The integrals on the right-hand side were computed graphically and then normalized by analytic evaluation of the integral for the threshold photon energy when $F_C^2(qR) = \text{const}$.

From the analysis of the experimental data[3] of π^0-meson photoproduction from protons,[5] it follows that within 10% accuracy, for the energy region of primary photons studied, one can neglect the contributions of the $M(\frac{1}{2})$, E1, and E2 photoproduction amplitudes to the total cross section σ_t^H. Hence taking into account only the amplitude $M(^3/_2)$ in the total cross section for π^0-meson photoproduction from protons, we can easily determine that $\sigma_t = {}^2/_3\,\sigma_t^H$. To calculate σ_t we use the total cross section σ_t^H measured in reference 6. The calculated curve σ_t^C/σ_t^H is compared with the experimental points in the figure. Calculation of the contribution of the E1 amplitude makes a small reduction in the calculated magnitude of σ_t^C/σ_t^H for $h\nu = 160$ Mev; however this change lies within the limits of the statistical uncertainty of the experiments.

As can be seen from the figure, there is good agreement between theory and the observed experimental results. Therefore at primary photon energies of 160 to 200 Mev the elastic photoproduction of π^0 mesons from carbon dominates. At higher energies it seems that inelastic processes begin to appear in π^0-meson photoproduction which show up as small deviations of the experimental ratio σ_t^C/σ_t^H from that calculated theoretically. This is consistent with the conclusion recently given.[7]

Similar measurements we have made of the π^0-meson photoproduction cross section from beryllium nuclei do not show any significant differences between the energy dependence of the total cross section for π^0-meson photoproduction from carbon and beryllium.

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CALCULATION OF ENERGY LEVELS OF Tl^{206} *AND* Bi^{210} *NUCLEI*

L. A. SLIV and Yu. I. KHARITONOV

Leningrad Physico-technical Institute, Academy of Sciences, U.S.S.R.

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1. To calculate the energy levels of the Tl^{206} and Bi^{210} nuclei we used the data of Sliv and Volchok[1] on the neighboring nuclei. The nucleus ${}_{81}Tl^{206}_{125}$ has one neutron hole and one proton hole. Using the single-particle neutron wave functions of Pb^{207} ($p_{1/2}$ — ground state, $f_{5/2}$ — 620 kev) and the proton wave functions of Tl^{207} ($s_{1/2}$ — ground state, $d_{3/2}$ — 350 kev), we can plot a zeroth-approximation level scheme for Tl^{206} up to 1 Mev. With the aid of these data for Tl^{206} we obtain the following multiplets with corresponding energies, spins, and parities: $(p_{1/2}s_{1/2})$, 0 kev, $I = 0^-, 1^-$; $(p_{1/2}d_{3/2})$, 350 kev, $I = 1^-, 2^-$; $(f_{5/2}s_{1/2})$, 620 kev, $I = 2^-, 3^-$; $(f_{5/2}d_{3/2})$, 970 kev, $I = 1^-, 2^-, 3^-, 4^-$. The problem is to determine the forces that split the levels belonging to individual multiplets. Such forces may be: 1) interaction with the surface of the nucleus, and 2) weak paired interaction of the neutron and proton holes, located in different shells. The interaction with the surface was computed in the weak-coupling approximation.[2] It was found significant that regardless of the choice of parameters, the interaction with the surface does not split the doublet levels, but shifts them as a whole. The magnitude of this shift is a function of the energy $\hbar\omega$ of the first vibration level and of the "hardness" C of the Tl^{206} nucleus. Calculation has shown that as C changes from 1000 to 1500 Mev, and as $\hbar\omega$ changes from 1 to 3 Mev, the relative distances between the doublets do not change substantially.

