absence of $A_\alpha^\pm$ in (1a) [because the function $\Phi(\mathbf{r}_1, \mathbf{r}_2) = \underset{\alpha}{S}\psi_\alpha(\mathbf{r}_1) F_\alpha(\mathbf{r}_2)$ satisfies the Schrödinger equation $(H - E)\Phi(\mathbf{r}_1, \mathbf{r}_2) = 0$].

The equality (4) becomes nearly obvious if we note that the exact solution of the Schrödinger equation need not be sought in an explicitly symmetric form but may be found first as an asymmetric solution and then symmetrized.

Further, if we calculate the cross sections using only $a_\alpha^\pm$,[1] they will be the same for both signs. However one cannot agree with such a definition. Indeed, if the wave function of the two electrons has the form $\Phi^\pm(\mathbf{r}_1, \mathbf{r}_2) = \underset{\alpha}{S}[F_\alpha(\mathbf{r}_1) \times \psi_\alpha(\mathbf{r}_2) \pm F_\alpha(\mathbf{r}_2)\psi_\alpha(\mathbf{r}_1)]$, we obtain the following expression for the radial component of the scattered flux:

$$j^\pm(\mathbf{r}) = 2\,\mathrm{Im}\Big\{\underset{\alpha}{S} f_\alpha^*(\mathbf{r})\frac{\partial}{\partial r} f_\alpha(\mathbf{r}) + \underset{\alpha\beta}{S}\psi_\alpha^*(\mathbf{r})\frac{\partial}{\partial r}\psi_\beta(\mathbf{r})\int f_\alpha^*(\mathbf{r}') \times f_\beta(\mathbf{r}')\,d\mathbf{r}' \pm \underset{\alpha\beta}{S}\Big[\psi_\alpha^*(\mathbf{r})\frac{\partial}{\partial r} f_\beta(\mathbf{r})\int f_\alpha^*(\mathbf{r}')\psi_\beta(\mathbf{r}')\,d\mathbf{r}' + f_\alpha^*(\mathbf{r})\frac{\partial}{\partial r}\psi_\beta(\mathbf{r})\int\psi_\alpha^*(\mathbf{r}') f_\beta(\mathbf{r}')\,d\mathbf{r}'\Big]\Big\}, \qquad (5)$$

where the f_α differ from the F_α by the absence of incident waves.

Hence we can see that the difference between the cross sections must appear not because of the inequality of a_α^+ and a_α^- but because of the exchange term in the flux, which Drukarev ignores, since he does not consider the flux produced by the functions ψ_α of the continuous spectra. Taking account of the exchange according to references 2 and 3 is simply taking account of the flux of atomic electrons excited to corresponding levels of the continuous spectrum.

*It should be remarked that Eqs. (1b) and (1c) are not wholly accurate in the case of ionization, in which case the asymptotic form of the wave function of one electron cannot be given independently of the other electron.

[1] G. F. Drukarev, JETP **31**, 288 (1956), Soviet Phys. JETP **4**, 309 (1957).

[2] N. Mott and H. Massey, Theory of Atomic Collisions, Oxford 1949; Russ. Trans. M., 1951; ch. 8 Sec. 4.

[3] H. Massey, Revs. Modern Phys. **28**, 199 (1956); Russ. Trans. Usp. Fiz. Nauk **64**, 589 (1958).

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ON THE PAPER OF V. A. BELOKON': "THE PERMANENT STRUCTURE OF SHOCK WAVES WITH JOULE DISSIPATION"

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In his discussion of the magnetohydrodynamic shock wave, V. A. Belokon'[1] writes down equations for the momenta and for the heat flow for the one-dimensional stationary motion of a non-viscous, non-heat-conducting, but electrically conducting gas. It is asserted that this system of equations leads, on the one hand, to the necessity of the existence of a maximum of the entropy inside the region of flow, and on the other hand, to the impossibility of a decrease in the entropy. On the basis of these facts, the author comes to the following conclusion: "In view of the absurdity of a continuous solution, we consider it unavoidable to postulate a Riemann isentropic discontinuity in the flow parameters within a compression wave of any amplitude, by analogy with the isothermal discontinuity for purely heat-conducting gases." The magnetic field at this discontinuity is considered continuous.

We cannot agree with this basic postulate. Indeed, if the magnetic field is continuous in the passage through the discontinuity and the gas is considered non-viscous and non-heat-conducting before and after the discontinuity, then this is a gas-dynamical discontinuity which always leads to an increase in the entropy.

