the showers can be explained by the effect of nuclear scattering, as shown by experimental data on the nuclear-active component at sea level.[4] The higher energy of particles at larger distances ($r \geq 500$ m) is explained by the fact that, at these distances, some of the electrons originate in the μ-meson decay.

A detailed presentation and discussion of the results will be published.

*For the distance of 0.1 m, we have used the data of Strugal'skiĭ.[2]

[1] Dmitriev, Kulikov, Massal'skiĭ, and Khristiansen, JETP **36**, 992 (1959), Soviet Phys. JETP **9**, 702 (1959).

[2] Z. S. Strugal'skiĭ, Dissertation, Moscow State University, 1959.

[3] K. Kamata and J. Nishimura, Suppl. Progr. Theor. Phys. **6**, 93 (1958).

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ON ANOMALOUS EQUATIONS FOR SPIN ½ PARTICLES

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A paper of L. A. Shelepin[1] argues that anomalous equations (obtained by one of us[2]) for particles with spin $\frac{1}{2}$ and with several rest masses are reducible. We want to call attention to the erroneousness of this assertion and to show where the mistake is in reference 1.

The proof of the reducibility of the anomalous equations was constructed by Shelepin on the basis of a theorem which asserts that if the Lorentz transformation matrix S for the wave function ψ which satisfies the equation

$$(\beta_\mu \partial^\mu - i\varkappa)\psi = 0, \tag{1}$$

can be written as a direct product

$$S = S' \times S'', \tag{2}$$

where S' and S'' represent the Lorentz transformations corresponding to the functions ψ' and ψ'' satisfying the equations

$$(\beta'_\mu \partial^\mu - i\varkappa)\psi' = 0, \qquad (\beta''_\mu \partial^\mu - i\varkappa)\psi'' = 0, \tag{3}$$

then the algebra $U(\beta)$ is given by the direct product $U(\beta) = U(\beta') \times U(\beta'')$.

The proof of this theorem in reference 1 is not complete. This assertion can be graphically demonstrated by repeating the proof by some other method, that is by using infinitesimal rotations instead of general Lorentz transformations. In this case the matrix S can be written in the familiar form $S = 1 + \frac{1}{2}\epsilon_{\mu\nu}I^{\mu\nu}$ (we have similar expressions also for S' and S''). Equation (2) then has the form

$$I_{\mu\nu} = I'_{\mu\nu} \times 1'' + 1' \times I''_{\mu\nu}. \tag{4}$$

From the requirement of the invariance of (1) and (3) under Lorentz transformations, the well-known relations for the matrices β_μ, β'_μ, and β''_μ result

$$[\beta_\mu I_{\nu\sigma}] = g_{\mu\nu}\beta_\sigma - g_{\mu\sigma}\beta_\nu, \tag{5}$$

$$[\beta'_\mu I'_{\nu\sigma}] = g_{\mu\nu}\beta'_\sigma - g_{\mu\sigma}\beta'_\nu, \qquad [\beta''_\mu I''_{\nu\sigma}] = g_{\mu\nu}\beta''_\sigma - g_{\mu\sigma}\beta''_\nu. \tag{6}$$

If we now represent the matrices β_μ in the covariant form

$$\beta_\mu = c_0(\beta'_\mu \times 1'') + c_1(\beta'_\nu \times \beta''_\mu \beta''_\nu) + \cdots + d_0(1' \times \beta''_\mu) + \cdots, \tag{7}$$

that is, symbolically $\beta = u(\beta') \times u(\beta'')$, where $u(\beta')$ and $u(\beta'')$ are general elements of the algebra $U(\beta')$ and $U(\beta'')$, then equation (5) will be identically satisfied on the basis of relations (4), (5), and (7). The proof of the quoted theorem in reference 1 is finished up by finding the solutions of Eq. (7) which satisfy Eq. (5) identically. However, this is not sufficient for a proof: it actually should be shown that the solution in the form of Eq. (7) represents a unique solution for the given operators $I_{\mu\nu}$. We have here a situation very similar to that in tensor algebra. As is well known, one can in the latter satisfy the transformation law for a second rank tensor by constructing a quantity equal to the product of two vectors. However, it does not follow from this that every tensor of the second rank can be described by the product of two vectors.

If such a proof did exist, then anomalous equations for particles with spin $\frac{1}{2}$ and with two or more rest masses could be completely reduced. Since, however, these equations do not decouple, they represent the case where the solutions of (5) do not have the form of (7).

In the anomalous equations $(\beta_\mu \partial^\mu - ik)\varphi = 0$, satisfying all the physical requirements, the matrices β_μ are represented in the following form:

$$\beta_\mu = \gamma_\mu \times \alpha_{(\mu)}, \qquad \beta^\mu = \gamma^\mu \times \alpha_{(\mu)} \quad \text{(no summation!)} \tag{8}$$

Here γ_μ are the Dirac matrices, and the matrices $\alpha_{(\mu)}$ are given in reference 2. Although expression (8) has on first glance the same appearance as (7), there is an essential difference between the two expressions. Neither the matrices $\alpha_{(\mu)}$ nor their products satisfy equations of the type (5),

Only in the case of anomalous equations for particles with a unique rest mass can the β_μ in (8) be represented in the form (7).