We subsequently calculated the weak paired interaction between the neutron and proton holes. Since the specific type of the potential can hardly affect the magnitude of splitting of the doublet levels, we chose a Gaussian potential. We also took into account the dependence of the potential of paired interaction on the spin variables. The operator of paired interaction W_p was taken in the form

$$W_p = \exp\{-(\mathbf{r}_n - \mathbf{r}_p)^2/r_0^2\}(V_s\pi_s + V_t\pi_t),$$

where V_s and V_t are the constants of the singlet and triplet interactions, π_s and π_t the operators of singlet and triplet projection, and r_0 a parameter characterizing the radius of action of paired-interaction forces. In view of the weakness of paired interaction of holes located in different shells, we used perturbation theory. Assume that for each doublet the level with the larger spin lies above the level with the smaller spin. Then the splitting Δ_i of the doublets considered is expressed in terms of the difference $(V_s - V_t)$ as

$$\Delta_1 = a(V_s - V_t), \qquad \Delta_2 = -b(V_s - V_t), \qquad \Delta_3 = c(V_s - V_t),$$

where the coefficients a, b, and c become certain linear combinations of the Slater integrals, and are positive and of the order of 0.01 for $r_0 = 1.85 \times 10^{-13}$ cm (customarily used in nucleon-nucleon scattering).

As shown by Golenetskiĭ, Rusinov, and Filimonov,[3] the level scheme of Tl^{206} actually has three doublets in the energy range up to 750 kev. The splitting most accurately measured was that of the second doublet, $\Delta_2 = (40 \pm 10)$ kev. This makes it possible to estimate the difference $V_s - V_t$ at (12 ± 3) Mev. Then the splitting of the two other doublets is found to be $\Delta_1 = (85 \pm 20)$ kev and $\Delta_2 = (70 \pm 15)$ kev, in good agreement with the experimental values. The experimental data available on Tl^{206} are insufficient to determine the constants V_s and V_t separately. Thus, on the basis of our calculations, we can propose for Tl^{206} the energy-level scheme shown in Fig. 1.

2. In the nucleus ${}_{83}Bi^{210}_{127}$ there is one neutron and one proton in excess of twice-filled shells. We use for the calculations the single-particle wave functions of the ground state of $Pb^{209}(g_{9/2})$ and $Bi^{209}(h_{9/2})$. The multiplet $(g_{9/2}h_{9/2})$ contains 10 levels with spins from $I = 0$ to $I = 9$. The parity of these levels is negative. The level shift of the multiplet due to interaction with the surface was calculated for a whole series of values of $\hbar\omega$ and C. In all the cases considered, the spins of the first three levels are in increasing order, namely: 0^-, 1^-, and 2^-, but the fourth level has spin 9^-. Moreover, the distances between levels are almost independent of $\hbar\omega$ for a given value of C. In taking account of the paired interaction, we used for the difference $V_s - V_t$ values obtained from the calculations of the Tl^{206} levels. The level scheme of Bi^{210} depends on the spin assumed for the ground state (0^- or 1^-). Thus, for C = 1000 Mev we obtain two versions of level schemes for Bi^{210} (Figs. 2a and 2b).

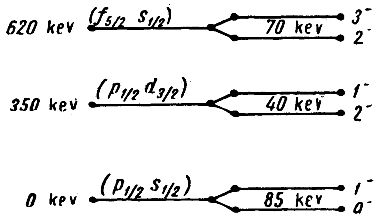

FIG. 1. Level scheme of Tl^{206}.

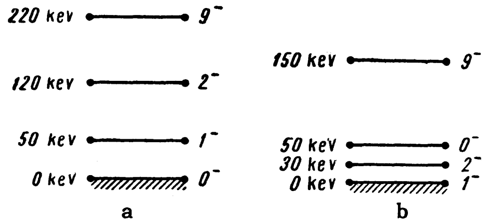

FIG. 2. Two versions of a level scheme for Bi^{210}.

3. The Tl^{206} nucleus is a fortunate case for the determination of the constants of paired interaction of nucleons (holes) located in different shells. It is therefore of interest to carry out the following measurements: a) determine the spin of the ground state and of some excited level of Tl^{206}, and b) measure the probability of E2 transitions in the schemes of Tl^{206}, Tl^{207}, and Pb^{207} so as to determine the parameters of interaction with the surface.

Very important in the construction of the scheme for Bi^{210} is the final determination of the spin of the ground state.

In conclusion, we wish to note that in our calculations we disregarded the effect of static deformation, since its influence can hardly be greater than the accuracy of the computations.

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