matrices β_μ are represented in the following form:

$$\beta_\mu = \gamma_\mu \times \alpha_{(\mu)}, \qquad \beta^\mu = \gamma^\mu \times \alpha_{(\mu)} \quad \text{(no summation!)} \qquad (8)$$

Here γ_μ are the Dirac matrices, and the matrices $\alpha_{(\mu)}$ are given in reference 2. Although expression (8) has on first glance the same appearance as (7), there is an essential difference between the two expressions. Neither the matrices $\alpha_{(\mu)}$ nor their products satisfy equations of the type (5),

Only in the case of anomalous equations for particles with a unique rest mass can the β_μ in (8) be represented in the form (7).

We introduce now a concrete example of the β_μ matrices (8) for particles with spin $\frac{1}{2}$ and with two rest masses. The matrix β_0 is equal here to

$$\beta_0 = \gamma_0 \times \begin{vmatrix} 2l & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \sqrt{2}\,l & r_1 & r_2 & r_3 \\ \cdot & \sqrt{2}\,l & l & -\sqrt{1/2}\,r_1 & -\sqrt{1/2}\,r_2 & -\sqrt{1/2}\,r_3 \\ \cdot & r_1 & -\sqrt{1/2}\,r_1 & k_1 & \cdot & \cdot \\ \cdot & -r_2 & \sqrt{1/2}\,r_2 & \cdot & k_2 & \cdot \\ \cdot & r_3 & -\sqrt{1/2}\,r_3 & \cdot & \cdot & k_3 \end{vmatrix}, \qquad (9)$$

where the coefficients are given by the expressions

$$r_1^2 = {}^2/_3 k_1^2 (k_1 - \lambda_1)(k_1 - \lambda_2)/(k_3 - k_1)(k_1 - k_2),$$
$$r_2^2 = -{}^2/_3 k_2^2 (k_2 - \lambda_1)(k_2 - \lambda_2)/(k_1 - k_2)(k_2 - k_3),$$
$$r_3^2 = {}^2/_3 k_3^2 (k_3 - \lambda_1)(k_3 - \lambda_2)/(k_3 - k_1)(k_2 - k_3),$$

$k_i \neq 0$, $i = 1, 2, 3$, $l = 0$, $\lambda_1 > k_1 > k_2 > k_3 > \lambda_2$, $\lambda_1/2 > \lambda_2$, $\lambda_1 + \lambda_2 = k_1 + k_2 + k_3$. The parameters λ_1 and λ_2 determine the rest masses of the particles and must be taken as given, so that only two of the three parameters k_i are independent.

With the aid of the β_0 matrices and the generators I_{01}, I_{12}, I_{23},

$$I_{01} = \gamma_0\gamma_1 \times \mathrm{diag}\{-{}^1/_2, {}^3/_2, -{}^1/_2, -{}^1/_2, -{}^1/_2, -{}^1/_2\},$$

$$I_{12} = \gamma_1\gamma_2 \times \begin{vmatrix} \cdot & {}^1/_2 & \sqrt{1/2} & \cdot & \cdot & \cdot \\ {}^1/_2 & \cdot & \sqrt{1/2} & \cdot & \cdot & \cdot \\ \sqrt{1/2} & \sqrt{1/2} & {}^1/_2 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & -{}^1/_2 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & -{}^1/_2 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & -{}^1/_2 \end{vmatrix},$$

$$I_{23} = \gamma_2\gamma_3 \times \mathrm{diag}\{{}^3/_2, -{}^1/_2, -{}^1/_2, -{}^1/_2, -{}^1/_2, -{}^1/_2\},$$

we can determine the remaining matrices β_k ($k = 1, 2, 3$) and the other generators I_{02}, I_{03}, I_{31}.

By means of a long, but not difficult calculation, one can convince oneself that the only matrix commuting with all the matrices of the anomalous equations given here is the unit matrix. Therefore it follows that the corresponding β_μ matrices are not fully reducible and that the anomalous equations for particles with several masses do not decouple.

Anomalous equations do not represent the only equations contradicting solution (7). If $l \neq 0$ is chosen in matrix (9), then by making the corresponding choice for the coefficients of the matrix one can satisfy all the physical conditions and construct irreducible equations for particles having κ_1 in a spin $^3/_2$ state and masses κ_2 and κ_3 in a spin $^1/_2$ state. Several similar examples could be given.

All Shelepin's work is based on the assumption that the solution of the form (7) to Eq. (5) has a unique character. Since this assumption is untrue, the method considered in reference 1 of constructing an arbitrary algebra $U(\beta)$ by using direct products of the Dirac algebras is not general enough.

[1] L. A. Shelepin, JETP **34**, 1574 (1958), Soviet Phys. JETP **7**, 1085 (1958).

[2] I. Ulehla, JETP **33**, 473 (1957), Soviet Phys. JETP **6**, 369 (1958).

