[6] H. Wulff, Z. Physik, **150**, 614 (1958).

[7] J. Holtsmark, Ann. Phys., **58**, 577 (1919).

[8] V. I. Kogan, Физика плазмы и проблема управляемых термоядерных реакций (Plasma Physics and the Problem of a Controlled Thermonuclear Reaction), IV, Academy of Sciences Press (U.S.S.R.) (1958) p. 258.

[9] W. Lochte-Holtgreven, Probleme des Plasmas in Physik und Astronomie, Akademie-Verlag, Berlin 1958, p. 156.

[10] I. I. Sobel'man, Usp. Fiz. Nauk **54**, 551 (1954).

[11] H. Griem, Z. Physik **137**, 280 (1954).

Translated by H. Lashinsky
303

THE CROSS SECTION FOR ELECTRON-ELECTRON SCATTERING AT HIGH ENERGIES

V. N. BAĬER

Submitted to JETP editor July 17, 1959

J. Exptl. Theoret. Phys. (U.S.S.R.) **37**, 1490-1492 (November, 1959)

SINCE it is intended very soon to make experiments on the scattering of electrons by electrons at high energies,[1] it is desirable to derive the general formula for the scattering of an electron by an electron in the case in which the charges of the electrons are smeared out in an invariant way.

The calculation has been made for the case of the exchange of one photon. As is well known, in this case the vertex operator for the interaction of an invariantly smeared out electron and a photon can be written in the form

$$e\Gamma_\mu(q) = e\left[\gamma_\mu f_1(q^2) + (1/2m)(\hat{q}\gamma_\mu - \gamma_\mu\hat{q}) f_2(q^2)\right]. \qquad (1)$$

Here q is the momentum transferred, and $f_1(q^2)$ and $f_2(q^2)$ are functions describing the distributions of charge and current in the electron.

If a particle of spin ½ has a point charge e and a point anomalous magnetic moment μ, then $f_1 = 1$, $f_2 = -i\mu/2$; for small values of the magnitude of the momentum transfer q the functions f_1 and f_2 are respectively the distributions of charge and anomalous magnetic moment in the electron. If, however, the magnitude of the momentum transfer becomes larger than or of the order of the reciprocal of the length characterizing the distributions of charge and anomalous magnetic moment in the electron, this simple interpretation of the functions $f_1(q^2)$ and $f_2(q^2)$ becomes incorrect, and both functions describe both the charge distribution and the anomalous magnetic-moment distribution. Just for this reason, although the anomalous magnetic moment of the electron is of radiative origin and is very small ($\mu \sim \alpha/2\pi$), the function $f_2(q^2)$ can be very important for the description of the charge and current distributions in the electron at small distances.

After averaging and summing over the spins of the electrons in the initial and final states we get the following formula for the scattering cross section in the center-of-mass system:

$$d\sigma / d\Omega = r_0^2 X / 4\gamma^2, \qquad (2)$$

where

$$\begin{aligned} X = {} & \alpha / 4(1-\lambda)^2 + \beta / 4(1-\mu)^2 - (\delta_1 + \varepsilon_1) / 4(1-\mu) \\ & \times (1-\lambda); \end{aligned}$$

$$\begin{aligned} \alpha = {} & 2[2 - 2\lambda + \varkappa^2 + \mu^2] |f_1|^4 + 8(1-\lambda) \\ & \times [4 - 3\lambda + \lambda^2 - \mu^2 - \varkappa^2] |f_1|^2 |f_2|^2 + 4(1-\lambda)^2 \\ & \times [7 - 2\lambda - \lambda^2 + 2(\varkappa^2 + \mu^2)] |f_2|^4 - 16(1-\lambda) \operatorname{Im}(f_1 f_2^*) \\ & \times [|f_1|^2(\lambda - 2) + |f_2|^2 (1-\lambda)(\lambda - 5)] \\ & + 48(1-\lambda)^2 [\operatorname{Im}(f_1 f_2^*)]^2; \end{aligned}$$

$$\begin{aligned} \delta_1 + \varepsilon_1 = {} & 2\{2 \operatorname{Re}(f_1'^2 f_1^{*2}) [\lambda + \mu + \varkappa - \varkappa^2 - 1] - 2 \operatorname{Im}(f_1 f_2 f_1'^{*2}) \\ & \times (1-\lambda) \times [5\varkappa + 3\mu + \lambda - 3] - 2 \operatorname{Re}(f_1'^2 f_2^{*2})(1-\lambda) \\ & \times [5\varkappa + \mu + 2\lambda - 1 - \lambda^2 - 2\lambda\mu] - 2 \operatorname{Im}(f_1' f_2' f_1^{*2})(1-\mu) \\ & \times [5\varkappa + 3\lambda + \mu - 3] + 8 \operatorname{Re}(f_1' f_2' f_1^* f_2^*)(1-\mu)(1-\lambda) \\ & \times (2\varkappa - 3) - 4 \operatorname{Im}(f_1' f_2' f_2^{*2})(1-\lambda)(1-\mu)[6 - 4\lambda - \mu - \varkappa] \\ & - 2 \operatorname{Re}(f_2'^2 f_1^{*2})(1-\mu) \times [5\varkappa + \lambda + 2\mu - 1 - \mu^2 - 2\lambda\mu] \\ & - 4 \operatorname{Im}(f_1 f_2 f_2'^{*2})(1-\lambda)(1-\mu) \times [6 - \lambda - \varkappa - 4\mu] \\ & + \operatorname{Re}(f_2'^2 f_2^{*2})(1-\lambda)(1-\mu)[\lambda^2 + \mu^2 - 5\varkappa^2 - 13]; \end{aligned}$$

the coefficient β can be obtained from the formula for α if we make the following replacements:

$$f_{1,2} \rightarrow f'_{1,2}; \quad \lambda \rightarrow \mu, \quad \mu \rightarrow \lambda.$$

