, 1959)

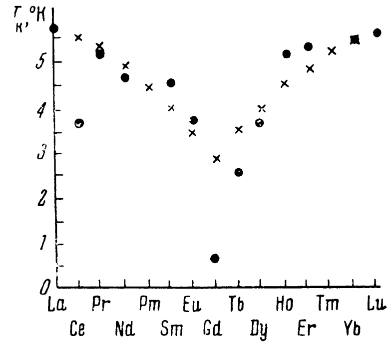
It was shown in reference 1 that the polarization of the conduction electrons, caused by the s-d exchange interaction, prevents the establishment of a superconducting state in typical ferromagnets with high Curie points (for instance, Fe, Co, and Ni). At the same time it was shown that superconductivity could in principle occur in metals of the transition groups, if the s-d exchange were sufficiently weak. This may, apparently, occur in the rare earths where the exchange interaction between the conduction electrons and the electrons of the incomplete 4f shell is, generally speaking, weaker than the interaction in the transition metals of the iron group. However, even in the rare earth metals the effective repulsion between the conduction electrons[2] induced by the s-f exchange (which counteracts the attraction caused by the longitudinal phonons) leads to a lowering of the critical temperature T_c of the transition into the superconducting state, while this lowering must depend on the magnitude of the spin S_f of the 4f shell. Such a dependence $T_c(S_f)$ has, indeed, been found recently by Matthias et al.[3] in one-percent solid solutions of rare-earth elements in lanthanum (see figure).

To ascertain whether we can explain the experimentally observed dependence $T_c(S_f)$ by an effective repulsion caused by s-f exchange, we consider the well known expression for T_c:[4]

$$T_c = 1.14\,(\hbar\omega / k)\exp\{-1 / N_0 V\}. \qquad (1)$$

The matrix element V of reference 1 has now the form

$$V = V_{ph} - V_c - V_{sf} = V_0 - V_{sf}, \qquad (2)$$



Transition temperature for superconducting one percent (atomic) solid solutions of rare earth elements in La: ● – experimental points according to ref. 3, × – theoretical values evaluated using Eq. (2) of this paper.

where V_{ph}, V_c, and V_{sf} are respectively the matrix elements of the interelectronic interaction induced by the phonons, the quasi-Coulomb interaction, and the s-f exchange interaction.

We shall, moreover, take the estimates given in reference 2 for gadolinium:

$$V_{ph} = 4.2 \cdot 10^{-12} N^{-1}\ \text{erg},$$

$$V_c = 1.1 \cdot 10^{-12} N^{-1}\ \text{erg} \text{ and } V_{sf} = 5.5 \cdot 10^{-11} N^{-1}\ \text{erg}.$$

We note now that Gd and La have the same inner electron shell structure and the same crystal lattice structure. To estimate the magnitude of V_{ph} and V_c for La we can thus take the same values as for Gd.

Moreover, since $V_{sf} \equiv 0$ in pure La with a completely empty 4f shell, for a one-percent solution of Gd in La we must substitute in Eq. (2) the quantity $0.01 \times 5.5 \times 10^{-11} N^{-1}\ \text{erg} = 5.5 \times 10^{-13} N^{-1}\ \text{erg}$ instead of V_{sf} for 100% Gd. We find then from (1) and (2)

$$T_c = T_c^{(0)} \exp\{-0.22 / N_0 V_0\}, \qquad (3)$$

where $T_c^{(0)}$ is the critical temperature when there is no s-f interaction. The value of N_0V_0 was determined by Pines[5] for La to be 0.37 at $T_c^{(0)} = 5°$K. However, if we take into account that $T_c^{(0)} = 5.7°$K was obtained for La in the experiments of Matthias et al.[3] we obtain easily by the method indicated by Pines[5] the close value $N_0V_0 \approx 0.39$. Substituting now into (3) the values $T_c^{(0)} = 5.7°$K and $N_0V_0 = 0.39$, we get $T_c \approx 2.8°$K, whereas according to the figure the value of T_c for La with 1% Gd in solution is considerably smaller, $\approx 0.6°$K. This discrepancy shows that for the case of a one-percent solution of Gd in La the total lowering of T_c can apparently not be explained solely by the occurrence of an effective repulsion induced by the s-f exchange interaction.