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THEORETICAL INTERPRETATION OF INELASTIC p-p AND p-n COLLISIONS AT 9 Bev

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INELASTIC N-N collisions can be separated, using the impact parameter as criterion, into those involving collisions of the central regions of the nucleons and those in which the periphery of one nucleon collides with the central portion of the other.[1] An optical-model analysis of N-N collisions in the energy range $E = 1 - 9$ Bev indicates that one type of collision takes over from the other at an impact parameter of $r_0 \sim 0.6 \times 10^{-13}$ cm. In the description of collisions of the central parts, in which most of the energy of the nucleons lies, the statistical theory of multiple production can be employed (see references 2 and 3).

In Fig. 1 the theoretical results, calculated from statistical theory of multiple production, are given by the dashed line, and the experimental histogram

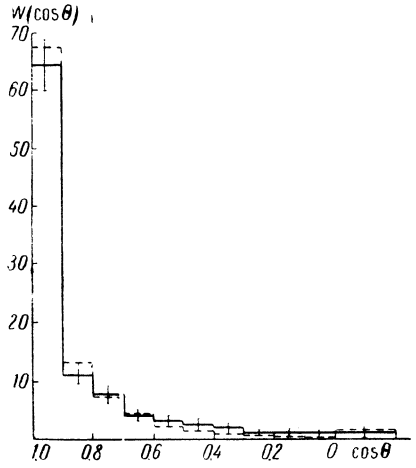

FIG. 1

for the angular distribution of charged particles produced in N-N collisions* (in relative units), by the solid curve.

In the table are shown the ratios η of the experimental number of particles emitted at small angles in p-p collisions to the number of particles calculated from the statistical theory.

Angular interval in degrees	η
0 — 3	1.9 ± 0.3
0 — 5	1.6 ± 0.2
0 — 10	1.2 ± 0.1

In Fig. 2 are given the ratios of the observed number of stars with n prongs (n = 1, 3, 5 for p-n and n = 2, 4, and 6 for p-p collisions) to the theoretical number, that is, $N_n^{exp}/N_n^{theoret}$. The mean theoretical multiplicities $\bar{n}_{(pp)} = 3.5$ and $\bar{n}_{(pn)} = 3.2$ exceed the experimental values[5] $\bar{n}_{(pp)} = 3.22 \pm 0.12$ and $\bar{n}_{(pn)} = 2.62 \pm 0.13$ only slightly.

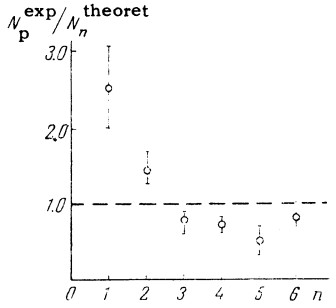

FIG. 2

From the figures and the table it follows that the theoretical and experimental values differ in the region of small angles and in events with a low multiplicity of particles.

Since the experimental angular distributions averaged over large angular intervals ($\Delta \cos\theta = 0.1$, see Fig. 1), the mean number of particles produced, and also the mean energy loss† follow the predictions of the statistical theory, one can conclude that the main proportion of the N-N collisions have to do with collisions between the central regions of the nucleons.

In order to understand the deviations, we consider periphery collisions. For a first estimate, we employ the Weizsäcker-Williams method.[1] The cross section for periphery collisions is

$$\sigma_p = 2\int \sigma_{\pi N}(\varepsilon)\, q(\varepsilon)\, d\varepsilon \approx \frac{5}{\beta^2} \cdot 10^{-27}\ \text{cm}^2 \approx 0.2\sigma_{\pi N},$$

where $\beta \approx 1$ is the nucleon velocity, $\sigma_{\pi N} \approx 30 \times 10^{-27}$ cm^2 is the total cross section for π-N interactions,‡ $q(\epsilon)$ is the energy spectrum of the periphery mesons, moving together with the nucleon.

The mean number of charged particles produced in the collision of a periphery meson of the incident nucleon with the other nucleon can be obtained either from the experimental data on π-N interactions for E = 1 — 5 Bev, or from theoretical calculations using the statistical theory (the results are in agreement). The mean multiplicity in periphery collisions turned out to be equal to $\bar{n}_{(pp)} = 3.3$ and $\bar{n}_{(pn)} = 3.1$.

If we assume that the nucleon losing a periphery meson remains in the excited state $T = J = 3/2$ (on account of the relativistic contraction of time, the lifetime of such an isobar is $\tau \sim 15\,\tau_0$, where τ_0 is the time of nuclear collision), then the number of charged particles emitted forwards (in the c.m.s.) in p-n collisions exceeds the number of charged particles emitted backwards by a factor of about 1.5. This asymmetry comes completely from the protons; the π mesons are emitted symmetrically relative to $\theta = \pi/2$. In p-p collisions the angular distribution in the c.m.s. is symmetrical relative to $\theta = \pi/2$. In both p-n and p-p collisions, the angular distributions are anisotropic in the c.m.s. Since the particles produced in central collisions are emitted isotropically (in the c.m.s.), then in the small-angle region the charge asymmetry may result completely from periphery collisions.

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*The experimental curve is the histogram averaged for p—p and p—n collisions, taken from Ref. 4.