Besides this, we have used the notations:

$$f_{1,2} \equiv f_{1,2}(q^2), \quad f'_{1,2} \equiv f_{1,2}(q'^2);$$

$$\varkappa = -m^{-2}(p_1 p_2) = -m^{-2}(p'_1 p'_2) = 2\gamma^2 - 1,$$

$$\mu = -m^{-2}(p_1 p'_2) = -m^{-2}(p_2 p'_1) = \gamma^2 + (\gamma^2 - 1)\cos\vartheta,$$

$$\lambda = -m^{-2}(p_1 p'_1) = -m^{-2}(p_2 p'_2) = \gamma^2 - (\gamma^2 - 1)\cos\vartheta,$$

$$\varepsilon = \gamma m.$$

ϑ is the scattering angle in the center-of-mass system; p_1, p_2 are the initial and p'_1, p'_2 the final momenta; $q = p_1 - p'_1 = p'_2 - p_2$, $q' = p_1 - p'_2 = p'_1 - p_2$.

The notations used in the formulas are chosen as in reference 2.

We remark that, generally speaking, we are taking into account the exchange of a "swollen" photon, for which the propagation function is $d(q^2)/q^2$, the function $d(q^2)$ being included in the vertex functions $f_1(q^2)$ and $f_2(q^2)$. On the basis of experiments on scattering it is in general impossible to distinguish a change of the vertex operator from a change of the propagation function of the photon.

It must also be noted that if new forces of a vector character (transferred by vector mesons) come into play at small distances, the formulas given here remain valid if we replace the photon propagation function $-1/q^2$ by the propagation function of the meson.

The well known Möller formula is obtained from Eq. (2) if we set $f_2 = 0$, $f_1 = 1$. All the radiative and mesonic corrections are included in the functions $f_1(q^2)$ and $f_2(q^2)$ (when we consider the exchange of one photon, and just in this case it makes sense to speak of the form-factors f_1 and f_2) and can be calculated. A deviation from the Möller formula (with account taken of the corrections mentioned above and of the exchange of a larger number of photons) would indicate the inapplicability of quantum electrodynamics at small distances, and the experimental determination of the functions $f_1(q^2)$ and $f_2(q^2)$ will give information about the distributions of charge and current in the electron.

---

[1] W. K. H. Panofsky, Report at the Ninth International Conference on High-Energy Physics, Kiev, 1959.

[2] J. M. Jauch and F. Rohrlich, The Theory of Photons and Electrons, Cambridge, Mass., 1955.

Translated by W. H. Furry
304

MUTUAL SOLUBILITY OF HYDROGEN AND DEUTERIUM AT 4.2° K

R. F. BULATOVA, V. S. KOGAN, and V. G. LAZAREV

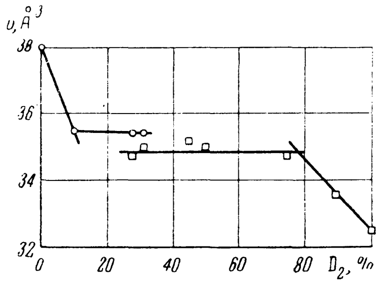
Physico-technical Institute, Academy of Sciences, Ukrainian S.S.R.

Submitted to JETP editor July 21, 1959

J. Exptl. Theoret. Phys. (U.S.S.R.) **37**, 1492-1493 (November, 1959)

We published previously[1] the general outlines of the H_2-D_2 phase diagram. The two-phase region for the H_2+D_2 mixture was roughly mapped out at helium temperatures for that concentration range over which the x-ray lines of both hydrogen and deuterium appeared simultaneously. From the x-ray diagrams and data on the structure of the pure isotopes,[2] one can determine the limits of solubility of hydrogen in deuterium and of deuterium in hydrogen at 4.2° K.

From this data on the pure isotopes it follows that a molecule occupies a volume of 38 A^3 in the H_2 lattice, and the volume in the D_2 lattice is 32.5 A^3. The figure shows the values of the volume v per molecule in the lattices of different solid mixtures. These results show that the limit of solubility of hydrogen in deuterium is 10% and of deuterium in hydrogen is 21%. It is interesting to note that the volumes corresponding to one molecule in both limiting mixtures are roughly the same (~ 35 A^3), and are close to the arithmetic mean of the volumes per molecule in the lattices of the pure isotopes.



The volume corresponding to one molecule; ○ – in the hydrogen lattice and □ – in the deuterium lattice, as a function of composition.

---

[1] Kogan, Lazarev, and Bulatova, JETP **34**, 238 (1958), Soviet Phys. JETP **7**, 165 (1958).

[2] Kogan, Lazarev, and Bulatova, JETP **37**, 678 (1959), Soviet Phys. JETP **10**, 485 (1960).

Translated by R. Berman
305