The same kind of problem concerning the structure of the shock wave was considered earlier by Burgers.[2] He showed that two cases are possible: a) in strong magnetic fields all parameters inside the region of flow, including the entropy, change monotonically. The entropy reaches its maximum value at the point corresponding to $x = +\infty$; b) in weak magnetic fields the region of flow consists of two parts, the region of continuous variation of the parameters, at the end of which the entropy reaches some value $S^* \neq S_\infty$, and a compression discontinuity with a constant field, at which the entropy increases from S^* to S_∞.

The problem proposed by Belokon', therefore, has a complete solution without any additional postulates, and the postulate put forward in his paper is incorrect.

It is furthermore impossible to accept the following assertion of the author with respect to the effect of viscosity on the structure of the shock wave:

"If dissipation occurs by way of viscosity in addition to Joule heating, then the isentropic discontinuity mentioned above will be smoothed out for all amplitudes, since for vanishing viscosity the curves for continuous evolution of the flow parameters pass arbitrarily close to the isentropic line S_{max}, coinciding with it only in a single point, at $+\infty$."

The problem of the family of integral curves for the one-dimensional stationary flow of a viscous and electrically conducting gas, corresponding to the problem of the structure of the magnetohydrodynamic shock wave, was discussed in detail by Ludford.[3] He showed that in the case of vanishing viscosity the corresponding curve approaches the curve obtained by Burgers.

[1] V. A. Belokon', JETP **36**, 1316 (1959), Soviet Phys. JETP **9**, 932 (1959).

[2] J. M. Burgers, Penetration of a Shock Wave into a Magnetic Field. Magnetohydrodynamics, Ed. R. K. M. Landshoff, Stanford (1957) (Russ. Transl., Atomizdat, 1958).

[3] C. S. S. Ludford, J. Fluid Mechanics **5**, 67 (1959).

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ON THE PHASE OF THE SCATTERED WAVE (A REPLY TO V. V. MALYAROV)

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THE possibility of determining the phase of the scattered wave by numerical methods with the help of the differential equation for the phase

$$d\delta / d\rho = -[l(l+1)/\rho^2 + U(\rho)]\sin^2(\rho+\delta) \qquad (1)$$

was discussed in reference 1, and it was shown that it is necessary in this case to find the solution which satisfies the condition

$$\sin(\rho+\delta) = \rho/(l+1) \text{ for } \rho \to 0. \qquad (2)$$

V. V. Malyarov[2] correctly observed that one and the same symbol is used in reference 1 to denote two proportional constants, and also that the condition $\int^{\infty} U(\rho)\,d\rho < C$ should be replaced by $\int^{\infty} |U(\rho)|\,d\rho < C$.

Then V. V. Malyarov proposes, incorrectly, to obtain the second term in the expansion for $\delta(\rho)$ from the expression for $\delta(\rho)$ of reference 1, overlooking the fact that this expression was formally obtained from (1) and (2). Actually this term can be obtained by substituting the series

$$\delta(\rho) = -l\rho/(l+1) - a_2\rho^2 - a_3\rho^3 - \ldots$$

in (1). We then obtain for $\delta(\rho)$ the expansion

$$\delta(\rho) = -l\rho/(l+1) - \gamma_0\rho^2/2(l+1)^3 - \ldots, \qquad (3)$$

while the method of V. V. Malyarov would lead to the incorrect expression

$$\delta(\rho) = -l\rho/(l+1) - \gamma_0\rho^2/2(l+1)^2.$$

Finally, V. V. Malyarov alleges that the note[1] "...is of no interest for scattering theory and can lead the reader into error," and he bases this assertion on the following argument: "The use of the additional condition (5) is justified for $\rho \to \infty$, when $A(\rho) \to$ const and $\delta(\rho) \to$ const. For $\rho \to 0$, however, any other supplementary condition could be used instead of (5). Different additional conditions correspond to different phases $\delta(\rho)$. The problem is indeterminate. It follows from the definition of the phase of the scattered wave that the phase $\delta(\rho)$ obtained with the help of such a supplementary condition is not the phase of the scattered wave."

This remark of V. V. Malyarov cannot be applied to the contents of our note, because there we obtained (6) and (7) by using the classical method of variation of the arbitrary Lagrange constants, and we were concerned with the determination of the phase of the scattered wave at infinity [i.e., finding the limit of $\delta(\rho)$ for $\rho \to \infty$].

Applying (1) and (7) of reference 1 to the known cases where the scattering problem can be solved exactly, one easily sees that this phase is indeed the phase of the scattered wave, according to the interpretation of scattering theory.

The remarks of V. V. Malyarov concerning the note[1] are therefore without substance.

[1] F. S. Los', On the Phase of the Scattered Wave, JETP **33**, 273 (1957); Soviet Phys. JETP **6**, 211 (1958).

[2] V. V. Malyarov, JETP **34**, 1039 (1958); Soviet Phys. JETP **7**, 719 (1958).

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