†$(\Delta E/E)^{exp} = (40 \pm 10)\%$; $(\Delta E/E) = (40\text{-}50)\%$ (see ref. 6).

‡In the calculation of $q(\varepsilon)$ we took the value of the meson charge to be $f^2/\hbar c = 0.08$ and the radius of the central region of the nucleon to be $r_0 \approx 0.6 \times 10^{-13}$ cm.

[1] D. I. Blokhintsev, CERN Symposium **2**, 155 (1956). Blokhintsev, Barashenkov, and Barbashov, Usp. Fiz. Nauk **68**, 417 (1959), Soviet Phys.-Uspekhi **2**, 505 (1960).

[2] V. S. Barashenkov and B. M. Barbashov, Nuovo cimento **7**, Suppl. 1, 19 (1958).

[3] Belen'kiĭ, Maksimenko, Nikishov, and Rozental', Usp. Fiz. Nauk **62**, 1 (1957).

[4] Bogachev, Bunyatov, Merekov, and Sidorov, Dokl. Akad. Nauk SSSR **121**, 617 (1958), Soviet Phys.-Doklady **3**, 785 (1959).

[5] Bogachev, Bunyatov, Gramenitskiĭ, Lyubimov, Merekov, Podgoretskiĭ, Sidorov, and Tuvdéndorzh, Preprint, Joint Inst. Nuc. Res. 356; J. Exptl. Theoret. Phys. (U.S.S.R.) this issue, p. 872 (Russ. p. 1225).

[6] Barashenkov, Belyakov, Wang Shu-Fen, Glagolev, Dolkhazhev, Kirillov, Lebedev, Mal'tsev, Markov, Tolstov, Tsyganov, and Shafranova, Атомная энергия (Atomic Energy), in press; Nuclear Physics, in press.

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SPECIFIC HEAT ANOMALY AND NUCLEAR RESONANCE IN CRYSTALLINE HYDROGEN IN CONNECTION WITH NEW DATA ON ITS STRUCTURE

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FOLLOWING Keesom,[7] the interpretation of experimental data on the specific heat anomaly[1,2] and nuclear resonance has been based[3-6] so far on a close-packed hexagonal lattice structure for crystalline hydrogen. Recently Lazarev[8] and co-workers have found an error in this previous work and showed that the x-ray lines obtained could be explained either by a hexagonal lattice, deviating a little from close-packing (a = 3.7, c = 6.42), or by a tetragonal lattice (a = 4.5, c = 3.68). It was therefore necessary to determine the degree of agreement of the experimental data on the specific heat anomaly with the new crystal lattices ascribed to hydrogen. Since the structure is not yet resolved unambiguously, it is no less important to calculate the anisotropy of nuclear resonance in a hydrogen single crystal* and to find out whether the structure can be elucidated by the nuclear resonance method.

1. From Hill and Ricketson's experimental data,[1] Nakamura[2] found that within a certain temperature range the dependence of the anomalous specific heat, C_V, on the temperature T and concentration ρ is given by

$$C_V = \frac{R}{T^2}(\alpha\rho + \beta\rho^2), \tag{1}$$

where $\alpha = 1.1$ and $\beta = 15.7$. He also obtained a similar formula theoretically, and found the coefficients α and β to be very sensitive to the crystal structure. For a close-packed hexagonal lattice Nakamura obtained α strictly equal to zero, with $\beta = 20$ to the first approximation and 18 to the second. For the newly determined tetragonal lattice we calculated for α and β values in better agreement with experiment: $\alpha = 0.3$ and $\beta = 18$ to the first approximation and 16 to the second.

2. In Van Vleck's formula[9] for the second moment of a resonance line due to intermolecular† dipole-dipole interaction, the crystal structure is taken into account through the sum

$$\sum_k r_{ik}^{-6}(3\cos^2\theta_{ik} - 1)^2, \tag{2}$$

where r_{ik} is the distance between the i-th and k-th molecules, and θ_{ik} is the angle between the magnetic field and the vector θ_{ik}. We have calculated the sum for an arbitrary magnetic field direction and obtain the following expressions for tetragonal and hexagonal lattices:

$$\sum_k r_{ik}^{-6}(3\cos^2\theta_{ik} - 1)^2 = c^{-6}(9.2 + 0.88\cos^2\theta + 0.24\cos^4\theta), \tag{3}$$

$$\sum_k r_{ik}^{-6}(3\cos^2\theta_{ik} - 1)^2 = a^{-6}(11.2 - 16.4\cos^2\theta + 19.7\cos^4\theta), \tag{4}$$

where θ is the angle between the field and the fourfold or corresponding sixfold axis. The considerable difference between the anisotropies for a tetragonal (3) and hexagonal (4) lattice can conveniently be used to decide the structure of solid hydrogen.

3. Moriya and Motizuki[6] proposed a theory of spin-lattice relaxation in solid hydrogen. Because of the difficulty of the calculation, the relaxation time, T_1, was derived for the simplest case corresponding to the magnetic field parallel to the sixfold axis. Since the determination of the orientation of a single crystal under experimental conditions presents considerable difficulty, we considered it necessary to carry through the cumbersome calcu-