To estimate in how far this repulsion is responsible for the decrease of T_c in solutions of rare earths other than Gd in La, we take for the one percent solution of Gd in La the value $T_c \approx 2.8°$K found above. Taking into account that according to

Kasuya[2] $V_{sf} \sim S_f(S_f+1)$ we find then easily from (1) and (2) (taking into account that the ratio $0.01\,V_{sf}/V_0$ is small, reaching a maximum value of 0.18 for the case of Gd) the following expression

$$T_c \approx T_c^{(0)} e^{-\alpha S_f(S_f+1)}, \quad (4)$$

where $\alpha = 0.043$* for $T_c^{(0)} = 5.7°K$.

Putting in (4) $S_f = 1/2,\ 1,\ 3/2,\ 2,\ 5/2,\ 3$, and $7/2$ we get the values of T_c for one percent solutions of all rare earth elements in La which are given in the figure. It is clear that an appreciable deviation from the experimental values occurs only in the case of Gd (if we disregard the drop in the experimental point for Ce, a possible cause for which was considered by Matthias et al.[3]). This shows that for solutions of rare earths, with the exception of Gd, in La the lowering of T_c is basically caused, apparently, by the effective repulsion induced by the s-f exchange interaction.

As far as the solution of Gd is concerned, there occurs here possibly an additional lowering of T_c caused by the "undermagnetization" of the conduction s-electrons by the electrons of the inner f-shells.[1] Such an action of the inner shells of Gd (a three-percent solution of which in La already displays ferromagnetism for $T < 1°K$) is very probable also in the light of the earlier expressed point of view[3] that the electron system of superconductors contains factors favorable for ferromagnetism.

In an interesting paper by Akhiezer and Pomeranchuk[6] the problem of the character of the interaction between conduction electrons induced by ferromagnons is also considered. In contrast to Kasuya[2] these authors are led to the conclusions that the above mentioned interaction is similar to the interaction induced by the phonons and has the character of an attraction and can thus "...help when the conditions for the occurrence of superconductivity are approached..." in ferromagnetics (see reference 6, p. 861). In this connection we must draw attention to the fact that the s-d exchange interaction (which is discussed here) is connected with a change in sign of the spin component of the "s-electron" which, because of the commutation relations for the electron Fermi-operators in second quantization, makes the sign of the terms in the Hamiltonian with matrix elements of the kind V_{sf} [see Eq. (2) of the present paper and also Eq. (3) for U_{if} in reference 6] opposite from that of the corresponding "phonon" terms in the Hamiltonian. A more detailed, quantitative discussion of this problem will be given by us in another paper.

*One may assume with a sufficiently high order of accuracy that the coefficient α has the same value for all rare earth elements. See in this connection, for instance, Eqs. (6) and (9) of Kasuya's paper.[2]

[1] S. V. Vonsovskiĭ and M. S. Svirskiĭ, Dokl. Akad. Nauk SSSR **122**, 204 (1958), Soviet Phys. Doklady **3**, 949 (1959).

[2] T. Kasuya, Progr. Theoret. Phys. (Kyoto) **20**, 980 (1958).

[3] Matthias, Suhl, and Corenzwit, Phys. Rev. Letters **1**, 92 (1958).

[4] Bardeen, Cooper, and Schrieffer, Phys. Rev. **108**, 1175 (1957).

[5] D. Pines, Phys. Rev. **109**, 280 (1958).

[6] A. I. Akhiezer and I. Ya. Pomeranchuk, JETP **36**, 859 (1959), Soviet Phys. JETP **9**, 605 (1959).

Translated by D. ter Haar
307