We introduce now a concrete example of the β_μ matrices (8) for particles with spin $\frac{1}{2}$ and with two rest masses. The matrix β_0 is equal here to

$$\beta_0 = \gamma_0 \times \begin{vmatrix} 2l & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \sqrt{2}\,l & r_1 & r_2 & r_3 \\ \cdot & \sqrt{2}\,l & l & -\sqrt{1/2}\,r_1 & -\sqrt{1/2}\,r_2 & -\sqrt{1/2}\,r_3 \\ \cdot & r_1 & -\sqrt{1/2}\,r_1 & k_1 & \cdot & \cdot \\ \cdot & -r_2 & \sqrt{1/2}\,r_2 & \cdot & k_2 & \cdot \\ \cdot & r_3 & -\sqrt{1/2}\,r_3 & \cdot & \cdot & k_3 \end{vmatrix}, \tag{9}$$

where the coefficients are given by the expressions

$$r_1^2 = {}^2/_3 k_1^2 (k_1 - \lambda_1)(k_1 - \lambda_2) / (k_3 - k_1)(k_1 - k_2),$$
$$r_2^2 = -{}^2/_3 k_2^2 (k_2 - \lambda_1)(k_2 - \lambda_2) / (k_1 - k_2)(k_2 - k_3),$$
$$r_3^2 = {}^2/_3 k_3^2 (k_3 - \lambda_1)(k_3 - \lambda_2) / (k_3 - k_1)(k_2 - k_3),$$

$k_i \neq 0$, $i = 1, 2, 3$, $l = 0$, $\lambda_1 > k_1 > k_2 > k_3 > \lambda_2$, $\lambda_1/2 > \lambda_2$, $\lambda_1 + \lambda_2 = k_1 + k_2 + k_3$. The parameters λ_1 and λ_2 determine the rest masses of the particles and must be taken as given, so that only two of the three parameters k_i are independent.

With the aid of the β_0 matrices and the generators I_{01}, I_{12}, I_{23},

$$I_{01} = \gamma_0\gamma_1 \times \operatorname{diag}\{-{}^1/_2, {}^3/_2, -{}^1/_2, -{}^1/_2, -{}^1/_2, -{}^1/_2\},$$

$$I_{12} = \gamma_1\gamma_2 \times \begin{vmatrix} \cdot & {}^1/_2 & \sqrt{1/2} & \cdot & \cdot & \cdot \\ {}^1/_2 & \cdot & \sqrt{1/2} & \cdot & \cdot & \cdot \\ \sqrt{1/2} & \sqrt{1/2} & {}^1/_2 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & -{}^1/_2 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & -{}^1/_2 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & -{}^1/_2 \end{vmatrix},$$

$$I_{23} = \gamma_2\gamma_3 \times \operatorname{diag}\{{}^3/_2, -{}^1/_2, -{}^1/_2, -{}^1/_2, -{}^1/_2, -{}^1/_2\},$$

we can determine the remaining matrices β_k ($k = 1, 2, 3$) and the other generators I_{02}, I_{03}, I_{31}.

By means of a long, but not difficult calculation, one can convince oneself that the only matrix commuting with all the matrices of the anomalous equations given here is the unit matrix. Therefore it follows that the corresponding β_μ matrices are not fully reducible and that the anomalous equations for particles with several masses do not decouple.

Anomalous equations do not represent the only equations contradicting solution (7). If $l \neq 0$ is chosen in matrix (9), then by making the corresponding choice for the coefficients of the matrix one can satisfy all the physical conditions and construct irreducible equations for particles having κ_1 in a spin $3/2$ state and masses κ_2 and κ_3 in a spin $1/2$ state. Several similar examples could be given.

All Shelepin's work is based on the assumption that the solution of the form (7) to Eq. (5) has a unique character. Since this assumption is untrue, the method considered in reference 1 of constructing an arbitrary algebra $U(\beta)$ by using direct products of the Dirac algebras is not general enough.

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[2] I. Ulehla, JETP **33**, 473 (1957), Soviet Phys. JETP **6**, 369 (1958).

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THEORETICAL INTERPRETATION OF INELASTIC p-p AND p-n COLLISIONS AT 9 Bev

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INELASTIC N-N collisions can be separated, using the impact parameter as criterion, into those involving collisions of the central regions of the nucleons and those in which the periphery of one nucleon collides with the central portion of the other.[1] An optical-model analysis of N-N collisions in the energy range E = 1 — 9 Bev indicates that one type of collision takes over from the other at an impact parameter of $r_0 \sim 0.6 \times 10^{-13}$ cm. In the description of collisions of the central parts, in which most of the energy of the nucleons lies, the statistical theory of multiple production can be employed (see references 2 and 3).

In Fig. 1 the theoretical results, calculated from statistical theory of multiple production, are given by the dashed line, and the experimental histogram